Probability Foundations for Electrical Engineers
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Lecture - 15
Part 2

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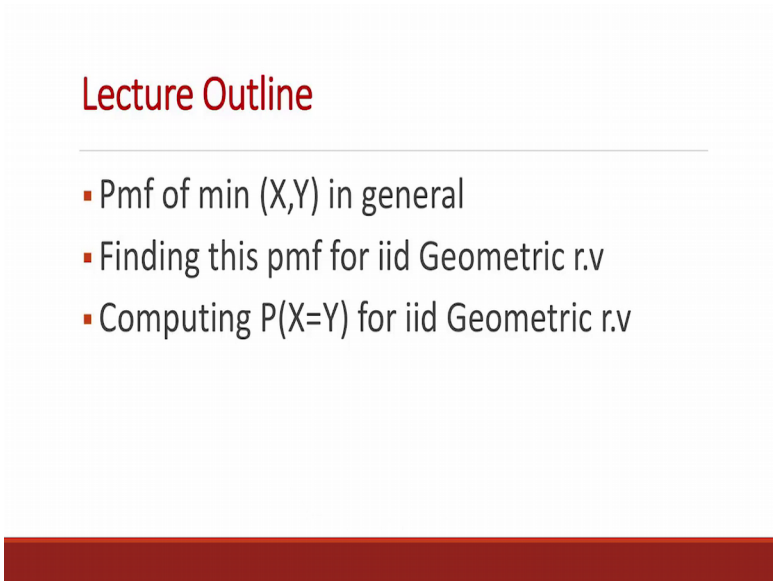

So, likewise for max if I sorry min, if I want to define W, I do not think I have use W.

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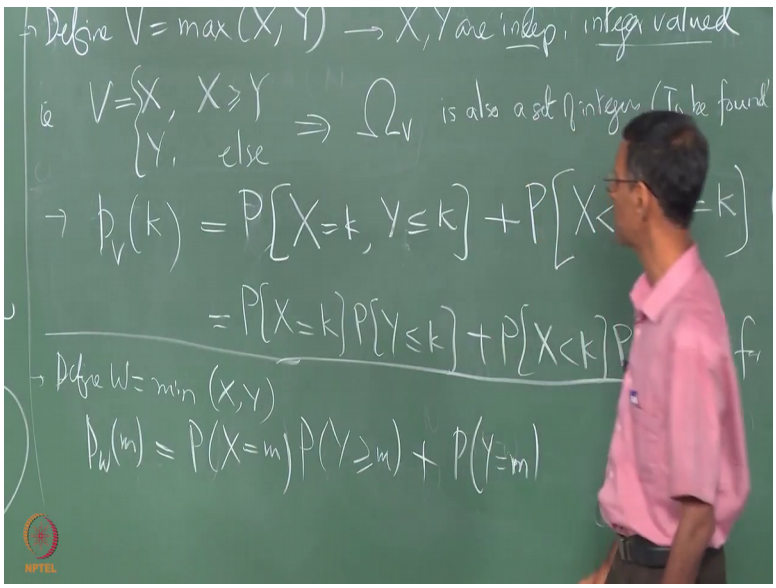

If I want to do min what do I do? I can do something very similar P W of m, since I do not like let me use a different index what is P, this is for the independent integer valued case it is (Refer Time: 00:50) I just write the result and we will explain it - P X equal to m P Y greater than equal to m plus P Y equal to m I am put Y equal to m what will I put here.

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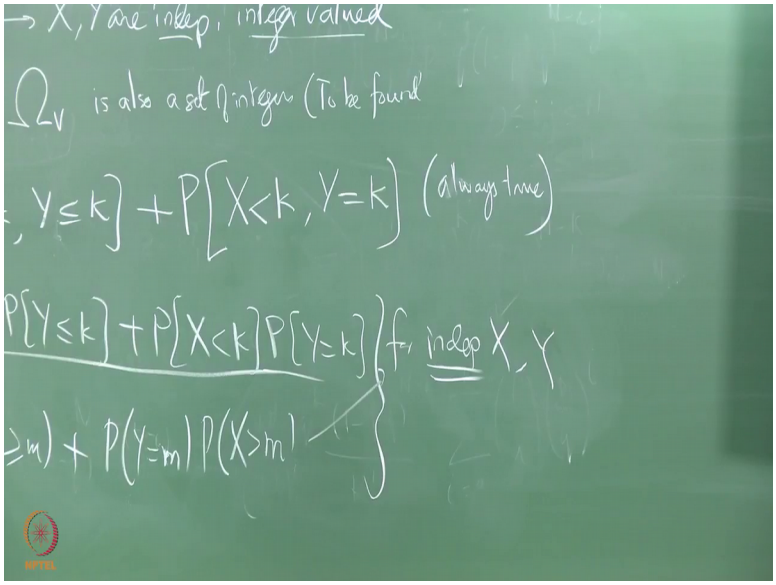

Student: (Refer Time: 01:06).

