Probability Foundations for Electrical Engineers
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Lecture – 03
Sigma Fields

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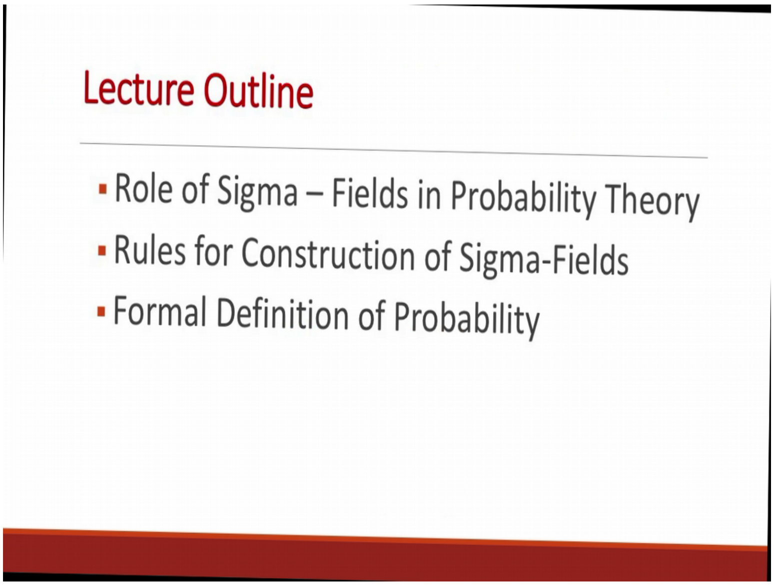

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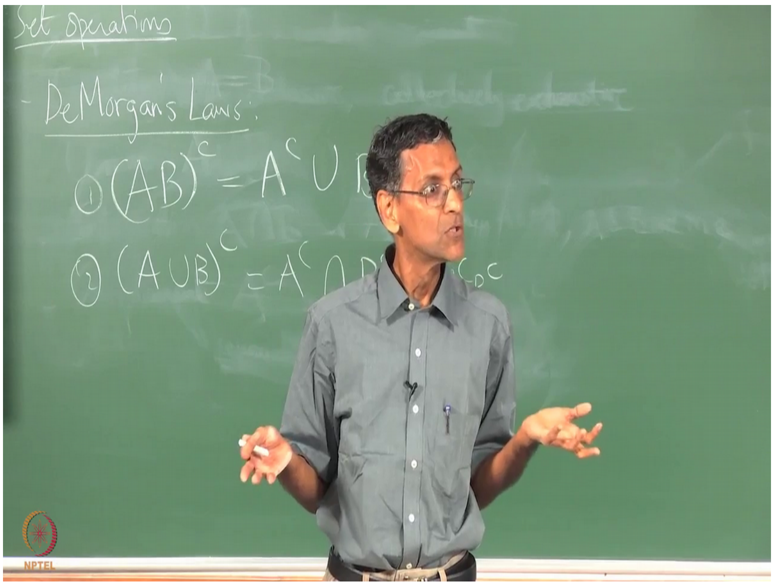

So, our set theory is very just basic. Now how does all of this apply to probability? Basically you have to keep; remember one very important thing right probabilities are fundamentally assigned to or attached to sets probabilities are numbers which are attached to sets right of any kind those sets can be can be sets that you start out with such as AB, they can be the unions intersections remember all of these operations like unions intersections and complements they give you a new sets, right. So, if you say you start with probability of A right, then you must also know think of the probability of A complement, right. So, that is formalised in this note in this notion of a sigma field or a sigma algebra, right which sort of says you have given a sample set capital omega what are the sets right to which I can assign probabilities right.

