Probability Foundations for Electrical Engineers
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Lecture - 46
Continuous Random Variables

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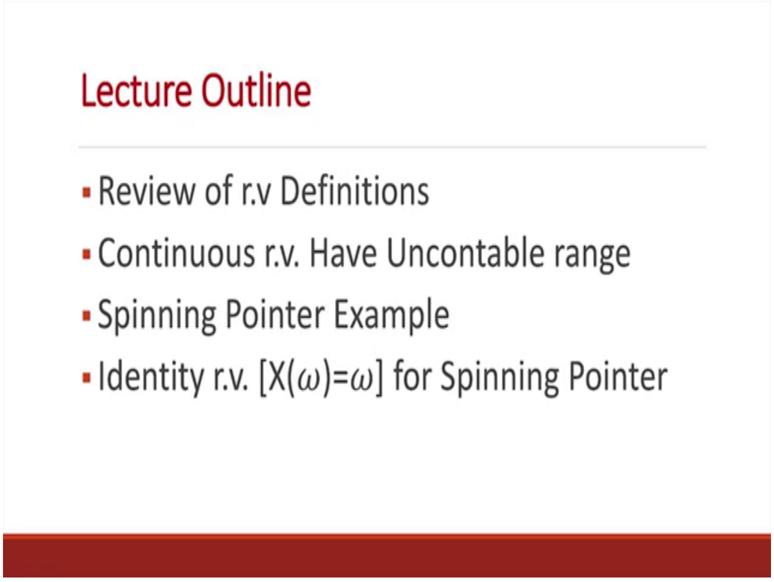

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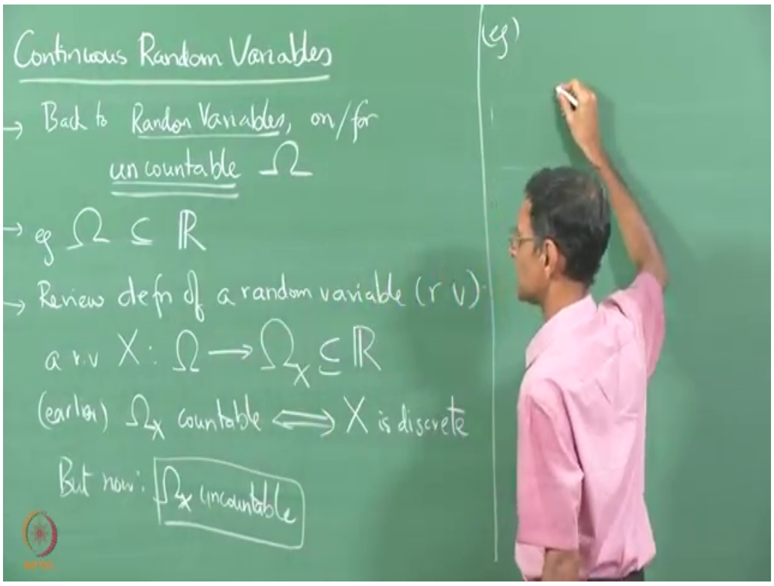

Now, we are going to come back to the topic of random variables, but we are now going to see what you know how these random variables behave when they are defined on continuous or uncountable sample space or let us say on or for the case of uncountable omega. Such as omega itself being a subset of R there is a real line, so this is the simplest such uncountable sample space that we have right.

So, to define a random variable we basically need a functional mapping between every sample point and the real line itself. So, let us review the definition of random variable. If you review the definition of a random variable what is it well let us use r v instead of saying random variable all the time right.

So, a random variable x is basically a mapping between the sample space and its range space at the set of values that x takes which is a subset of the real line. So, if this omega x is kind is countable which is what was the situation in all the earlier examples then we call an x is a discrete random variable right. So, earlier we had the case of omega x countable which is identical to the case of discrete random variables. In other words x can take only a countable number of values such as 0 1 2 3 whatever we have seen enough examples of that.

But now I am going to move on to the situation where omega x is uncountable which is certainly possible when the underlying space is a portion of the real line right. So, so omega x is uncountable means that you are basically dealing with a fundamentally different situation and a discrete random variable. So, but now, so let me write here now your omega x is uncountable the set of values that x can take is basically not countable and therefore, we need to deal with this situation carefully.

