Probability Foundations for Electrical Engineers
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Lecture – 52
Joint pdf and joint CDF

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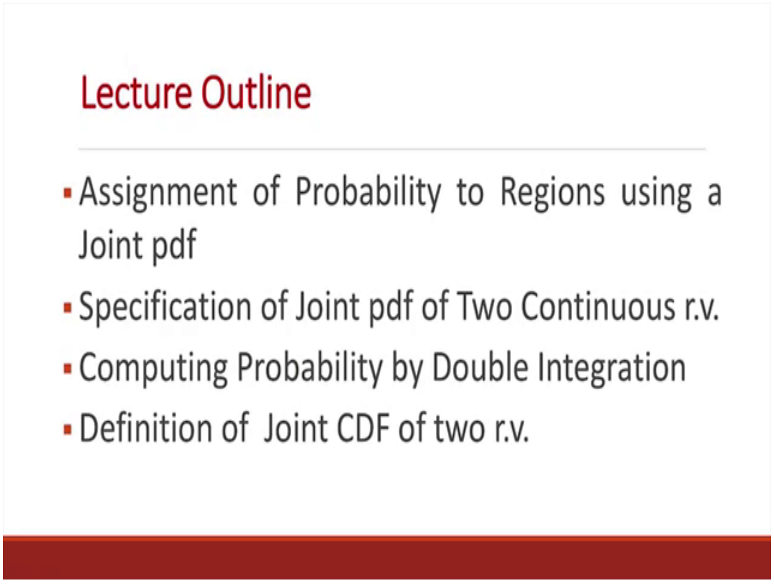

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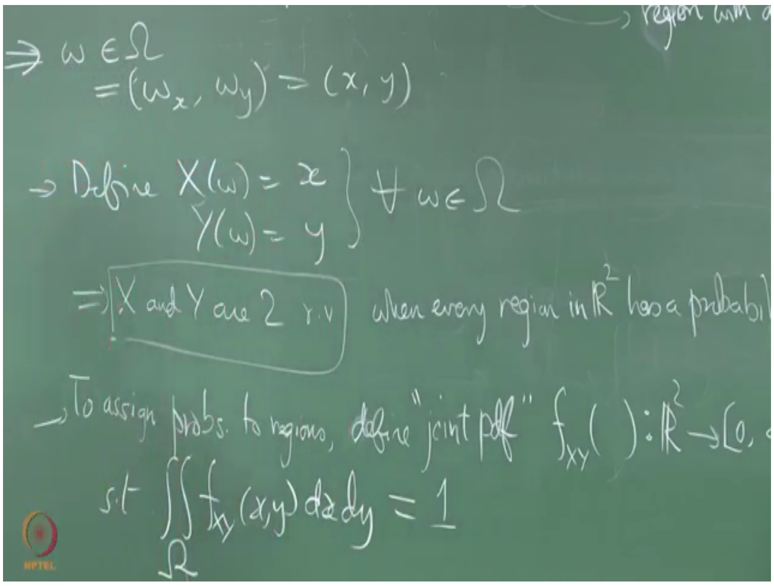

To assign probabilities to regions let me to other regions. Of course, omega is probability 1, always it has to have to regions we start with or we define a function which we call the joint pdf. So, what is this joint pdf? It takes you from basically all of R squared to the interval 0 to infinity I mean it is a non negative function on xy, such that what? The double integral, only just I have to bend down a little bit.

So, this double integral over omega see you have defined already defined this; this is the sample space for the experiment. So, we have to make sure that this function is has unit volume when you do a double integration. And also there smoothness constraints on this F, you know want it to be some totally crazy function which jumps around all over the place; some discontinuities here and there especially the boundaries is as we will see we look at examples you know.

So, F is not completely is not allowed to be totally funny it has to be minimally smooth in some sense, but I do not have a mathematical way of putting it down. So, I am not I am not going to write it here, but just keep it in keep that in our heads f cannot be arbitrarily allowed to vary. Specially this double integral must make sense and not only for over omega right. So, we do domain extension, first of all we defined it how we are going to define this f xy or the entire plane; obviously, by saying that it will have non0 values only in omega and outside omega it will have 0, that is the first thing you have to do right.