Strictly greater than m correct. So, this is again for independent X Y. Of course, if you want to remove the independence you will have to club this is joint event like with it here otherwise it is as long as their integer valued you can writes, you know you can write the equivalent which I did not write, but integer valued and dependent gives you something like this. And it turns out that for the geometric case this simplifies very in very interesting way there is when X and Y are both geometric, independent geometric let say with both are iid.

So, will just do that one calculation, I hope let me before proceeding further minimum is. So, how do I define minimum? W equals X for X smaller than equal to Y and its Y otherwise. So, once you know X and Y you know both the maximum X Y and the minimum X Y. Remember one of the X and Y have to be the maximum and other one has to be the minimum let us (Refer Time: 02:21) thing always keep in mind because when

you look at joint X and distribution max and min one of them has to be X the other has to be Y.

Unlike the case of X plus Y and X minus Y when we generate new values when we do in taking maximum minimum you are not generating some new numbers you are take the existing numbers and re relabeling them, so anyway.

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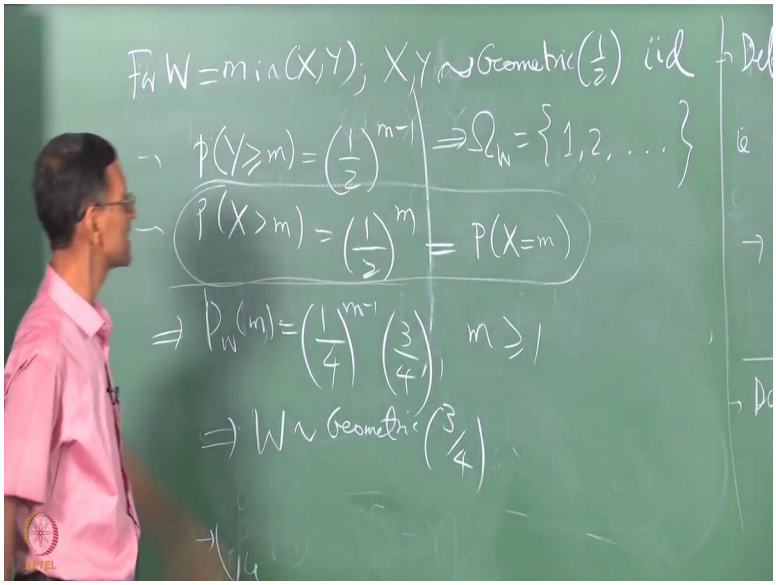

For W equal to min X Y let say X and Y are I mean say tilde geometric half iid in otherwise there independent and identically distributed parameter geometry with the geometric parameter half. Then you can show that what is P Y greater than equal to m will be half power m minus 1, what is P X greater than equal m probably half power m. These things come from the geometric distribution who have we did this earlier, did we or did we not; I am sure we did it.

This is the probability that there is no success up to the trial m minus 1; this is the probability that there is no success up to the trail m including m. So, that is why you got m minus 1 and here you got m here. Now if you put it all together maybe I should ask you to do it you do it and tell me what is P W of m. First of all what is omega W that we should do a first, omega W is what X can got from 1 to infinity not 0 to infinity. So, when geometric case we starting the trials (Refer Time: 04:23) from 1. So, omega can be 1 itself and there is no limit on how large omega can be potentially. So, you have to

allow all values all integer values for non negative integer values there, all positive integer values 1 2 up to.

So, what is this? If you plug it all in there what is a X equal to m. So, the probability that X equals m is half power m minus 1. So, if you do all this together you will find that P W of m you get this interesting expression this is half power sorry 1 by 4 to the power of m minus 1 into 3 by 4, for m greater than equal to 1. So, P remember P X equal to m is what is half, for the geometric case the probability of with geometry half geometric by the success parameter half what would be P X equal to m it will be half power m minus 1 into half, just half power m no sorry.

Let me, this is that, that is true is a. So, this is equal to the probability that X equals m interestingly for the half case you get is X equal to m and X greater than m has the same probability. This been no success up to event m up to event m, up to event up to m minus 1 that is when. So, the X equal to m means what? There is no success in the first m minus 1 trials. So, you get the probability of half power m for having the success at the ampoule or you get half power m for succeeding in the future.

So, say the geometric (Refer Time: 06:21) has some very interesting properties like this also, these to have the same for that of course, you need half it though it does not work for any P as you can check. So, this is this and similarly P Y equal to m is also because here iid P Y equal to m is also exactly half parameter. So, if you manipulate you get this what is this pmf?

Student: Geometric.

Geometric; with what success parameter?

Student: 3 by 4.

It is 3 by 4. What is intrusion behind this? The minimum of 2 geometry in the independent geometrics iid in this case half is, when, if you toss 2 coin separately and you count the number of trials did you get the first head. What is W going to be? W is going to be the minimum of the 2 counts.