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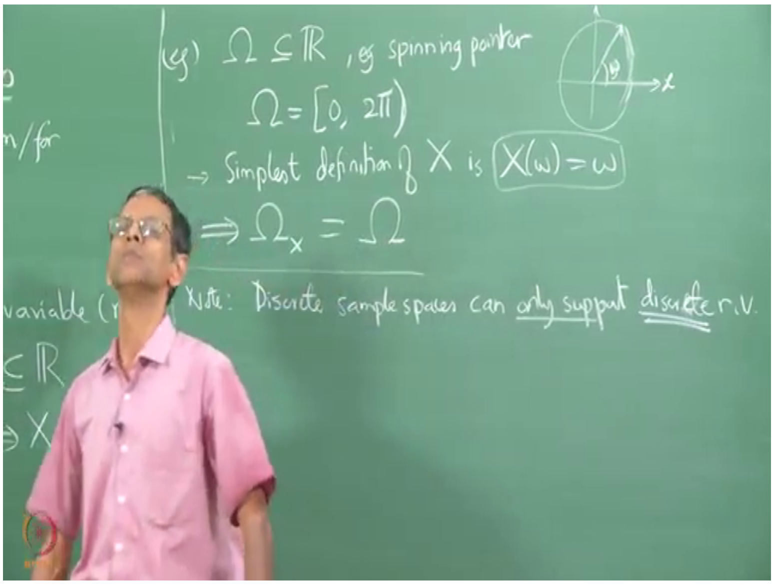

Simplest example I said it is the case of the sample space itself being sums up some portion of the real line again a spinning pointer. This example comes up a lot in our study of random variables because it is such a simple, but very often and commonly encountered experiment where you spin a pointer and it comes to rest at some, the pointer comes to rest at any arbitrary position on the unit circle. The radius, the spot point, the radius of the pointer is some let us say some number could be even normally taken to be unity does not matter all and the pointer comes to rest at any point on the circle and the angle that it makes, let us call this angle as omega.

The angle that it makes with the positive x axis or some predefined axis is defined to be the output of the experiment and the collection of all these angles is this set the continuous or the uncountable set capital omega. So, for the capital omega for the spinning pointer case capital omega is basically the set of all number real numbers between 0 and 2 pi and for mathematical consistency we will say that 2 pi is basically the same as 0 so we will use the not included notation for 2 pi, but anybody close 0, it is just, I just convention. So, we spin the pointer and it comes to rest it sum any angle, angle can be here in the first quadrant or a second or third or fourth basically in a angle between 0 360 degrees or 0 and 2 pi in radians.

So, for this spinning pointer we have defined the omega, but we have not defined the random variable x. So, now, the simplest definition of x of random variable x in such situations is to take x to be the output of the experiment itself. This is a very useful important identity mapping which essentially says that the angle at which the pointer comes to rest is the out value of taking by the variable directly. So, there is no processing of that angle the angle, the angle is directly assigned try to the value of the random variable. So, what is; in this case whenever you take x of omega equal to omega then automatically the set of values taken by x is basically equal to the sample space itself which is obviously uncountable whenever you are talking of any interval of the of the real line.

So, and it need not be necessarily only spinning pointer I could have a linear experiment where I could be measuring the distance to from some origin and that could be measured in an uncountable fashion instead of in a discrete manner. And if I define this x of omega to be omega then no automatically the set of values taken by x becomes the same as the sample space itself ok.

So, now earlier we use the concept of probability mass function to assign probabilities to x, but now we cannot do that because x takes an uncountable number of values and so we have to use the earlier, the concept of CDF and pdf that we did in the earlier lectures and we have to propagate them or move them to this setting or this case of this x. But the important thing that we should realize is that a discrete space such as tossing of a die or putting balls into bins those discrete examples cannot support or discrete sample spaces cannot support continuous random variables right. So, discrete sample space can only support a discrete random variable or discrete I can only support, in other words you cannot define continuous random variables with uncountable omega x for discrete spaces.

So, if you want to define a continuous random variable whose range is infinity, whose set of values that it takes is sorry not infinite uncountable then you must have a finer or a more or a more continuous sample space and that it is not and the simplest starting point for such spaces where is where the sample space the set of values are possible in the experiment here is some itself some portion of the real line. Then in that case if you are starting with this then you can automatically define the identity mapping itself a system as the most basic random variable on of that experiment and then this x is obviously, a continuous honorable continuous random variable.