So, to for uniformity sake all pdf, like all univariate pdf were defined on the whole real line all double this double joint pdf will also be defined all the whole plane.

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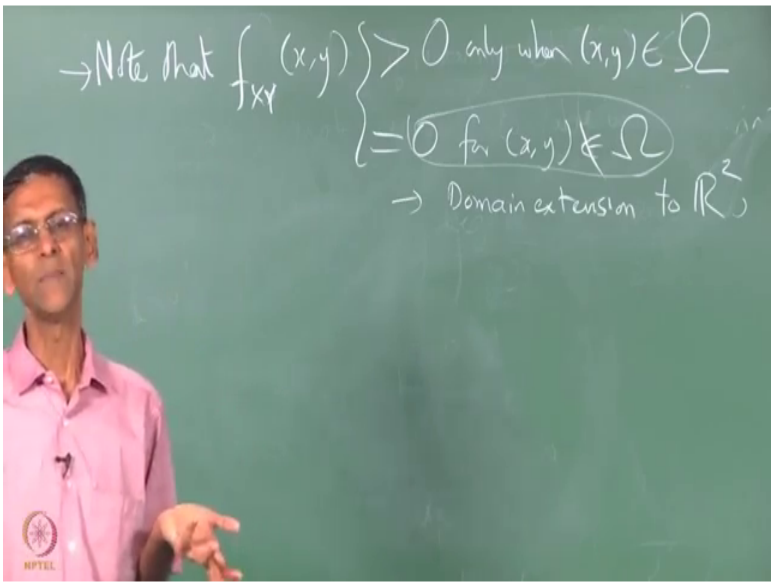

So, what do we note? We note that f xy, x of the value of this is, its first of all as like in the one d case it is not a probability it is only a density function. So, this f xy, xy is greater than 0 only of course, it can also be 0, but it only when xy is in omega right. So, and, so you do this, this is domain extension to R square for example, when you consider the that is experiment it makes no sense to say that the sample space is the entire xy plane typically taken to be some circle of with some finite radius.

So, what you say the joint density function, it takes nonzero values only inside that circle such that this is satisfied. So, now, can we claim this statement every region of R squared will have a probability where it can be 0, can be unity yeah we can say that now without any problem; every region of R squared can be assigned a probability. If the region of R squared that we are interested in is totally outside capital omega then it corresponds to a null event probability 0, so as simple as that. If that region includes omega completely then it is a certain event is probability 1, is not it. So, every region of R squared will be assigned a probability a based on this f this joint pdf.

So, one more thing I want to say is that this is not the only way in which you can do it this is the most practically important way that for engineers as we are, we care a lot we care mostly about situations where regions in R squared have probabilities assigned through a joint pdf, but that is not the only way.

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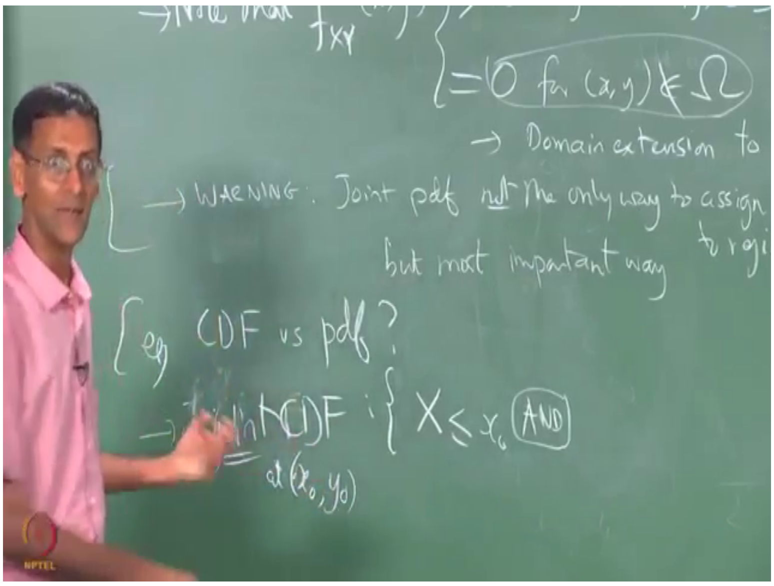

