Probability Foundations for Electrical Engineers
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Lecture - 14
Part 1

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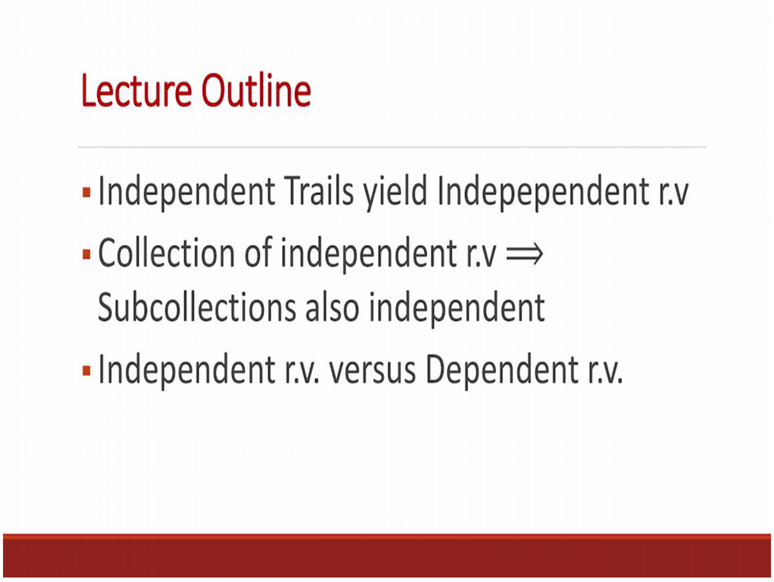

Yesterday, we were talking about this this topic right the independence of n random and move aside. I thought it is a good time to be a little more formal about it or go a little more into depth in this.

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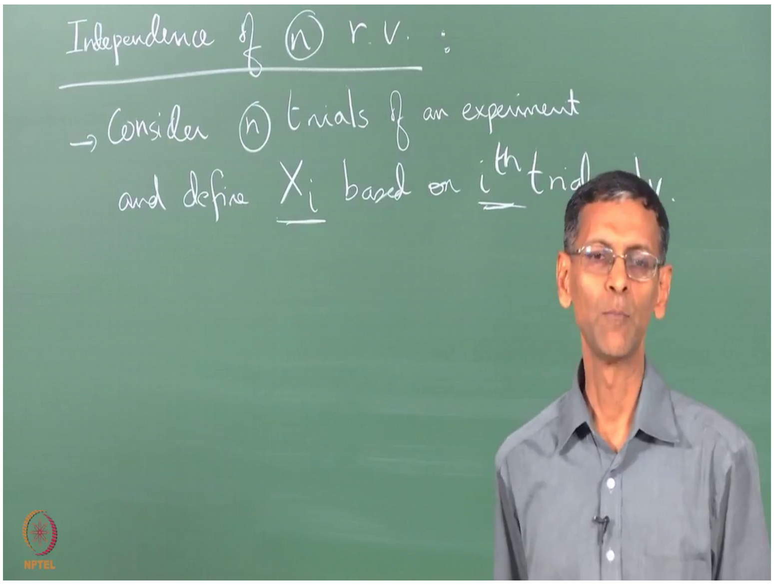

So, what I am asking you to do is consider n trials of some experiment and define the random variable X i pay only for the n i-th trial right you in other words i do not want this X i to be defined as the sum of all the numerical observations that you may make from different trials or anything of that sort right the i-th each trial is associated with one random variable if you toss and i are n times the natural thing to say will be the right i X i will be the outcome on the i-th trial for example, but I write; so I want to define X i only for i-th trial.