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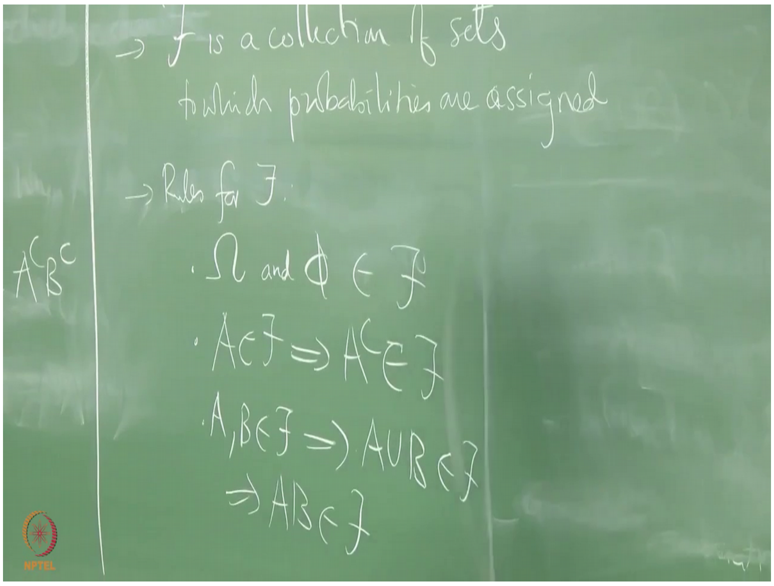

So, we that is the next important point that we are going to look at; let me start here the concept of sigma algebra right sigma field. So, these are; so, what the f is basically the collection of sets to which probabilities are assigned. So, as such it occupies a very important theoretical right position in probability theory. In fact, you talk of tried right you talk of omega you talk of discrete f and you talk of the probability function P right.

So, very probability experiment is omega f P right. So, what is this f right what are the rules for to form this collection is f and basically it right it f should follow some rules which allow it to be called a sigma field right. So, rules for f, again I am saying this is a very intuitive level right not very rigorous right one is that omega and phi are always in this f note this use of this notation here element means it belongs to should have said it earlier itself small omega belongs to capital omega right that f that that notation we use a lot ok we will use it here now right.

So, the note right capital omega is just as I said. So, universal set like anything else it always has to belong to capital F. So, the simplest f is only some something which only contains as it turns out only these 2 sets the universal set that they certainty event omega and the null event or the null event, right. So, if you want to assign probabilities the standard thing to say well is that I will always assign 0 to this and one to this right, but this is not very helpful because it does not talk about anything in between right you all you know for example, that the null event never happens right and the certainty event happens always big deal right.

So, you have to go further. So, what if you want to include a certain element A in a set; that means, A in f; that means, you also have top include a complement right. So, A if you include a you must also include a complement; that means, that you cannot say that I will only assign a probability I will not assign a complement of probability right, then if a if you want to include A and B in f then you must include a union B also and by de Morgan's law right with a fact that you have to complements you can automatically mean or whatever show that must include a intersection also.

So, up to this point well I mean. So, what if what if somebody says that I have a 1, a 2, a 3, again, right you know that you can form the unions of a 1, a 2 with a 3 and so on, right; obviously, the collection will keep growing as you as you increase the number of sets in f what in you know. So, that this is basically this framework is basically sufficient if you to deal with a finite collection of sets in f, but we also need to deal to separately list out the right the case of accountably infinite collection of sets right.

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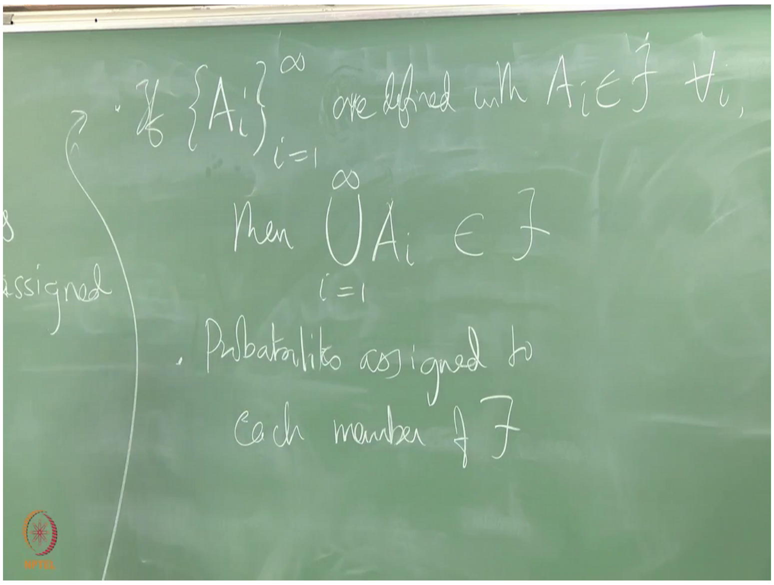

