Probability Foundations for Electrical Engineers
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Lecture - 56
Examples: Two Independent Continuous Random Variables

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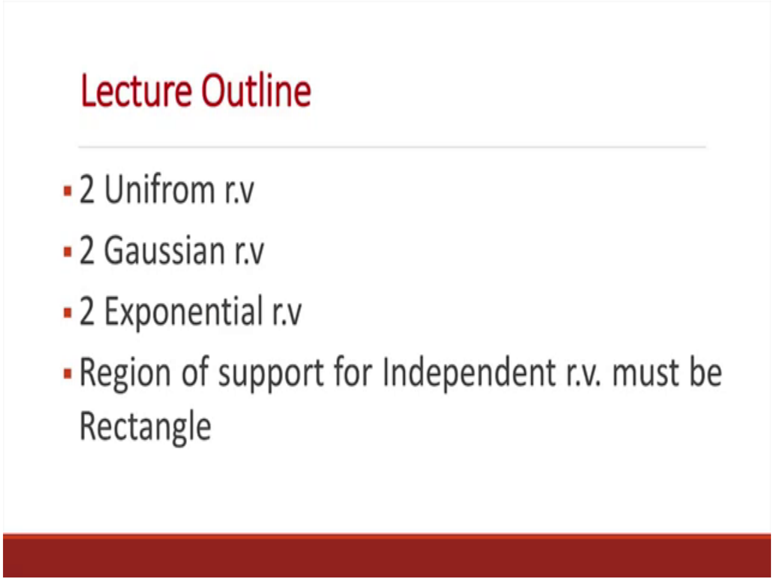

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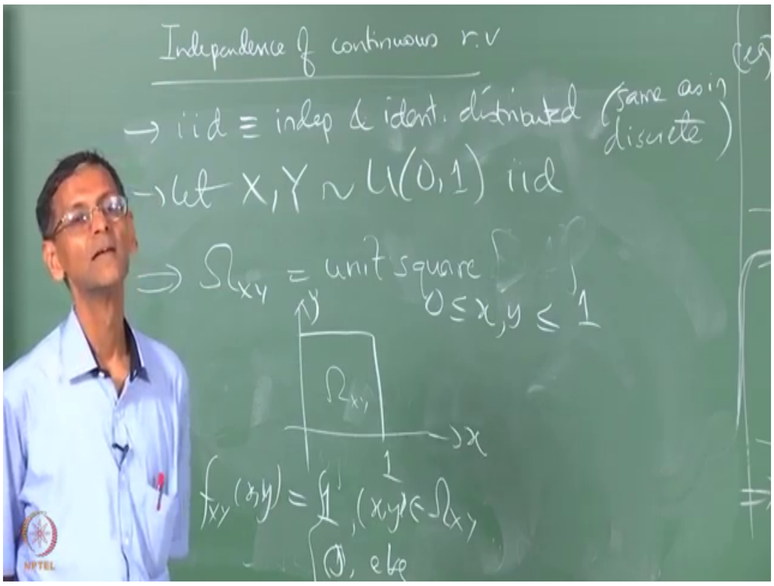

What happens if you combine two u 0 1? Before we do that let me say that the concept of iid is obviously, the same here also. What is iid always mean?

Student: Independent (Refer Time: 00:34).

Independent and identically distributed, so this exact same thing that we had with discrete happens here with this is valid here also. So, iid same as a discrete right, let x and y be u 0 1 two continuous iid. So, what is a joint pdf going to look like? Where is this joint pdf go and have it support if I want to combine them. So, these are two random number generators independently spinning away and this gives you a number x this gives you a number y we pull take them together and plot them on the plane. So, what is going to be; the omega x y is going to be the unit square 0 1. You can think about a something like the 0 ask I am not sure this notation is just we just we just put it maybe is basically or it may be better to write it like this, ignore this, write it like this.

So, remember when you plot these things you are always plotting values taken by x along the x axis and always plotting the values taken by y along I mean along the y axis and any point here is the x coordinate is the value taken by x and the y coordinate is the value taken by y that is all. That is the convention we are following uniformly everywhere and it is obviously, not just true of xy I could have zw. So, whatever I num whatever random variable I put first will always be horizontal axis whatever random variable comes second will always be vertical convention do not need to change.

