Probability Foundations for Electrical Engineers
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Lecture – 47
pdf and CDF of Continuous random variables

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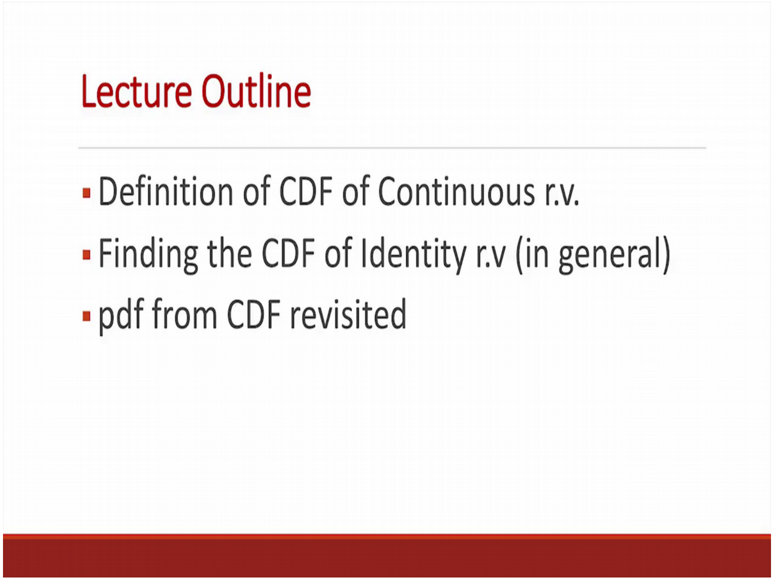

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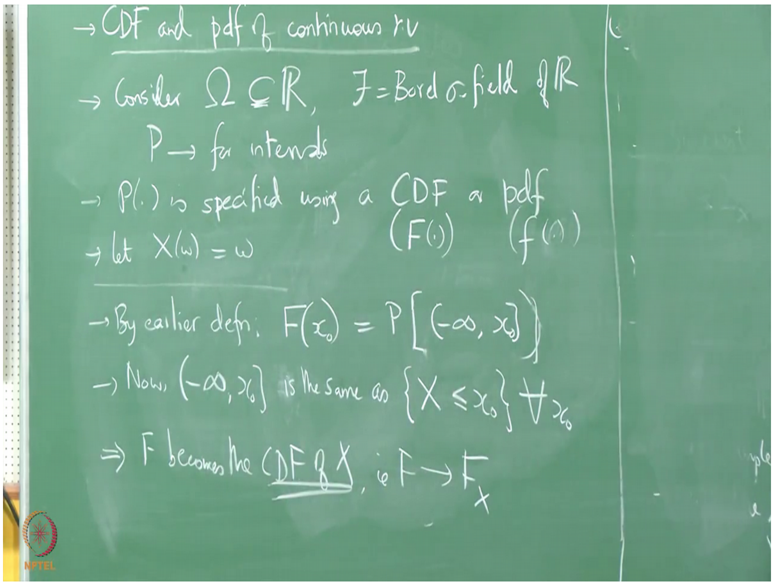

Now, the most important point which I want to get to here is the CDF and pdf of continuous random variables. Earlier we have defined the CDF and pdf as way in which to you of assigning a probability measure to an experiment whose sample space was a portion of the real line. But with a more commonly encountered as properties of random variables and so we have to make the connection between what we said in the previous lectures and our definition now of continuous random variables. Just remember that continuous random variable means omega x is uncountable that is all and so we still we really do not know at this point what kind of probability measure this continuous random variable is going to inherent.

In fact, we cannot say beforehand because we have not talked about the probability in all the only up to now only defined, how to assign values of random variable right. So, we have we have to now bring the idea probability and of course, we for that we need the CDF and the pdf which is the I mean in the case of omega equal to also let us start with the simplest example. Consider the sample space omega and some subset of R which is again the same thing as here which is well I am not necessarily going to restrict myself spinning point could be any experiment where omega is some subset of R right.

And then of course, I have the script F being the Borel sigma, Borel sigma field on whether are particular subset of or you know you can take it to be all set of intervals does not matter whether R or sigma it does not matter anyway. So, then you have the probability measure P for intervals. This is what we spend a lot of time doing in the couple of lectures back. So, now, we have the probability measure on this you know defined on intervals of capital omega and let us say P is specified using either CDF or pdf, does not matter.

