Probability Foundations for Electrical Engineers
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Lecture - 59

CDF Method

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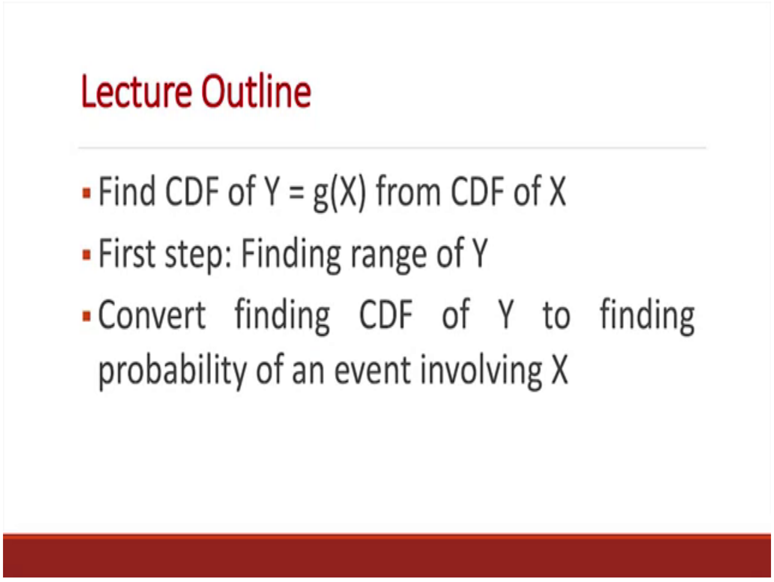

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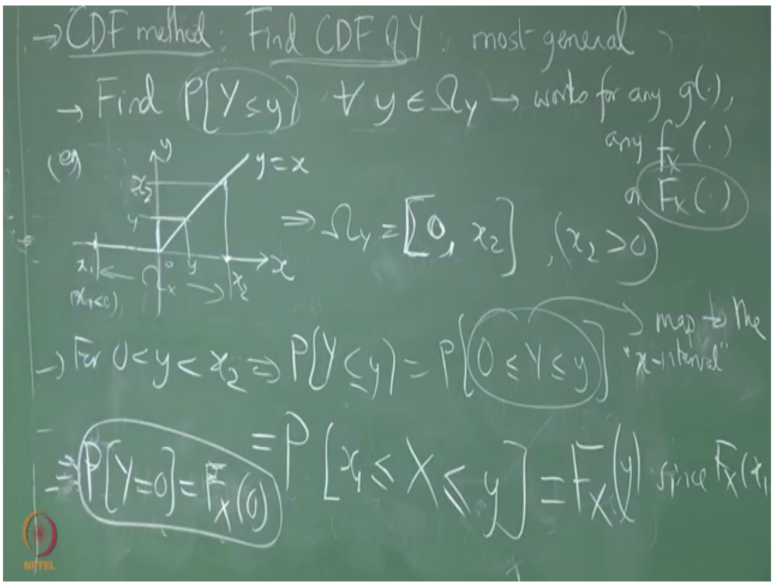

So, the CDF method find the CDF of Y, this is most general. Remember we have set time and several times that the CDF is certainly more general than the pdf, it exists for any random variable discrete or continuous is just that analytically it is not convenient, but Maths, does not care about analytical convenience it can cares only about what what can be written eventually. So, or how well behaved is it.

So, this is always a very well behaved function of the argument and bounded between 0 and 1 and so on and always exists. So, no matter what why you are looking at. So, for all of these reasons the all textbooks always start with the CDF of Y. So, that actually it turns out I mean I even though. So, let us say illustrate with some example graphically rather than. So, basically I want to find the probability that y is less than some number y for all y in omega y.

So, here we are going to assume that this omega y is a continuous interval and not discrete as I said if you have discrete to discrete you are only looking at pms. So, that is completely outside the purview of this discussion. We are not going to use CDF for that again we are going to use CDF as a general tool which may which turns out this going to only way to handle this flat the example of the flat g, the g the CDF does not care about kind of g you have essentially works for any g, any f x or f x.

