Probability Foundations for Electrical Engineers
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Lecture – 51
Two-dimensional Real Sample Space

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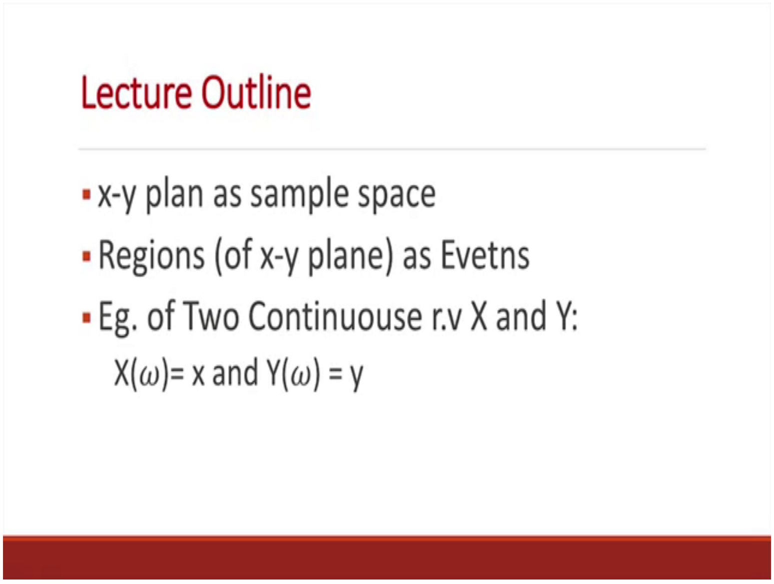

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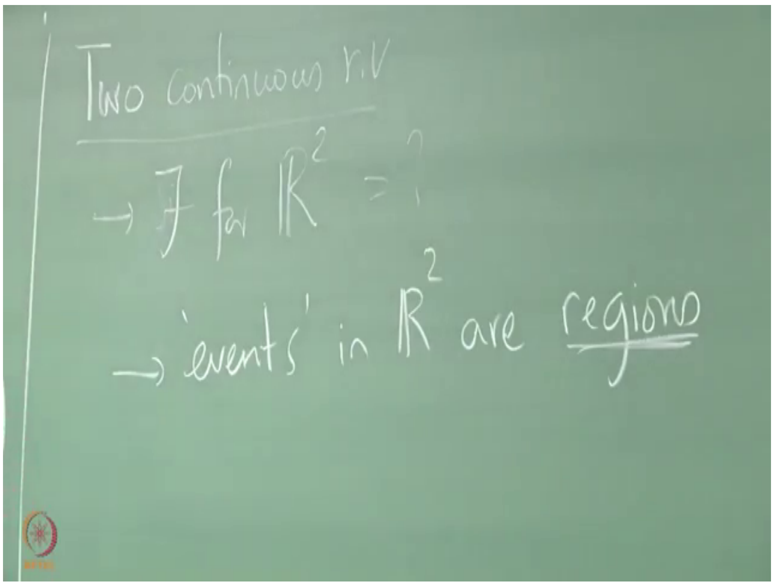

So, we start today's lecture pretty much with two continuous and while doing this you have to look at this as what is the F for R square, a suitable sigma field for R square what is this that is what we first have to look at. Why are we interested in R squared basically? We are interested in R squared because R square is a minimal space that we require to define two continuous and variables. If you have only R you cannot and you sell it let us say we define x of omega equal to omega that is you defined our standard random variable on a single for omega is a single real measurement.

Then once you observe the measurement there is no more randomness right, you cannot define yet another random variable and expect it to be truly random you knows I separately of x. So, we just R your very your completely limited in that you cannot really go beyond one random variable. So, if you want to go two if you want model situations where there are two measurements each which can vary some to some extent independently or separately from each other and where looking at one measurement you do not know any I know you do not know the other one fully and completely, you know something about the other one, but not everything. For that you need to consider at least R squared, if not bigger spaces like R cubed or R whatever right.