So, here of course, they are iid. So, it does not matter, but anyhow we have to keep the convention x y. So, this is omega x y the unit square. What is f xy by construction? Obviously, unity the square 1 into 1 this 1, 1 into 0, 0 the simplest multiplication anybody can do. So, for x y in omega x, this omega x y and 0 otherwise, so you can say instead of saying well I start with this and I form this. It need not always be said you can just simply start directly with this x and y be two random variables defined on the unit square and such that the such that the density function is constant then obviously, with their whole thing holds. You do not have to say, you do not have to start with this all the time.

How would I combine two Gaussian pdfs just to say this was kind of, it is a very important example no because u 0 1 it turns out is a basic random number generator. So, is we have to understand it like everything I mean all the things we can do with it we need to know we will do lot more things with it, but let us move away from u 0 1. And look at Gaussian. Supposing we have two both 0 mean for simplicity, but let us say different variances and let us say they are, we know that they are independent to begin with and how are we going to form the joint pdf. So, this is example.

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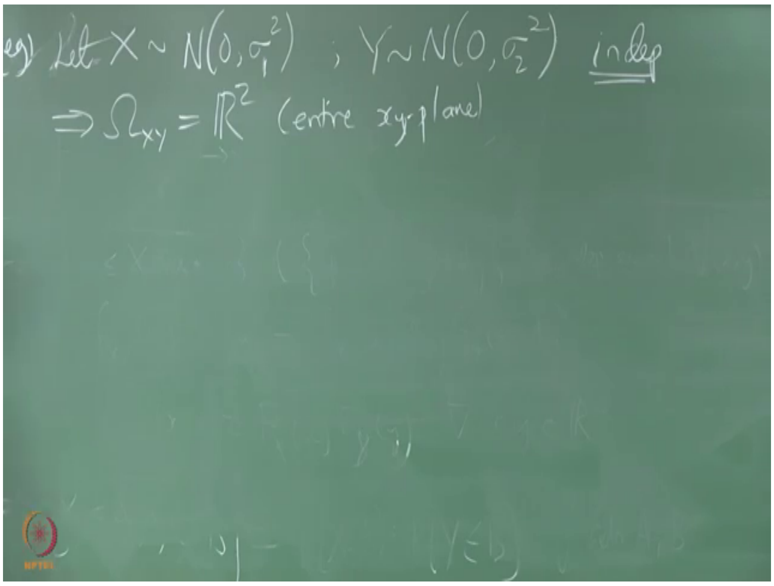

An example again, let x be some Gaussian sigma 1 square I need to have different variances so I will put 1 and 2, y is another Gaussian sigma 2 square independent. Now, I cannot call them iid because I am not saying that sigma one is equal to sigma two so; obviously, they are not in general iid. If sigma 1 equals sigma 2 then they become iid. Remember iid is not only independent also identically distributed means have after the same pdf or pmf, but I am just saying they are in the two independent random variables and; obviously, means they are coming from two independent sources two independent experiments.

So, if you now want to combine them. So, in this case what this omega x and omega y both of them are the entire real line. So, when you combine them the omega xy will be the whole plane, is the entire xy plane which is to say that any point a plane can occur in the in the joint experiment which you why you consider both observations. Here one should mark this also this is 1. That is this first part. The second part is the in way the easy part which does not require any mathematics I mean this is recommending calculations. What about f xy?

Student: (Refer Time: 06:42).

When you multiply the two Gaussian what happens?

Student: (Refer Time: 06:45).

Well, of course, you get this 1 by the square root of 2 pi will give you this, so it will become 1 by.