So, it does not matter how I write the input specification I can write it as a CDF, I can write it as a pdf, if a pdf is the you know it is possible g can be anything flat whatever, the CDF can always be written. So, for example, you take that you take this flat take this simple example let us say this is my omega x. So, that now omega y what is supposing. So, what is omega y? First thing as I said always find is to find this, what is that in this particular in this particular example.

Student: (Refer Time: 03:11).

Omega y is clearly you you know, you say you go up and down this way and see how what interval you are going here. Obviously you are going only supposing it is x 1 to x 2 the this x 1 x 2 is not the same as this x 1 x 2. So, please excuse the reusing of x 1 x 2, keep the separate from this. So, like if this goes from x 1 to x 2, but x 1 is less than 0 let us say or you know it does not matter, let us say x 1 is less than 0 just for the sake of argument right. So, and if this is g of this is let us say y equal to x again I am taking the simplest case of the half wave rectifier as this as this found as it is called here right.

So, remember there are no diodes and no cutting voltages and alright half wave rectifier is ideal is 0 for; less than 0 and it gives you the same value for x greater than 0 right. So, this is the point y equal to. So, this is x 2, this will be x 2 here again right. So, what is omega y? Let us say is a closed interval 0 to x 2 obviously 0, now in this case is very much counted it has to be closed you cannot by any way a stretch with the imagination 0 is an open interval in this case right.

So, what now, so this is what we want. So, we pick a y here. So, we are assuming x 2 is positive. So, for y in the ray somewhere in between 0 and x 2 we are asking for this probability which is what, this interval only. You can say well minus infinity to y, well we have to qualify actually the CDF has to be specified for all real numbers y, but in this case by inspection you can say that if this small y is less than 0 then; obviously, this becomes an impossible event and it has 0 probability that I think does not need to be said to you know we need to hop on that too much.

So, we only care about the increase that is the interesting portion of the CDF always occurs for y only in omega y, if this y becomes more than omega y then again it saturates at one it becomes a certain event right. So, we are only interested the in the pole in this portion where, where what something interesting is happening in this output CDF it is not 0 or it is not 1. So, supposing I pick y for 0 less than y less than x 2, I pick some number in this range which is here then probability that y is less than equal to y will be the probability that 0 is less than equal to y less than equal to y, is not it.

I pick a number y, this, the probability of this entire finding y in this whole range which is the output CDF is basically the probability of this closed interval. So, this has to be done with care now because actually it is not that much case. So, what range of x is now going to give me what range of x is now going to give me this range output range of y?

Student: x (Refer Time: 07:03).

Yes. So, now, this has to be mapped this range must be mapped to mapped to interval on the x interval let me call it x interval, equivalent x interval, that is you map it to the interval which map whether you have one interval on the x axis mapping to another interval of the y axis. So, that is what we have to. So, we have to find the interval on the x axis which maps or a region of the x axis which maps to this region that you care about.

So, this is clearly the region in this case it is a region x 1 to y. So, I have to instead the in place of. I start with this y I go down I obviously, get the same y here and so it is this. And this can be written in terms of the CDF of x which is why I am saying, so once I have this I do not need this and of course, g I have used, I am not, I am not; obviously, claiming that this point is going to work for. So, for any g this all of this, this happening y here putting y here in the, this y is only for this particular g.

So, conservation of probability now, this CDF is equal to this, this can be written this is now nothing, but capital F x of y minus, well in this case it may only it is possibly only this because we are assuming that this say actually x 1 can be go going on minus entry also it does not matter in this case right, can be there is no reason to in this case you do not have to subtract anything it is just simply f x of y, is not it. Since we will write here f x of x 1 is 0. Assuming that it is this is the leftmost point of omega x.