So, let me write it here maybe should I use the word here is writing warning. Joint pdf not the only way to assign probabilities right, but this is a most practically significant most important from an engineering point of view. So, what does this remind you of? it reminds you of this the pdf and the CDF conflict that we had it with the single do you use a CDF to sign probabilities value you can do that, but it is in general it turns out to be not as important as starting the pdf from an engineer, again from an engineering point of view all are engineering problems we and we will work we love pdfs, we do not care. So, much about and even here there is a CDF that we have to look at right.

So, what is cell (Refer Time: 06:55) CDF versus pdf. So, here in the 2D case all books will not talk about a straight way talk about the joint pdf instead they will say we will start off with something called a joint CDF. What is the joint CDF? So, as they say, so we have to. So, now, that we have understood the joint pdf does anyone have any problem with the joint pdf; yes, you have a non negative function which is reasonably smooth you defined it for you know. So, that it has this property you first of all identified omega you identified then you identified a suitable function which has this property and its nowhere negative by the way why cannot pdf be negative I think the answer should be obvious right. If you look at a very small interval or a small region around where is negative you can get negative probability. So, you cannot ever allow negative values in pdf that is all. Even though it is not itself probability you cannot allow negative values has to be positive.

Then of course, this one is just a normalization all you care about is a this integral is finite. So, everyone is with this right. So, what is the joint CDF? I am sure those who have a studied what particular region are we looking at for a joint CDF for two random variables. The joint CDF is this event this region minus infinity at for joint CDF at let us say or for x naught some you pick a point x naught y naught in the plane, it is basically minus infinity to x naught or maybe I should use, I should not use this I will use random variable notation itself capital X less than x naught and I will put an AND here which is all basically a comma does not seem to have quite the same impact this writing the AND, but please remember that right. So, anytime you use a comma is a very firm AND; AND what?

Student: y (Refer Time: 09:24).

Right. So, what is this? It turns out that this is what region, it is a not the region. What is that region? So, this region is completely oblivious to capital omega it just you just define some you know you know if the definition is does not care about the actual experiment at all, it just says well somehow it is supposed to look at that and that too you know its vary this x naught for you know you supposed to do this for all x naught y naught in R squared every supposed to do somehow magically define this number. I mean I have not defined the CDF here. So, it should be the probability for that region right. So, let me write it here.

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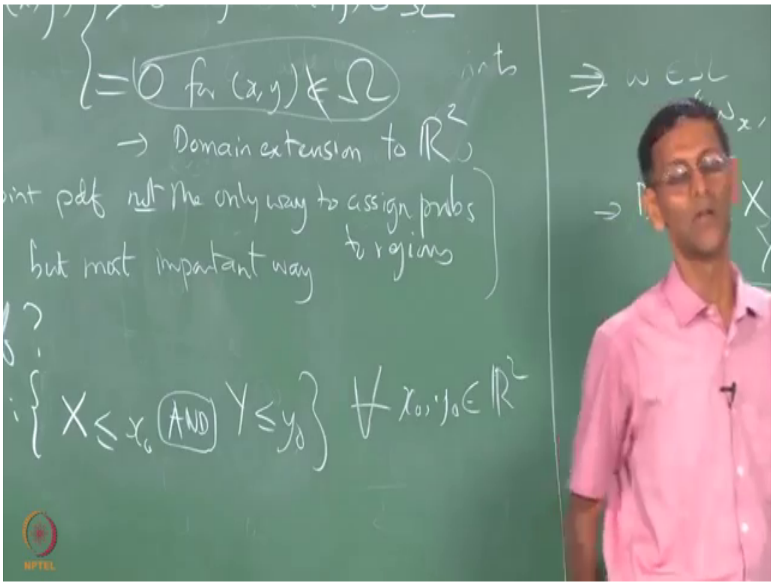

So, I have picked some point x naught y naught here without any regard to capital omega if I just consider this region and say that this capital F xy.