In other words we have of our entire complement of omega F and P which is the basic probability experiment that is central to any construction of probability. So, we have omega f p and I am not is specifying which CDF or pdf to use as we can see we are we can use many different types of pdf or CDF or pdf models and all of them will be consistent and therefore, at this point we are saying the we are not, we are say we are letting things be the as general as possible.

So, now we will bring the random variable. So, let again the simple we let us look at the simplest case of x of omega equal to omega. So, together with this capital omega and this x we have defined the random variable x such that which is exactly what I wrote out here and now we want to now look you know attach this CDF this CDF. Now, we have I need to bring in the notation here for CDF. So, CDF we will we will denote in general with the script capital F and the pdf with the small F. So, these are functions as we as not putting the name of an argument here.

So, the CDF has is denoted by this capital F. So, what is this CDF? So, basically as we pointed out earlier the interval minus infinity. So, the CDF by earlier definition the probability or the value of the CDF at any, for any real argument is basically the probability of this interval right, going go up to and including the value of the number that we pick. So, if we pick some to make it even clearer I will, let me some x naught here if you think that will make will, if it will be of some help like we can picture the CDF defined at the num right, at the value at the x naught to be the probability that we are assigning to this interval which is a set of all real numbers smaller than or no bigger than x naught.

So, now this is the definition of the CDF regardless of without before we bring brought in this random variable. Now, we bring in this random variable and it is clear that this interval now, this interval minus infinity to x naught is basically identical to this to the event that or let me not put equals is a same as or is identical to which is the same as the event that the random variable takes values no bigger than no larger than x naught because of this way of defining the random variable right.

So, you get some number between minus x sorry minus infinity and x naught that this is the value taken by the random variable and therefore, the random variable has to be no bigger than x naught. So, if you define this for any in experiment where the sample space is some portion is the x. So, you define the x omega equal to omega then this is identical to this are the same, these two are the same events for all values of x naught, any real number x naught you pick then these to have to be the same. And therefore, the probability of this of this event is the same as the now you can go from the, you can attach the random variable to the CDF right.

So, therefore, the F becomes the CDF of X, the F becomes F with the subscript X. So, here of course, I have given primacy to the CDF because the CDF is as we as I will discuss in a couple of minutes from now is a more mathematical general way of assigning properties and the pdf, but still right. So, if this may the sound like a kind of the simplistic argument, but it is really not I mean, it is because you know this if this it is based on this fundamental idea or this basic idea that you define you understand how an arbitrary define CDF like this could be attached to a random variable and therefore, once you attach the CDF you can differentiate it and then and the density function will follow automatically assuming that when you are CDF is differentiable or a I mean this and we have said that continuous functions also quote and unquote can be differentiated; so we will.

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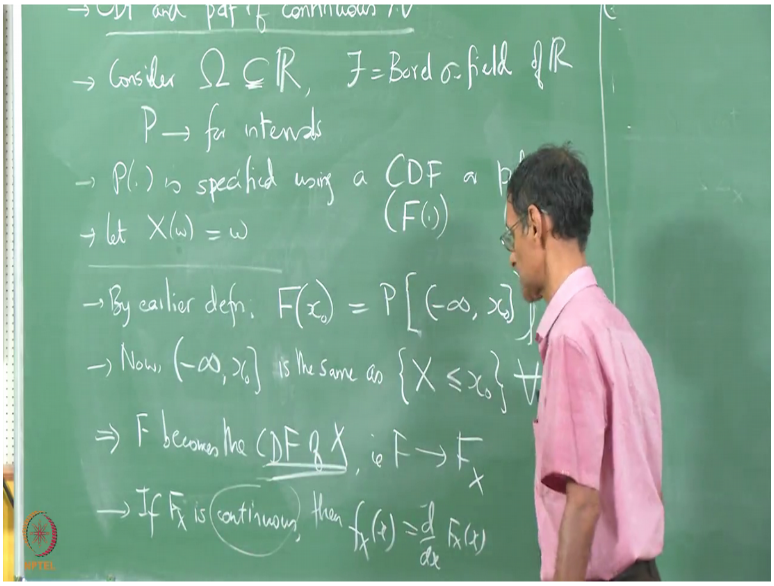

So, ref is even (Refer Time: 08:46), if capital F X is continuous then small f x of x that is a CDF pdf of x becomes that they we are going to in the sense that here we are not going to worry about the differentiability angle so much is we are already said in some previous lecture. Continuity; I mean continuous us and reasonably smooth the sense that the number of kings in capital F is going to be are going to be small right.

