Probability Foundations for Electrical Engineers
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Lecture - 57
Prob[X > Y]: Computation of Probability of a Non-rectangular Region

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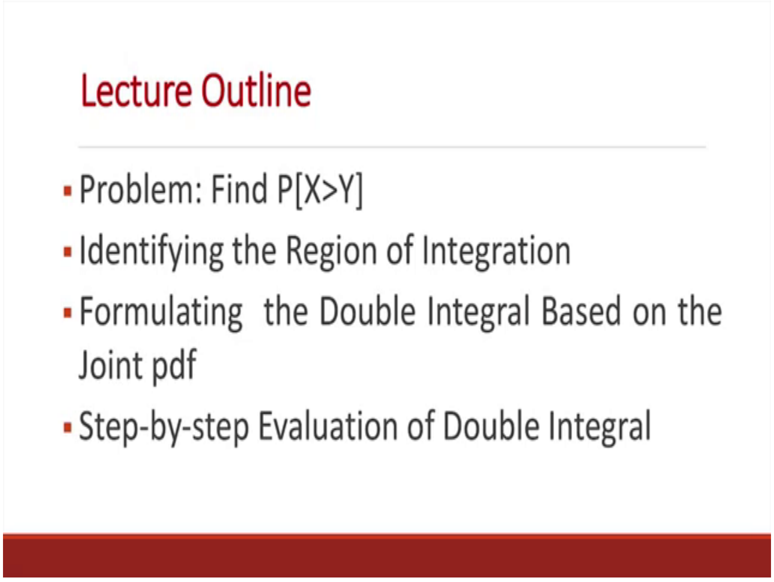

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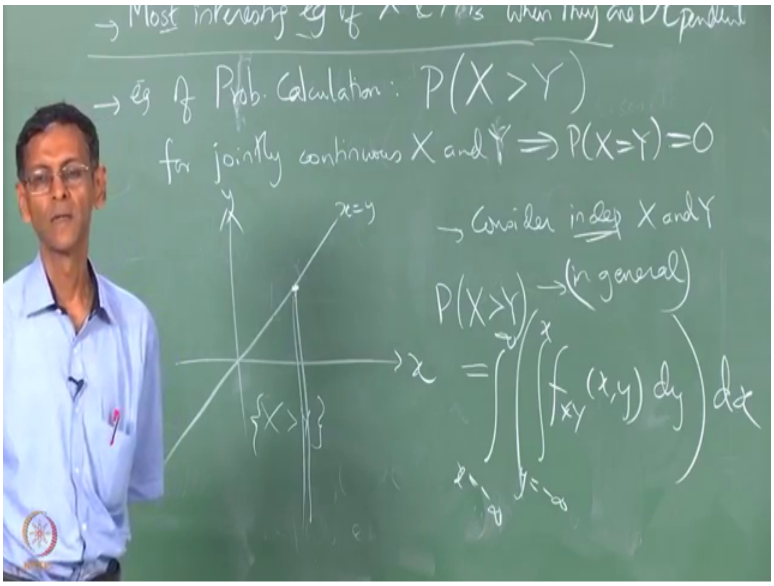

Anyhow, this is an example of a non trivial probability and I am looking to do a probability calculation over a non aligned rectangle or, a non aligned region in other words. The aligned regions are easy because, why are aligned region so trivial to calculate not trivial, but very easy to write down? Because X and Y limits are numbers independent of each other, the X limits do not depend on the Y limits etcetera and vice versa.

So, now let us look at this probability X greater than Y. So, X and Y for what? Jointly continuous X and Y, in this case this is what I want to do, I want to do, I want to do this calculation I am going to straight away assume that they are jointly continuous. When they are jointly continuous you know as an aside that probability X equal to Y 0 we already saw that yeah sometime last week, probability of any line or any curve is 0.

Student: 0.

No matter what, again what is the physical intuition behind this? In the case of independent X and Y the probability that you will get the same identical reading for two spinning pointers is very very small vanishingly small if you end this 0 and the limit if you consider infinite precision right. So, the partition on the X Y plane which corresponds to X equal to Y X greater than Y X less than Y now reduces to just X greater than Y and X, what I say X greater than Y, X less than Y you do not have the you do not have to consider X equal to Y, is not it. So, I have this and I have this straight line X equal to Y. So, what does this region correspond to? You still have quite a bit to the day to go. So, please wake up a little bit. So, what is this region correspond to?

Student: X greater than Y.