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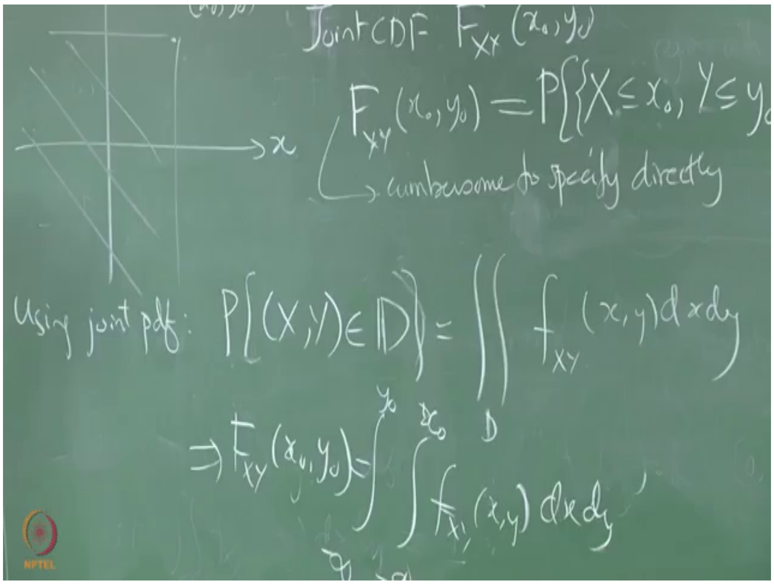

So, are this is joint; let me write here will joint CDF, capital F xy, x naught comma y naught again I am just using random variable notation here not set notation which is of that event with and; you should put the braces here for our consistency right.

So, this event right, somehow magically that region is supposed to be assigned a probability and that to for all x naught y naught in R square, not just for some points. So, it turns out that this is not the practically what use a simple way of assigning probabilities. Yes, theoretically you know it can take care of curves it can take care of that pdf approach has the limitation that it cannot handle points and curves so nicely, but it can handle regions perfectly well, that is what we want. This is more general, in that yes it can handle everything. But it is not very in my humble opinion it is not a good starting point for from, for any engineering problem. Just like in the 1D case nobody ever starts off with this is the CDF. So, what we do with it in most engineering problems you want you know you ok.

So, just for completeness sake we should be aware right. So, this is basically of course, when the joint pdf is specified you can always find this probability by integration it is not an issue what would be the limits of the integral minus infinity to x naught right. So, I did not write maybe I should not, I should have written that here. So, the probe using a joint pdf what is the probability of a region? You write this xy element of d some region d is basically the double integral over D f xy, x y, dx dy.

So, maybe this should have come up little ahead of this, but and I want to say lots of things I forget. So, I have only defined, I think everything is there on this piece just remember that this should have gone here before talking about joint CDF I should have written this. So, this fine, now coming back to this, so if you have a joint pdf fxy how do I find this probability I would integrate from minus infinity to the limit should be minus infinity to x naught minus infinity to y naught, that is all I have to do. So, I can always get this from this.

So, this basically, f xy, x naught, y naught, let me write it very clearly is basically minus infinity to x naught minus infinity. Again we have to be careful about the limits here if I say minus infinity to x naught; that means, the inner integral was on x, so f xy. So, when you do multiple integrals. So, please review your material or multiple integrals. You are not allowed to write this totally independently of this, this inner integral is dx and the outer integral is over y. So, that order has to be maintained because if you do not maintain that order you are going to run into trouble when you try to change variables and so on, all of this we have to do now. We have to come bring go back to our notes or whatever material that we picked up on multiple integration and keep it all ready for use right.