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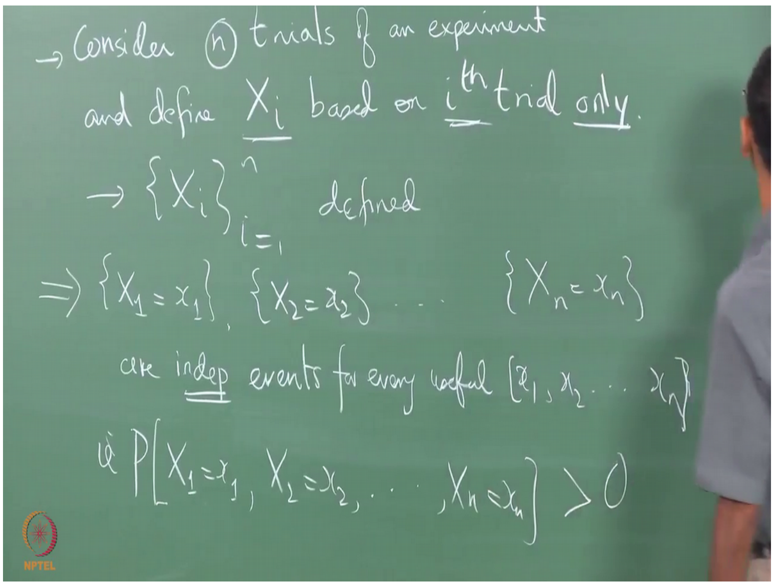

Therefore, I have defined X i all the way from i equal to 1 to n right. So, these n random variables what they have they actually turns out that they satisfy a very interesting property in that because the trials are independent when I say n trials automatically the n trials are independent. So, the events X 1 if you look at these events now you might see ask you. Sir, I mean ask the question why do we repeat X ; this X 1 is just for notation i could have written; write a b c d up to n if i want it, but that way I do not get enough a b c.

So, it is the normal thing to do the index these things also and just to keep the confusion i as small as possible whatever that you repeat the X , but write it as lowercase X here for the object for the values that they take right that is what we have been always been doing. So, we have these events X 2 equal to X 2. So, these n events these are all independent for what choice of vector small x for any vector for any for every right useful or every i useful what I have to say what I mean by useful choice of X 1 X 2 X n.

So, you have n an n vector that you are going to look at what do I mean again by useful this probability joint probability has to be positive I do not want this joint probability to be zero this is a joint probability that I want to choose it x; small x is such that this joint probability is positive and we have.

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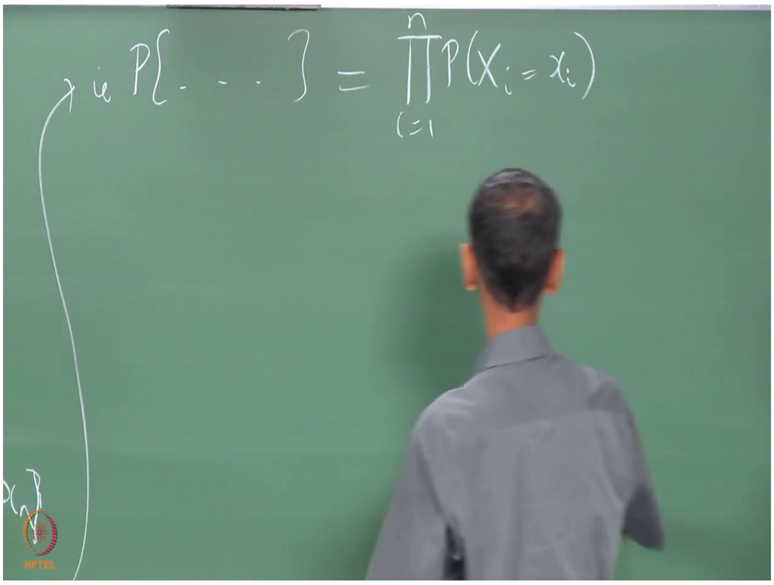

That quantity being equal to the pi product of X i the individual the problem a product of the probabilities of the individual events remember I said X i is based only on the i-th trial.

So, this small x i is a number which comes out only are they i-th trial on nothing else not only this. So, and whatever what do we mean by n events being independent this is; obviously, not the only condition right we have to have this condition being satisfied for any sub collection which is also true because when you have n trials you can; obviously, close your eyes to the trials you do not want.