X greater than Y and this is X less than Y. So, if I want to calculate the probability that X is greater than Y what do I have to I have to be integrate the joint density function of what region this entire region using the external definition. So, we are asking are the question what is the probability that the point will occur that X and Y we will take some any joint value in here as opposed to here. To simplify the calculation I will assume that X and Y are independent, consider also independent, not necessarily identically distributed by just independent X and Y that is, but before, before we bring the independence I can write the general expression P of X greater than Y is what is. So, how do I integrate? So, I start with f xy.

Student: (Refer Time: 03:51).

xy. So, even in the independent case in general this is not only true for independent it is also true in general I start with f xy. Let us say if I want integrate some function over this region what do I have to do I can integrate first vertically and then horizontally. So, if I want to integrate vertically which is with respect to Y what will be the limits?

Student: (Refer Time: 04:10).

We will assume that in general it is minus infinity using extended definitions of f xy and so on, it will be minus infinity to up to here. So, I am freezing the x remember how to do double integration your freezing you are integrating or one variable keeping the other one fixed. So, I am keeping x fixed and varying y from minus infinity to x only. So, y will go from minus infinity to x and then x in general can go from minus infinity to infinity the most general case it cannot be anything you cannot they cannot be a case which can go beyond this, this is in general, is not it.

Now, let me let us to simplify this let us bring in the assuming that let us assume they are independent of course, you cannot proceed beyond this unless you know the f xy, but in some symmetric cases we will see. For example, in the darts case what is this, what is this it will have to work out to be what half. Why? Because you can always you can look at it by symmetry and say that the circle is nice and symmetric about the axis. So, if I draw this line it will exactly divide the circle into two parts of each with equal area therefore, equal probability. So, symmetry should always be at the back of your minds.

But let me I am I am taking this a symmetric case where X and Y can have different pdfs, but let them be independent. So, how do I get here? So, this should have been here right, but anyway I wrote it there. So, it considered written here.

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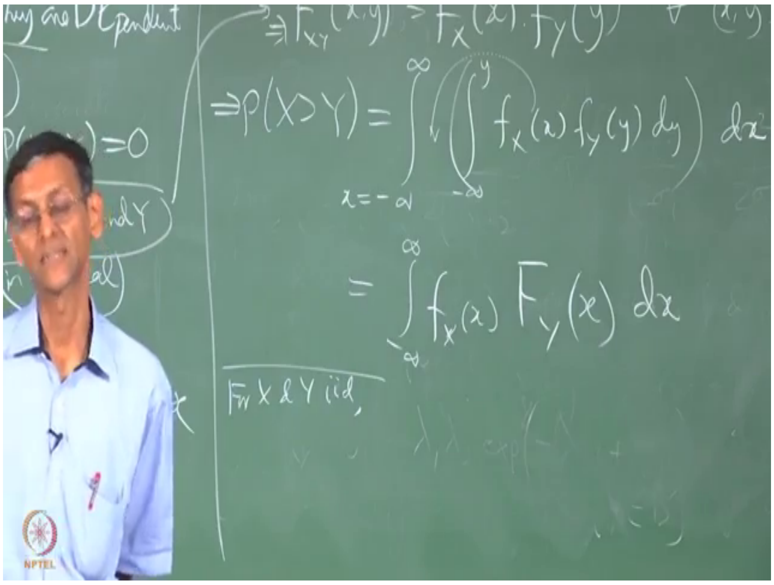

So, this implies that f xy, xy is some a is always a product of the individual pdf as a marginal pdf for all xy, no matter what. So, therefore, this probability of X greater than Y becomes this rewrite the same thing first write the integral on Y minus infinity to I that, the integrand I will split like this dy then I have dx and then integral from minus infinity to infinity. Now, what can I do with this integrand?

Student: (Refer Time: 07:16).

I can pull out the fx of x is it not this can be pulled out, what we see inside, the inside it turns out its minus infinity to infinity and this is x you can just. So, automatically that will be y. So, I have f x of x here this integral of f y of y dy from minus infinity to y is minus infinity to x is what if essentially this is capital F y of what, of.

Student: x.

Of x, it becomes the CDF. So, this is an interesting integral now it involves the combination of the CDF of the one and a pdf of the other this is true all you require here is the x and y independent of x and y you do not require them to be identically distributed of course, you cannot proceed beyond. This you have to leave, you have to stop here because you do not know what. And obviously, in the Gaussian case for example, this does not have a close form expression. So, you may have to you know do numerical integration all that is not you know is the case by case basis kind of thing, but for the exponential case.

For example where if you do exponentials and stuff like that remember the image will have to be appropriately adjusted it will always be 0 to infinity and not minus infinity to infinity you can. So, you are in a if you looking at two exponential random variables X with lambda 1, lambda 2 you would have to forget about this part on the plane and only integrate over here. But the 0 to, the 0 to x will also this will also gives you here f y of x only thing you have to remember is this x is positive not negative. But you know you have a closed form expression for this and this and the integral you can do by yourselves, is not it. So, please brush up all integration especially the exponential for simple things.