So, you have to make the jump to R squared from R which is basically the plane, xy plane and we have we need to look at the events the sigma field alright what a events we are going to define R square for which we going to assign probabilities. And those events are basically regions. The events in R squared are like we had intervals in R in R squared we have regions. And what kind of regions are these? These regions are basically anything that you can get by taking a countable union of rectangles.

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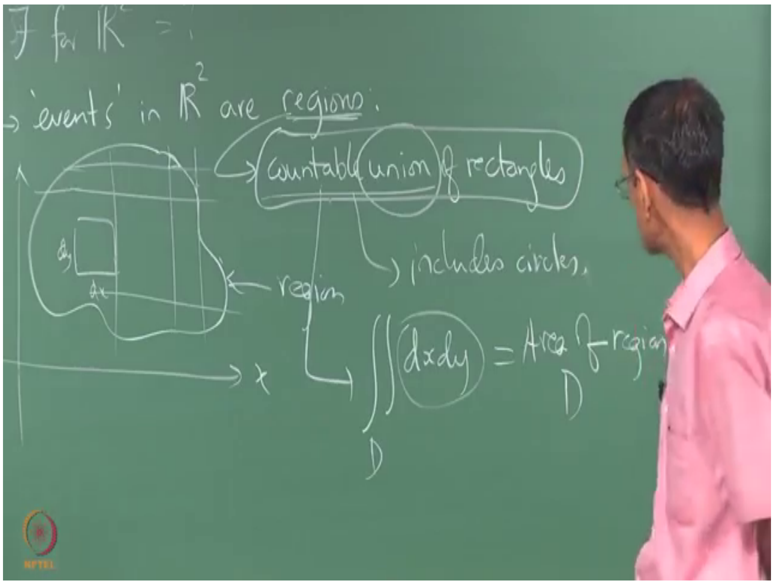

What is the region? And again to make life simple for us will start out with the most basic rectangle that we can think of which is basically dx dy. You start with again I repeat that kind of figure if I am if I start with dx dy this is an elementary rectangle and not only it is an is an rectangular it is also aligned what do I mean by aligned it is its sides are parallel to the x and y axis right. So, this is a simple aligned rectangle. And what kind of sets can you build with such rectangles? It turns out you can build pretty much any arbitrary region right.

So, if I have something like this some region like this how do I express this that that set of points in the xy plane as a countable union of such rectangles turns out I can divide, divide the thing like this and in these portions I can look at finer and finer divisions of the xy plane and ultimately obtain any core any funny shape almost any funny shape as a union the point is that we need is countable union. Of course, there may be an uncountable infinity of points in the plane just like there are uncountable infinity of points lines. So, you cannot think of, you know taking the union of individual points that is not a countable union.

But if you want a countable union we start, we allow these rectangles to be arbitrarily small. So, again I am not at expert in this kind of thing at all I am just I mean two or three places that I have read this business they say that with such rectangles you can generate just about any region we care about. So, I am just going to repeat that argument here standard does counter into you to statement is a circle is a union no rectangles.

Now, again you would take that forever its worth, but well actually you have to take it as that is that it is true right. So, we are going to assume that statement is correct, a circle it is the union of rectangles. Now do not ask me where as rectangle is union of circles, but anyway. So, this includes circles. Basically I think this goes back to when you study multivariable calculus you can, if you do the integral, what is this integral dx dy? Basically by do a countable union we are doing this, this is a countable union just like integral dx is a countable union of intervals.

But integral dx does not seem to have the you know it is too trivial you can just say well it is going to integrate to the length of the light interval or whatever, but here this is this is something more interesting than just take a linear putting together of intervals is not it this dx dy. So, what is this double integral give you if you integral if you perform this over a certain region d?

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It gives you the.

Student: Area.

Area of the region. Go back to your multivariable calculus class and check it out.