So, therefore, the problematic points in differentiability or small capital F s are not going to matter and basically more or less continue continuity of F will be enough to ensure the differentiability of F in most engineering cases right. So, we do not strictly in you know we should include any continuous case even though it may not be strictly differentiable in the mathematical sense. So, at the points where you cannot assign the unique the unique derivative as we said earlier we can define, you can assign a consistently you can take either the left derivative or the derivative it does not really matter.

So, this is a simple case of, this x of omega equal to omega.

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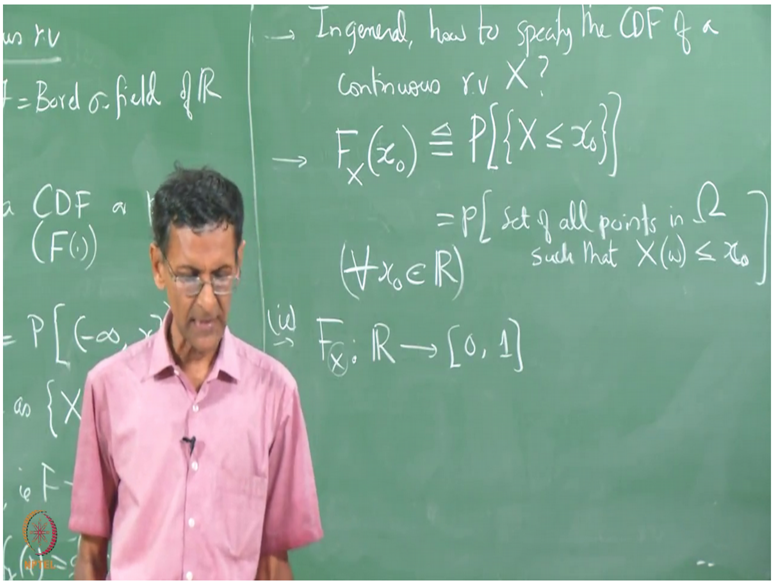

What do we do in a more complicated case in general right, how to, this is what we should look at how do we calculate or specify the CDF of a continuous random variable x. Why do (Refer Time: 10:36) this that I am saying CDF because as I said conceptually the CDF is more important than the pdf all he computationally is reverse. So, pdf is more important than this CDF right.

So, how do you define in the general case the CDF of a continuous random variable? So, basically you do it using the definition of a CDF, this is F, the c f of x which is written by the denoted by capital F subscript X at any, for any real argument x naught being any real argument is basically define to be them probability of this event, x. There is a defined to be the probability that X takes the value no bigger than x naught. So, this basically is nothing, but the probability of the set of all points of omega now because I am talking about general.

So, this all points are omega that this is general omega such that what X of omega, capital X of small omega there is measurement or there is a mapping or measurement on small omega is no bigger than x naught. So, this be, and this probability is not in general and easy thing to calculate. It only becomes an easy thing to calculate in some very simples cases like this. In general if you look at some complicated space and you define a complicated function capital X this is not an easy calculation to do in practice and therefore, we cannot talk about a general calculation of this kind because we do not even know how to assign probabilities for example, to arbitrary sets to a arbitrary omegas in R power k whatever.

But anyway, we have only you know take one thing and a time right. So, we will make the definition and then apply it as we go along that is a simplest thing. So, only thing we can really do. We can say, we will we will keep this is an operation definition of the CDF and you know of this x right. So, essentially we are saying that the CDF of the random variable x at any, for any x naught any real number x naught this is said of all points in the space such that the measurement, capital X now is a basically a measurement on the sample point omega real measurement on the sample point omega is no bigger than the number we have selected which is namely this x naught. So, if you do it we are going to do this. Let me write it here for all x naught in which is for all real numbers x naught.

In other words this capital F as we said earlier is basically is a mapping from the entire real line to the interval 0 1, because and it always takes values only between 0 1 being a being probability we have said this several times earlier, but anyways. Now, the only thing we are doing is we are attaching this subscript to the F. So, this is then, this is general procedure for doing this. And now let us say see we, let us take the case of the, let us take the case of this spinning pointer go back to the spinning pointer example and see how we can mathematically go through this where it is easy enough do to right. So, we do not have to break our head on how to calculate this probability.

In general I have to warn you that this is not an easy calculation you doing general, but only in some simple cases it is easy. So, we will pick an easy example and do this calculation and then try to we know talk about a little bit, but this not much more we can say at this point because this becomes highly specific to the particular experiment under consideration.