But here supposing I take the iid case, now I have said they are independent now to make it even simpler supposing now I want to bring in, I want to bring one more restriction I want to say that their iid. Now, iid what happens is f y of x becomes what supposing I just want to look at only x, f y of x is identical to f x of x.

Student: Sir, second integral (Refer Time: 09:57).

Yes, sorry about that, yes you are right.

Student: No, No.

Yeah, here it is, I mean as usual always talking and writing slips are possible thank you very much. Yeah you are right; obviously, there has to be x I wrote it correctly here that is what matters the first thing has to be correct second thing I am allowed to make mistake. So, supposing they are iid, now what happens fy is the same as fx for all points.

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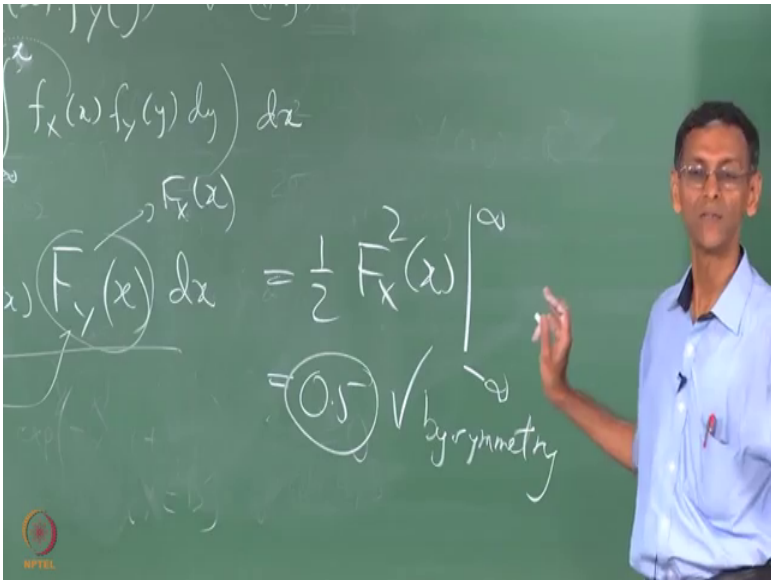

So, this will become if I change this to fx of x without let me do it this way without any waste erasing or without any write without writing it again.

Student: Sir, expected (Refer Time: 10:50).

No, no its we are not jumping let us not jump ahead of ourselves not doing expectations, but this integral it turns out is evaluatable no matter even if f y f x does not have a close form because this is the derivative of this. So, what is the integral going to be?

Student: (Refer Time: 11:09).

The indefinite integral is half of f.

Student: f x square.

F x square. So, what is the value of this?

Student: (Refer Time: 11:22).

This is, what is the value of this?

Student: Half.

Half. Does everyone see this that this is the derivative of this? So, when you have this the derivative and I mean multiplied by the function in I mean the product of the derivative of this is always the integral indefinite integral is this if you have a difficulty just differentiate this you should; obviously, get this. So, this is exactly 0.5. Why does this result make perfect sense?

Student: (Refer Time: 11:56) this is symmetric (Refer Time: 11:57).

This is symmetric case, this is 0, so in the symmetric case you have; obviously, exact same probability of X greater than Y and X less than Y. So, now, am I seeing some more some more a little bit a more spark here. Let me not, that the maths is actually and whenever the maths is consistent with the intuition we really we have a, new respect for the whole thing do not we. So, exactly what we get by symmetric right. So, in general of course, if an X and Y were not they were not id then you would have Y this would not be the same as this and you cannot proceed unless you know the exact distributions, you can proceed only if you know the form of this and this.

In the Gaussian case for example, you could not evaluate this in close form because you do not even have an expression form this in close form you only have this you do not have this for. So, you would have to do a numerical integration. Have you done numeral integration for any in any course, I guess probably not so far. But if you take any computational engineering course you take the first thing they would ask you to do is numerical integration right. So, this also you can think of as examples of numerical integration, very very important examples as far as we are concerned. Unfortunately we do not have the bandwidth in this course to send you off on a on a hunt or evaluation of these things. So, I just have to point out what you would have to do a leave it at that I mean. So, if you ever need to do it you know what to do. So, do not be shy to numerically integrate, numerically evaluate is not it.

So, this, I think are more examples of probability calculations like this for dependent or independent I think it is a you got in general drift instead of taking some funny example which may does not end up with any intuition like this I can always I have look at some region. So, the more regular the region here the region is really nice and regular because it is a still triangular, if the more irregularly make the region the too more complicated it is to evaluate the integral.