So, you take a region which is a subset of the plane the fundamental identity for finding the you the area such a region is to do this double integral where of course, the boundary could be defined by some, for some function y as y as function of x and so on and that would come into the limits of this integral.

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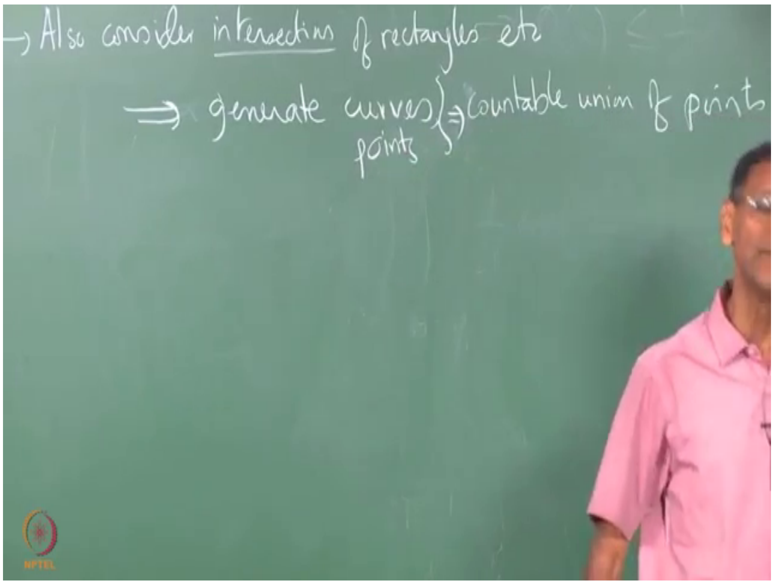

So, also consider intersections. So, what do you get, when you do intersections of rectangles in the unions of these intersections? So, what can you generate from this? You can generate curves, generate on the, not just curves you can also generate individual points sets I would say; what did I write here? Countable union of points, you can also generate a countable union of points you can generate curves comma points and then from this you can generate countable union of points. How do I get a point? My point is the intersection of 4 rectangles if you wish where they boundary includes is included in each rectangle.

Just like a single point in the on a real line you can think of as a intersection of two intervals exactly intersecting that point, similarly a point on the plane you see you can think of as a intersection of 4 rectangles whatever. And so the case that we looked at earlier the two discrete random variables it is obviously, included here except that we do not need to do all this for that where we only looking at only points in discrete random variables we will know only look at points you are not interested in intervals, you are not interested in common more complicated a regions and so on.

So, for that you do not need it you know if you only, if you going to stop with that you do not need all this at all because you will say every countable, every set is a subset is a valid event and, you know you do not have to worry about anything basically. So, this is a much bigger thing; obviously, it includes the union of points, it includes curves and it includes any complicated combination of regions and points and so once, so obviously, you can you can generate you know as an a enormous variety or whatever almost a everything of engineering interest can be generated this way right.

We are most interested at lee at least for now in regions because that is where continuous. So, our main interest is regions or not curves (Refer Time: 10:19) and that to regions with let me say area greater than 0, not equal to 0.

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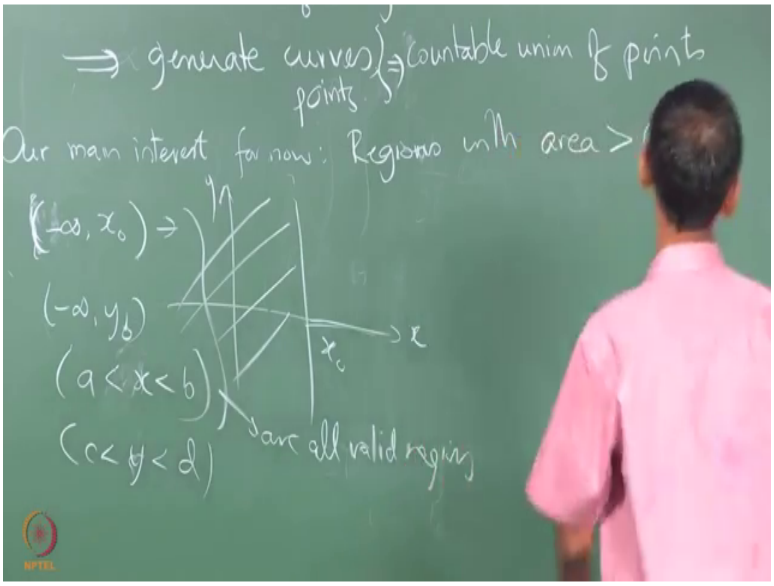