So, if you have this, this is no issue at all, you can integrate using if you cannot integrate analytically you can always integrate numerically it is not a problem. So, it turns out that this joint CDF in general specifying it directly is a very cumbersome business, especially that it has to have certain properties for example, it has to be non decreasing in both you know if you look at this it has to be non decreasing in both arguments and then if you of course, it is a probability. So, this has to be always between 0 and 1 right, but mainly it has to be non-decreasing.

But it can you are you know in general you can allow it to jump in a way that this cannot jump. If you do it like this, this will be a smooth function of x naught y naught if this is well behaved, if the joint CDF is where pdf is well behaved this is not going to have jumps just like the 1D case. But in general let me say this is cumbersome to specify directly. If it was difficult enough in a 1D case is almost very very painful to do in the 2D case when you have possibly. For example, you know if I have a funny shape omega this is my omega, you can imagine now how this CDF is going to vary in this region for example, how would you specify even if this omega is a nice circle it turns out its not at all easy to write it down.

And then if you look at some intersections of this half plane with omega then you will get even funnier functions of x naught and y naught. So, all told just I do not think I write I you can fight me on this issue, that is this, this from my experience is an absolute nightmare even when your capital omega is anything which is not aligned with the x and y axis. It is easy to do only in, CDF is easy to deal with only if capital omega itself is a nice rectangle aligned with the axis. Any other case right, because this capital F has no does not look at omega at all it is an absolute mess to specify for any other case. I have just written cumbersome, but I can say worse things about it verbally because I have tried you know, I have tried writing it and I found that just for maybe bringing it out as an example in a class and I just give up it took me one whole you know several pages to drive specified correctly. Once I start doing it I want do it fully and correctly and I found was not were the trouble.

I think maybe; one last thing. Supposing this capital F is reasonably smooth and differentiable how would you go from supposing somebody gave you this function with the like all the requisite probabilities that it is and I am not going to you can go read it up in a book it has to basically it cannot decrease as you keep increasing x naught or y naught it has to either stay the same or increase. Because the region is always getting bigger as you increase x naught or y naught. So, how do you get this from this? Basically the double derivative right, given capital F xy the density function if the function, if it is reasonably differentiable and so on right.

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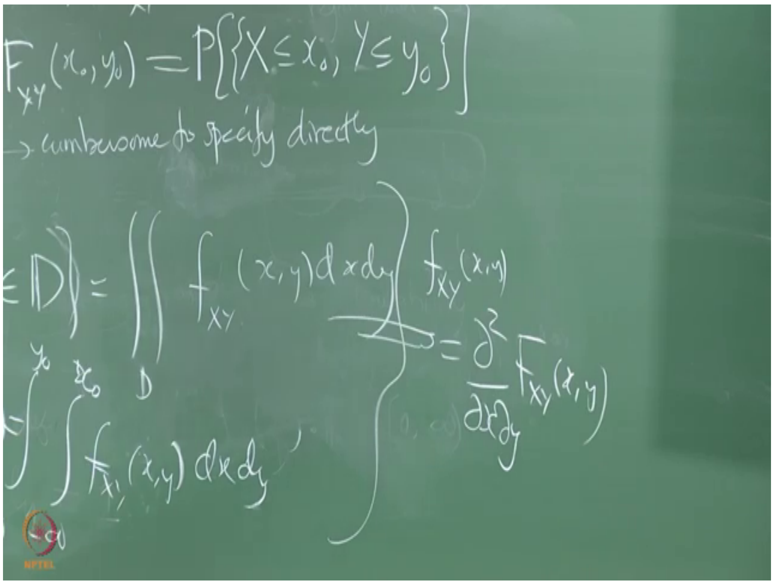

Let me write it here this a density function at some point is basically how do you go, what do you say I know, how do you write that or how do you say that d square by dx dy its just d square by dx dy only when you write it you write it using partial derivatives of what? Of.

Student: (Refer Time: 19:17).

That this of course, assumes this should not jump at any xy. So, we will stop here.