So, let me clear that point here, if A I; again this is a notation which we will use right if A i 1 to infinity is a short form of saying a 1, a 2, a 3, up to infinity f each ah are defined we define as infinite collection with A i in f all I no matter how right now how do I define you know; what would be a good example of this first; of course, you must have right as a experiment which supports an accountable infinite collection for coin tossing until you get a head type of experiment right I will just simply write the result and then we can look at it then the union of these sets must also be in that remember I might write expressions like this, but this capital omega is sitting on top of all of these unions and intersections right.

So, this countable countably infinite unions, that I have written out there right; obviously, has to be no bigger than or can be no bigger than capital omega. So, capital omega is well defined is no to get afraid of that left hand side right for example, getting a head on a second an even number of I would say well A i could be second toss fourth toss sixth toss or right; so on right that what would be you would what would the union be it would be sitting ahead on the on an even number toss right. So, that also has to be in f.

So, this is stated sometimes as a distinct rule right apart from this this would really come into effect only if you had the right and accountably at least accountably infinite number of elements in in omega it also right applies right when you have an uncountable number of elements in omega as well; that is also a case that you can form an infinite collection of sets, but remember even if omega itself is uncountable this is a countable collection anything with u index with an integer has to be countable right and by contrast or whatever you cannot index an uncountable set with an integer right you cannot use integers to count uncountable quantities right. So, if I write A i means that the collection A i is countable.

So, these rules right are the bedrock again that that used to form I define a sigma field capital f right and we will assign probabilities to every element of in continuation of this probabilities assigned to each member I would say I will not say element to each member of f which is some set right that is importance of f right yesterday we talked about the total number of sets right when the number of elements in omega is capital n all of you said 2 power n right which is perfectly correct. So, the simplest construction in such a case would be to say f consists of all subsets of capital omega right.

So, to make our life easier right we are not going to discriminate right in the case which is what is done without putting it in. So, many words right in all examples of probability theory they implicitly do it that is what I am looking for right the implicitly say without saying exactly saying. So, that any set can be assigned a probability right. So, when f is countable not even necessarily finite.

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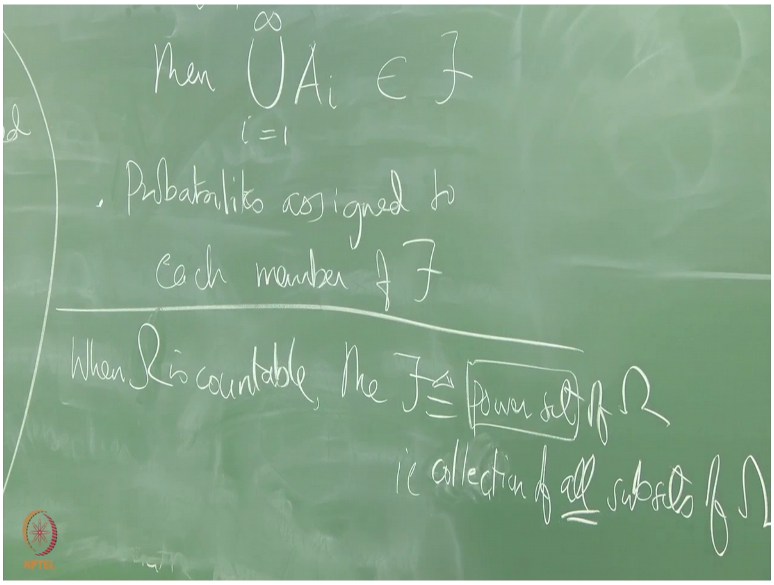

Sorry, when omega is countable right that is discrete and f is will be. So, f will be defined as this equal to with a triangle on top you going to use for definition, right by definition then f will be by definition the power set of what is its power set I e the set of all or let me not use; what set here I will just say the collection of all subsets. So, this terminology power set is very important and occurs quite often.