How do you distinguish a curve and a region? A curve has what? A area equal to 0, I cannot if I do a I mean this type of a double integral if I write it in the limiting sense for a curve it will collapse to 0.

So, we are now interested only with a regions with of having non; it could they where could infinity, but it is not 0. And we also note that even I mean the set of points let us say how do I write this minus infinity x naught; supposing I just look at minus infinity x naught this interval this corresponds to what this entire. I am not talking the random variable now assuming I have the axis x y and I pick some number x naught this interval minus infinity means interval just by looking only at this only at the variable x and not looking at variable y generates half a plane I mean the portion of the plane that looks like this, that is also a valid region.

So, this is so minus infinity x naught minus infinity y not a comma b on a; a less than x less than b all these such all such things are all valid, can also add c less than y less than d see all of these; I am so, we if we think of it as an event we should put the curly brace around it, but for now I am just looking at a valid region. So, I am just writing it with ordinary brackets or parentheses right.

Later on we are going to look at the event version of this. So, what happens is that because you know these are all regions now if you look at the random variable extension you can clearly say that there will not be any difficulty in the assigning you know if you can assign probabilities to any region in the plane then these also will have probabilities assigned to them and therefore, all the theory of study so far will not get upset by there by this kind of a definition.

So, the question is how do we assign probabilities to arbitrary regions once you answer that question, then we are home right. So, this is, after know I am not talk talked about any random experiment or anything the solving just looking at the plane and how to look at regions of the plane that is all.

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Now, we bring in the plane as the sample space of an experiment. Some omega is a subset of the plane, this is let us say this has finite area; this itself is a region, is omega is itself some it need not be the hole that the set of points that we get in the experiment need not be the whole plane just like in the real line case we could get some portion of the real line.

So, like that we you know we start with some sample space omega which is itself a countable union of rectangle say allowed region in other words right; obviously, you do not want to start with something which is a some funny thing right. So, this omega itself is a well behaved region in the in this sense that we talked about all along and so here is a region with area greater than 0, right.

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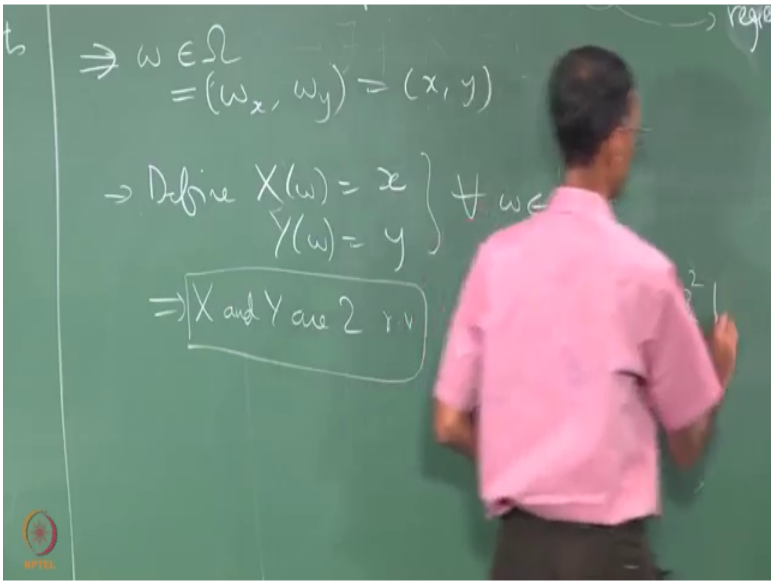

So, each omega in this is what is basically has two components omega x and omega y if you will write has an x component or it is basically we just write it as x comma y and right. So, every omega is basically has an x coordinate and y coordinate you assume that the axis are marked.