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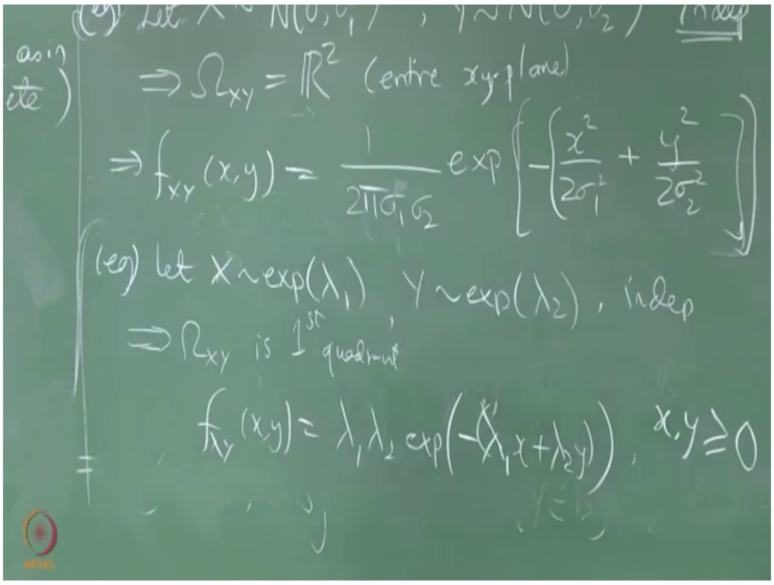

Student: (Refer Time: 06:51).

Sigma wait a minute, sigma 1 sigma 2. Now, the interesting part is it is algebraically that when you take the product of two exponentials you can add the arguments that is a crucial thing here in terms of simplifying this. So, later on we will do a lot of manipulations with this. So, in this case what is a manipulation tell you? It is 1 minus 1 by x square by 2 sigma 1 square and you can put one more bracket out here because this is the common way in which this is written. Of course I can pull out the half also. So, pull out the my half I get minus half of the x squared by sigma 1 square plus y square by sigma 2 square and I need one more bracket to round it off otherwise mathematically its incomplete so we will put that. So, this is for all xy.

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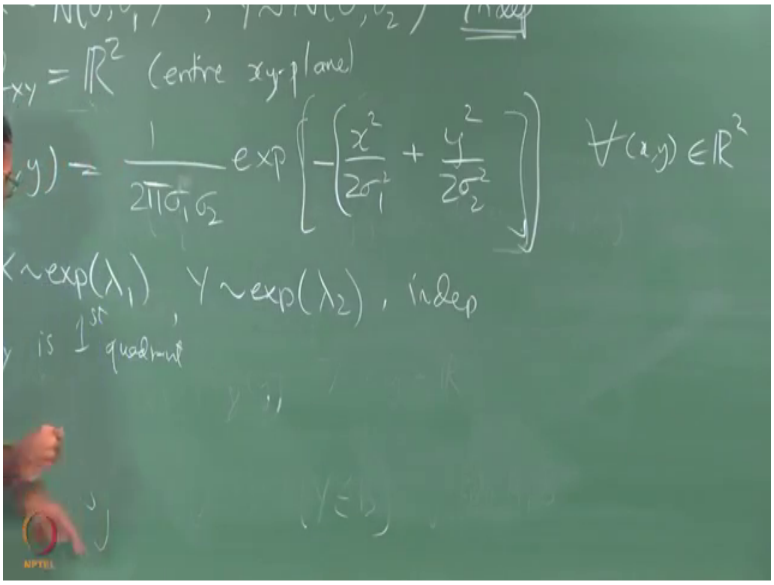

So, this is an example of a joint pdf which is asymptotically decaying to 0, I see move away from the origin. If you have nonzero means it is exactly the same thing said there is a little more algebra you will get as I said m parameter x minus my minus whatever m 1 and m 2, but the principle is the same.

So, what happens in these Gaussian cases? That you have this constant term here you have x of minus and then you that half is usually pulled out and you get what do you what is this look like, this is this is going to be an interesting quadratic in x and y. If they are independent the quadratic not have an xy term, it will just have x square by.

Student: (Refer Time: 008:41).

Some constant plus y square by another constant and it also like supposing means are also there it will also have some x by some constant you can always complete the square if we have terms like x square y square x and y without the x y you can always write it as an square on x and the square on y just by completing the square. All of you can complete squares, hopefully you know if not forgotten that.

If you have one xy then it turns out is fundamentally different well that is again something we are going to study later on, that turns out to be a very interesting case of joint Gaussian. Well x and y are not independent, they are dependent. We will come to that maybe in a homework problem for now, but later on we will look at it in more detail anyway.