So, it turns out that W equal to m will be getting X equal to m or Y equal to m I mean the first one anyway. So, I cannot think and talk at the same time. So, let me, so you can go

back and think about this, this is makes perfect sense intuitively let me not try do find words to explain what I mean let, but please go and think about it; some. So, (Refer Time: 07:55). Basically 3 by 4 is the probability is what, is what probability? The probability of getting a head at the any toss either X or Y is it not. Supposing you look at the probability now I can say correctly if you toss 2 coin 2 independent coins fair coins what is the probability of getting a head here or here, it is 3 by 4 is it not; that is the intrusion I am asking you to get.

So, the minimum is, when do you say the W takes a value 3 you get a head here or head here first one. So, that experiment will succeed with probability 3 by 4 now, it is not going to when you are tossing 2 coins and you are looking head here or a head here then if it you get neither then you toss one more time; so obviously, that is geometric the composite (Refer Time: 08:54) experiment is geometric with probability 3 by 4 where you will only looking at head here or head here, that is what is reflected in this.

So, again and again let me emphasize you all these probability calculations wherever possible they must be capable of explain being explain you know intuitively of course, not everything is possible is intuitively expandable, but certainly this is. This is ok, any questions on this.

So, we were successfully manipulated a few things. So, far let me with this I am pretty much done with whatever I wanted to say in for discrete random variable, but we do have a few minutes in today's class. So, what I am going to wind up with this bit known of one more manipulation regarding no, let me also may be this there is a manipulation here this the same case of X and Y are being independent geometric iid half.

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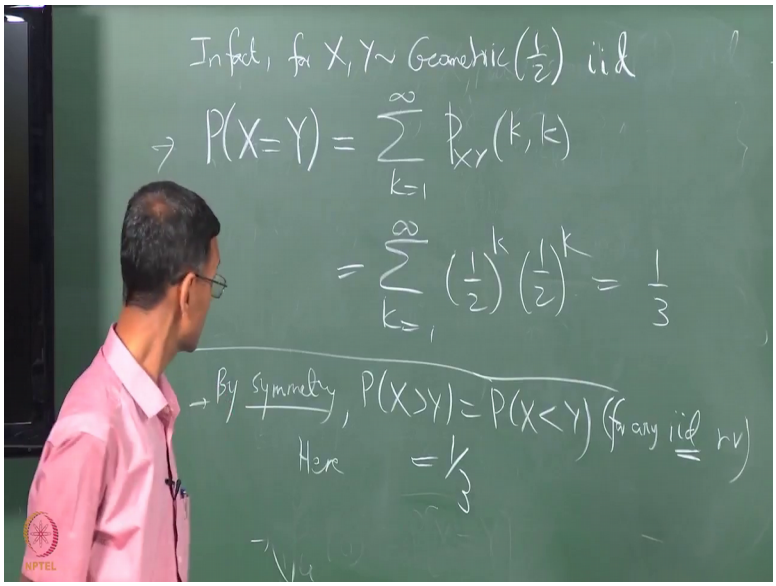

I wish I am derives all this, but anyway this geometric half iid there has some very interesting probabilities. So, what is the probability that X equal to Y? If somebody ask you this question what would you say, how do you this write this out.

Student: (Refer Time: 10:53) half.

This is X equals Y. This is not a geometric you know we are knowing looking at an event here we are not looking at a random variable in the sense you can write an indicated random variable for this, but that does not help you find the probability. So, you have to actually find it.

So, this will be I have to take this P X Y of k k and sum over X equal to k, Y equal to k. So, what values of k X and Y, what values of k do I need I go from 1 to infinity? So, this is physically you are tossing these fair coins 2 of them and you are looking at having the same number of trials in both camps, one person tossing in this corner of the room the other person tossing that corner of the room and you are looking at the event that the same number of trials is required to get a head.

What is this? This is a probability X equal to k Y equal to k because they are independent there will factor in somebody actually said the answer it is 1 by 3, fine. What is the probability from this can you infer that X is going to be greater than Y and less than Y? 1 by 3, 1 by 3 remember I said symmetric an always be use to tell you what is, when you

have iid random variables probability X greater than Y will be equal to probability X less than Y.

So, here remember I am not using equals anywhere X great strictly greater than Y and X strictly less than Y and I am not unless we there will be leaving on the equality because equality is as I re said earlier it is not cannot added to either side then you destroy the symmetry.

So, here, all 3 events who have the same probability in this case which is which very specific to geometric half, but in generally if somebody ask you to find this X probability of X greater than Y should we able to said it up from first principles, you add over the triangle the open headed triangle where X is greater than Y. Assuming X is, X and Y are both you know integer know positive integer value whatever there will be a portion of the X Y plane that will correspond to the event X strictly greater than Y you have to add up the probabilities of all those points in that portion and now going to.