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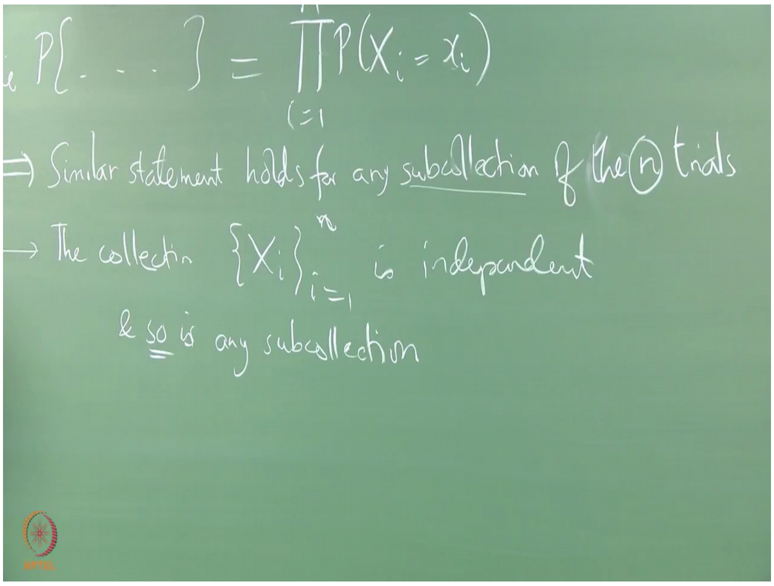

So, a similar statement or equivalent statement holds for any sub collection of the n trials. So, may all of this follows just from d n a and independent right if you have any independent runs you can always disregard some or as many of them as you want wanted disregard you can focus your attention only on what you want to look at.

So, if I want to look at only the first 3, then I have these just the 3 and I get the product of the first 3 things and the small x i. Remember again for every useful such vector where the joint probability is positive. So, in the case of die rolling i; i should be able to substitute in the place of small x i if i said that X is i-th number I should be able to substitute any number from 1 to 6 out here and this joint probability will be equal to this product of these individual probabilities.

So, when this happens you say that. So, this is a very important special case of having any independent random variables. So, the collection you say that the collection X i; i equal to one to n is independent which means that any sub collection of this is also independent and so is any sub collection. So, here we are looking at all n of them you can also look at. So, write the English word. So, used to refer to you know copy this property on to any sub collection.

So, this is a very very important concept because it allows us to mathematically bring in this call you know repeated trials put a mathematical face on the repeated trial and say- you can define in random variables of each trial note that we do not have to straight away assume that the X is must have the same distribution. For example, I can say on the first trial I am taking the number itself the second trial I am going to take the square root of the number the third trial I am going to take the third root of the number so on and so forth I do not need not have the same X i; you mean the same definition for how I measure that I will write the random variable it could be different on each time, but in the most useful case you have exactly the same way obtaining the X i the way same way of defining right the capital X i the random variable itself as the number that comes out.

Therefore, in most cases the X is will have the same pmf.

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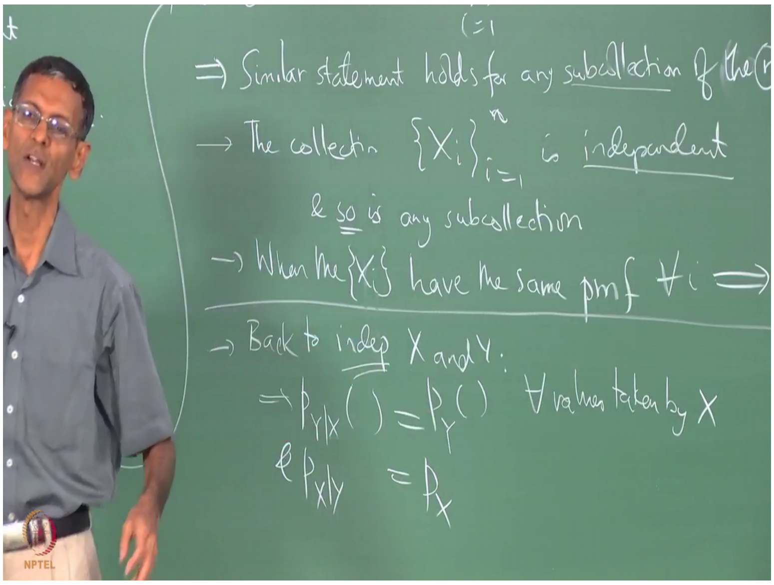

First we understand that they are independent then when the X is have the same pmf what do you get.