Here of course, you can say well at this point there is no axis, but for this for making the measurement you need the axis you need the origin you need the x axis and y axis without that you cannot define these say the x square, the x value the measurement of the y value of the measurement. So, if you for example, throw a dart at a plane we just more standard or they start one of the simplest. So, the dart comes lands here, what is right. So, unless you have the axis may define beforehand you cannot talk of the x coordinate of that for hitting point on the x and so on.

So, this is for, so every omega as I said with some x and y and now we define the two random variables capital X and capital Y right. So, we say this implies site fee, this is true then omega equal to omega y you know you can just we forget about their I know you can just directly write as x y you do not even have to look at this right; so now which means that I can define X of omega as a small x and Y of omega as small y.

So, if every omega I have defined the values taken by the two random variable. So, if I do this for all omega small omega and capital omega I have defined the two random variables x and y and obviously, this can always be done. So, are these now they are, therefore, now x and y; are they jointly I know are they two random variables now or not clearly as we said earlier.

Now we bring in the region. So, we are going to assume that every region is going to have a probability how are we going to calculate with that probability will come to a little later, but since every region has a probability this intervals on the x axis, interval in the y axis also have probability. So, therefore, there is no problem here in claiming that x and y are two random variables you can always say that if every region has a probability then every interval on the x axis and every interval of the y axis also will have a probability right.

So, therefore, these are two random variables when every region, every this is not just omega also the whole plane by extension, we are going to do the domain extension thing that we did earlier right.

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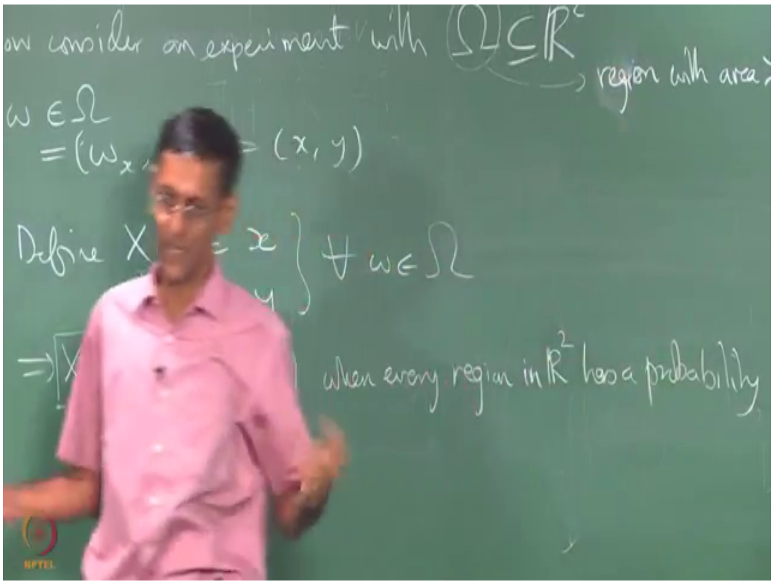

So, this is the most important idea here you can always take components of the space as random variable when you going to assign every region a probability right. So, there will not be any theoretical issue that you are going to run up against.

So, the question is how do you assign probabilities to regions. You have to start out with the; you assume that the region as I said for now we are going to assume that region has area 0 that omega itself is a, the capital omega is has region, has area greater than 0 and for every nonzero region we are going to do a double integration, fine.