Notice however, that this is not the only way that continuous random variables arise there are of course, many more complicated ways to define canoes and variables. For example if omega is bigger than R, let us say it is some portion of the plane or is a portion of 3D space, so omega can be bigger than R. R is when I say R without any superscript means up it means a real line and the R squared means plane. So, omega can be some subset of the plane.

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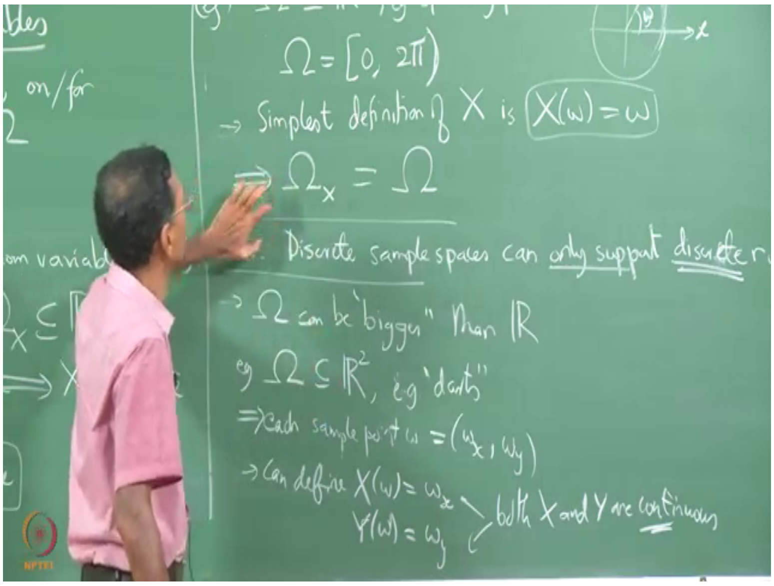

So, this situation where omega is some portion the plane is the classical setting for the darts experiment. The most important example here being darts. What is the darts experiment do? You say like some portion of the plane as it target then throw a dart at it from a distance and because of the physics is a problem the dart can land anywhere on the inside their target area and therefore, the hole all points in the target area become sample points of the experiment.

And there and again the number of such points is uncomfortable and therefore, random now you can easily define continuous random variables on such an experiment right. But now let us just take the case of you know let us not specify darts all we will just take the more general setting of omega being some subset of R square for in some way. I mean you could drop an object blindfolded onto a plane on the floor or something, you can do this in any number of ways not just darts anyway.

So, let us say that omega that the sample space of the experiment omega is, it is some portion of a square which means basically that each sample point omega has two components that is each outcome of the experiment sample point is basically the composed of an x coordinate and a y coordinate, provided you define the axis or once you define the axis which we presume that we have done by at the started experiment. So, there is an x coordinate to the point of the sample point and a y coordinate and together they define the sample the outcome of the experiment. So, both of these now are continuous numbers as opposed to here I have only one number of output by the experiment. So, here that you get two numbers omega x and omega y and therefore, we can define or mean by extension of this idea. So, we can define two random variables here x of omega as omega x, small x and y of omega as omega y.

So, this basically gives you two different random variables assuming that there is some that degree freedom in that that omega x and omega y can sort of vary independently of each other which is true. For example, in the darts case I mean there is no if you pick this as the x and y axis you throw it here, throw it here, throw it here, threw it here. There obviously, there is a there are two degrees of freedom and the value is taken by omega x do not tell you anything about the value omega y in a particular trial, anyway.

So, we have in such cases you have you can begin or it is clearly it is easy to see that you can define two random variables and both of them will be continuous. And both random variables specify this fundamental property of all random variables namely that there are mappings from the sample space, they are mapping from the sample space to some portion of the real line.

So, if you have for example, a more abstract setting where this could be some k dimensional subspace sorry space of R, some subset of R, I mean I am saying that it could be R power k for example, then you could have k components right. So, each of those components would be a different random variable x 1 to x k or you could take the maximum of the k components. So, that would be again a single random variable with uncountable sample space, because the maximum itself would be where you could not count the number of values taken by the maximum, it is again uncountable and so on and so forth.

There are many many ways in which continuous random variables can arise in practice and all of them are important in engineer. Most of them are important in engineering so we have to take some time to understand and appreciate and how they, why they are, why they arise and why it makes so much engineering sense to go through this exercise of defining them, because precise definition of random variable is very important in many engineering problems.

But unfortunately we do not have the time now to go deep into example, into examples. So, let me keep going with the mathematical consequence of having continuous random variables and later on maybe if time permits we can look at examples.