So, yesterday we said if capital omega has n elements then the it has 2 power n subsets the those will be those will form the power set when n becomes infinity right on that is accountably infinite case you still have the same construction except that it is an interesting point here this 2 to the power of countable infinity is our bridge from countable to uncountable; it is an interesting side note that you should be all aware of right have you seen this construct.

This point 2 power n as n becomes infinity is actually the number of points in the in some segment of real line right go on and read it up if you if you if you are right if you find; hear hearing this for the first time right that is the bridge you go to committee or where I do not know who is the mathematician I am a very I am not a mathematician. So, I to throw up my hands and say right the these are the things which I picked up just by you know reading this any number of times, but right even this morning I checked up something and the same result was there right is 2 power n nice that capital n goes to infinity that becomes that number becomes uncountable and it actually makes sense right.

If you look at all that is you know binary representations of the points between say 0 one what would you do you put a binary point and then you write strings of 0s and ones right to infinite precision right. So, how many decimals how many binary digits would you use you use infinite number of them right what is the total number say number of such binary digits 2 to the power of that infinity right not any infinity, but the infinity, which is the say number of financial numbers right and you claim that every real number has a representation of that kind a bit representation of 0 and ones say right has a representation of a kind point 0 one 0 one something right going up to infinity.

So, that is why you know intuitively you can understand right that n the power n infinity does give you a representation of that right count of some in in a way in the lose sense it gives you tells you how many uncountable numbers are there in the basic right set that we start with that uncountable set that we will use right I am not going to let me just not write that down here, but.

So, it is when omega is countable right we always use the power set of this omega as the f so; that means, a lot f work is saved for us, we do not have to scratch our head you know in every particular say probability experience what is this f for what are these f sets to which I am supposed to assign probabilities, right which says am I not supposed to assign probabilities this you do not have to add confusion for the time being.

However, I have to warn you this statement cannot be used right for the mathematical reasons this statement will not work in the case of omega itself is uncountable and that is where we get into really slippery ground as far as the you know the theories go right luckily they worked out you know what we need to use and we will just stick to that I will come to that point later in the course, but it is all being worked out for us right that is the nice thing about it.

We do not have to get into any many more complications that we need to, but this statement will not hold what will hold we will say that for them, right. So, since I have a few more minutes; let me continue every minute is precious remember right. So, are you so far does anyone have any questions you can please ask questions you know does not matter if you are on camera or not right as I said camera is going to focus on you it is only focusing on me; are you so far what is then probability quantitatively.

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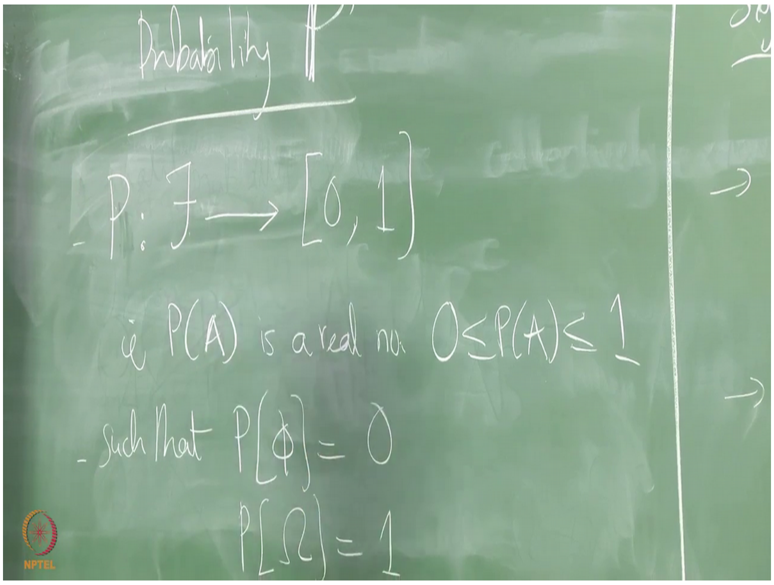