So, the independent Gaussian case should easily be able to tell it you know look at it and say. So, instead of x I can have x minus m 1 whole square instead of this y I can have y minus m 2 whole square and you should be able to expand that out and you may get one more constant which is independent of x and y I can pull that out and multiply to this way all of those things are obviously, possible.

If I want to combine two exponentials which is another very practically significant, one last example we will do. Supposing I say let x be some exponential of lambda 1, this means an exponential distribution with parameter lambda 1 and y is exponential with parameter lambda. This also is a case where the arguments and explanations will combine. So, what happens here? The omega xy is only the first quadrant now because x and y are both are so called non negative random variables and they do not take negative values you call them non negative. So, omega xy is the first quadrant something of course, I should say that independent. And what is f xy? Lambda 1 lambda 2 x for what minus.

Student: Lambda 1.

Lambda 1 x plus.

Student: Lambda 2.

Lambda 2 y for xy you say it like this x comma y greater than equal to 0 that is also obviously, important; understood that when either one x or y is negative the joint pdf is 0. So, again here by looking at this if somebody starts off with this you should immediately recognize this as.

Student: (Refer Time: 11:39).

As a x and y being independently exponential because immediately you have product and this is going to be the always a correct normalizing factor no matter what. If it is not well then I mean you might have to tweak it to make it the correct normalizing factor. But you know in some books of papers you are now more poor and some poorly written paper than a book hopefully all you might find some authors working with a some either a wrong normalizing factor or just some k whatever you can always find out the correct value for that.

So, before I move on supposing here if they are iid this sigma 1 equal to sigma 2 then you get further simplification here that is obvious is not it the denominator will be common and it will just be x square plus y squared by 2 sigma square. Just do that also possibility you should be alert to that is a simplest possible case.

So, this I think should be sufficient for now, to illustrate independent continuous random variables. The important factorization I think we have explained very clearly is anyone have any questions or anything regarding this. So, before that let me also say that most interesting cases of two random variables you do not want them to be independent you want them to independent. They just independent in fact, it turns out that the joint pdf is in some sense a redundant statement you can as well only limit yourselves to the imaginers.

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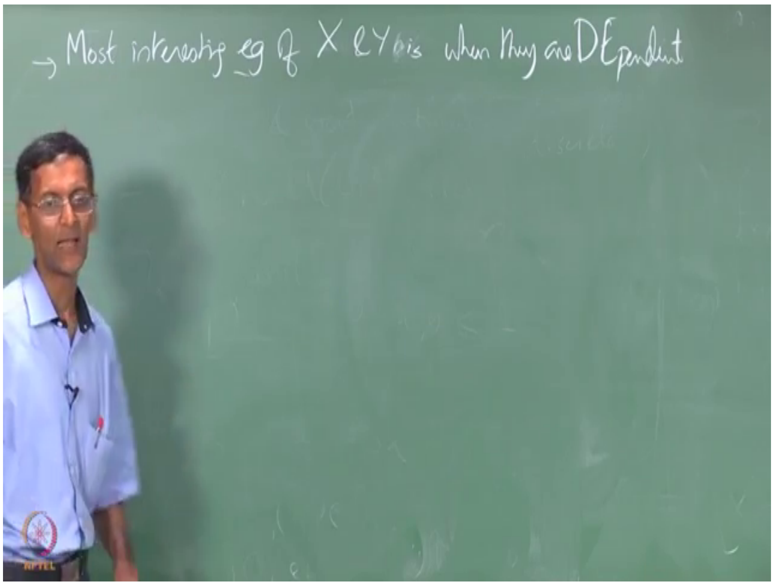

So, most interesting of x join x and y are is the dependent case this is exactly I sent discreet. So, how do you straight away how can you tell if the necessary condition for omega xy is not satisfy what is it necessary condition omega xy it has to be a regular with respect to the x and y axis some, if that is not satisfied and automatically you know that x and y are dependent. For example, in the trinomial pmf we could straight away tell because x and y by limited to some triangular region. So, whenever something like that happens I mean they may not it may not be the full triangle, but at least a points all lie lay inside a triangle and not the nonzero probability points were not outside that triangle. Likewise, here if your x and y jointly are in a circle or any triangle or any such thing clearly it cannot be independent they have to be dependent.