So, and of course, what is P f x of that is, what is P of Y equal to 0. This is what? The probability that Y is 0 is now the probability or it is just F x of what 0, now that is a separate point y of 0 is f x of 0, is it not. So, y is no longer purely continuous y has this finite probability at 0 and therefore, the CDF is the only tool we have in the situation. So, it turns out for simple cases like this the CDF yes can be used and then of course, I do not have to say that for y more. So, what is a plot of the output CDF? The plot of the output CDF shows a jump at 0. So, again I write the size of this board is always an issue for me, but to keep constantly erasing and so on, but does not matter.

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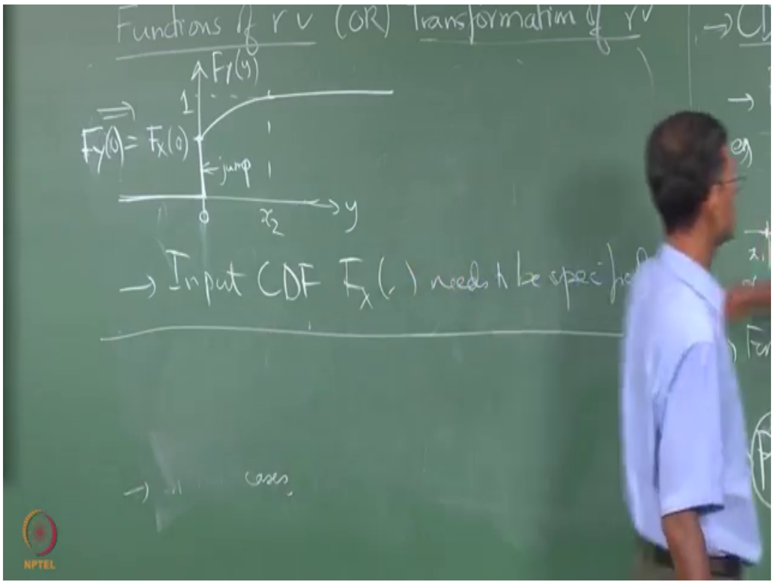

So, here if I plot F y versus y I do not need to look at the of course, any negative values right. So, if I plot this it will be up to 0 it is going to be 0 then at 0 it will jump to F x of 0 which we do not know what this I am not going to say its half or anything I do not know what it is depends on the statistics of x jumps to this.

And then it linearly increases does it not or no sorry it increases are not necessarily linear it increases according to F x of y up to x 2. So, it could be some non-linear increase whatever, up to x 2 in this particular case. I am assuming that x is not going to take a values a bigger than x 2 input random variable x is not going to take values bigger than x 2. So, therefore, at this point itself what do I get? I get 1. So, I get this I can be jump, this is equal to F y of 0s also the 0 the CDF always includes their limit by our definition right. So, therefore, F, this is F y of 0 will become equal to F x of 0 capital F y of 0 becomes the same as F x of 0, which is some finite quantity in this case.

So, this is a simple illustration of a of the CDF technique which works as I said in general. But it is it requires you to have a full specification input CDF right. So, input CDF F x is needs to be specified and if this input CDF is a pain in the neck to specify whatever reason the output is also not going to be a pretty CDF to write down.

So, what we are going to do is starting today and going into tomorrow is to look at is there a direct way to write the pdf. So, we will yeah, I am going to assume that you can use this if needed in whatever situation you might find yourself in later on in the future and I am not going to take any more specific examples of this, but basically remember to do this you need to look at this, this number is what you need to find. So, you have to map this interval to the x interval equivalent x interval. So, this technique CDF technique maps intervals to intervals and finds it carries over the probability of one in the interval on x to the probability of the equivalent interval in y, that is all you are doing here.

The pdf technique on the other hand directly at goes for the pdf without worrying about the CDF and its widely applicable whenever the input is a well behaved continuous random variable and the output the function g is also well behaved function that is whenever I try to do this these flat portions are not there.