It is probability function P which we will write remember this P is going to be sometimes its written in over 2 vertical lines, but its little difficult to keep carrying it on. So, we will just mostly stick to one vertical line for the P right, but I just wanted to when the first time, I am writing in this course right to emphasize the importance of this function I am writing it like that.

So, fundamentally this P is a function that maps you takes you from this right set f to what is it; it has the P itself has a domain and range right you all know about domains and ranges of functions right. So, domain of P is this f which means every set in f has to be assigned a number and this number is in the closed interval 0 2 0 1. So, that is basically probability right in a just stripped of all right all other you know these are to connections or whatever right, it is just a number at this point right.

Now, of course, it comes vey it is a very loaded number, it is not just a simple number, but right this is what you have to first understand. So, I P A right the probability of an element of an event A which belongs to f is real number between 0 and one such that again we have rules for assigning probabilities right. So, what are the rules that we have for assigning Ps right P of phi is 0 P of omega is one. Now since we want probabilities to reflect some useful measureable quantity in the real world or real approximation or estimator of some probability of some number in the real world right. So, an impossible event will never occur right phi as I said right in probability is impossible event right the null event or the impossible event. So, that always has to have 0 probabilities.

Now, why are we giving the set omega the universal set by convention we give it in probability right although the theory will just has will work reasonable work I am sure it can be reworked if you have any positive number as the probability of omega, but there is a important physical reason for making that maximum number equal to one you want it to be some finite number right n and there is a lot of intuitive sense right in in I mean a lot of things fall in place if you make this equal to one. So, that is the universally used convention which we are going to; obviously, follow here right.

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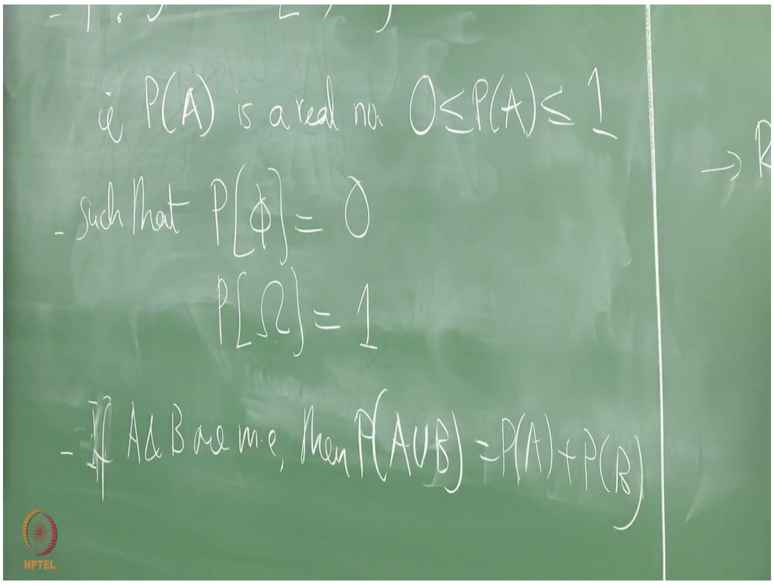

So, therefore, no probabilities can exceed one by definition right and then the other important rule is that right we will just say it and close for today if right if A and B are that is a 2 sets in f are exclusive are m e, then of A union B the union that is the probability that is attached to the union must be what the sum of right. So, this these are the basic rules; we will expand on this tomorrow right we are running out of time today. So, we will stop here.