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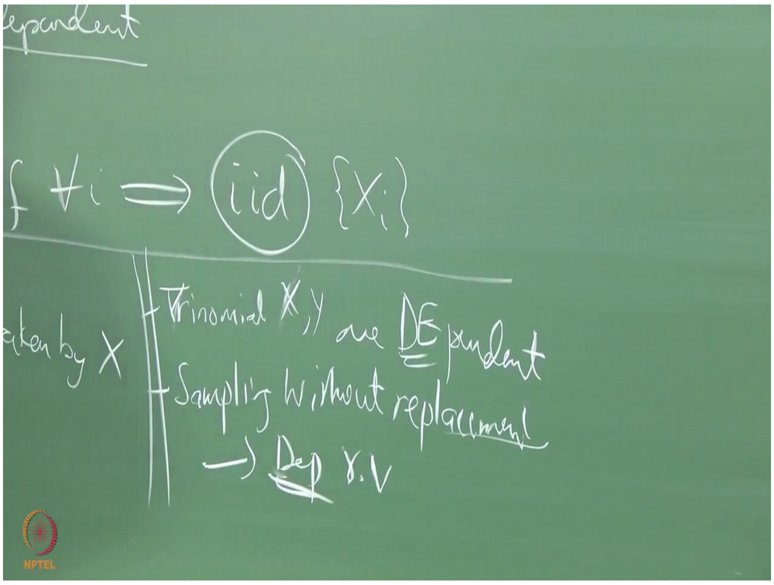

You get i i d the collection is in independent and identically distributed. So, this is again i stress this is an extremely important and useful contact concept and we will use it a lot when we look at asymptotic behaviours like the law of large numbers and things of that sort. So, I can what I am trying to say is I do not have to go through all this song and dance each time right to justify talking about n i i d random variables that is all I can just straight away start with that let me start with a collection of any i i d random variables immediately I can write I can go to that.

So, now let us continue with the 2 random variable case back to the 2 random variables X and y back to independent X just 2 random variable, if you have just the 2 random variables you know in this means that the product sorry there are joint is a protocol marginal. So, that also means that the conditional densities what are conditional pmf s p r y given X for example, will be what.

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Just the unconditional the marginal pmf for all values taken by X. Nowadays no matter what value you use for the for the conditioning quantity you get back the same unconditional pmf in general remember as we saw in the trinomial case if you put different values for X you get different pmf s no trinomial case please go and look at it more carefully right in each for each value of X equal to i the pmf was different it was not the same, but when you when they are independent you not only get the same value same kind of the same uncanny; I mean same pmf for every value of the conditioning X u and it is also equal to the unconditional pmf itself.

So, in other words well and truly observing X; X tells you absolutely nothing about y and vice versa since they are functions i am just writing this is a functional equality in some sense right the whole entire mass function is the same. So, omega X, in this case the omega y will be unchanged also observing X does not tell you does not increase or decrease omega y in any way it keeps it unchanged. So, it is not you get no change at all in the situation as far as y is concerned. Similarly here if you observe y you get no information about X. So, both if X and y are independent both of these will hold; obviously.

So, it is more than just simply writing joint as a product of marginals its lot is lot more going on that we have to we have to be aware of clearly they are independent I mean the dependent case what is the opposite of independent dependent. So, let me just say if it X n when X and y are the trinomial case for example, are remember they are de pendent without stress on that word on the syllable d independent versus de pendent.

So, independent random variables are more difficult to in the sense of being more mathematically complicated because a joint pmf is not obtainable from the marginals in the dependent case you have to do a lot more work to either get it numerically or model it. Mathematically you can just simply close your eyes and multiply the marginals and say this is a joint, not only that the random variables that you associate with sampling without replacement. They are typically dependent the first trial you get something then you throw it away, you do not add it back to the mix then you take out a second one in unless they write the colour is the whatever you are taking off is.

So, big that the first they are taking off the first one does not decide statistically change what is happening there right in general sampling without replacement gives you dependent random variables not independent like for example, the other case that we studied right of drawing to 2 numbers from 1 to n, so sampling without replacement right. So, it gives you dependent.

So, we can always shrink independent to i n DEP and depend to DEP of course, i i d is another very important acronym; we will be using a lot.