Probability Foundations for Electrical Engineers
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Lecture – 13
Part 2
Independent Random Variables

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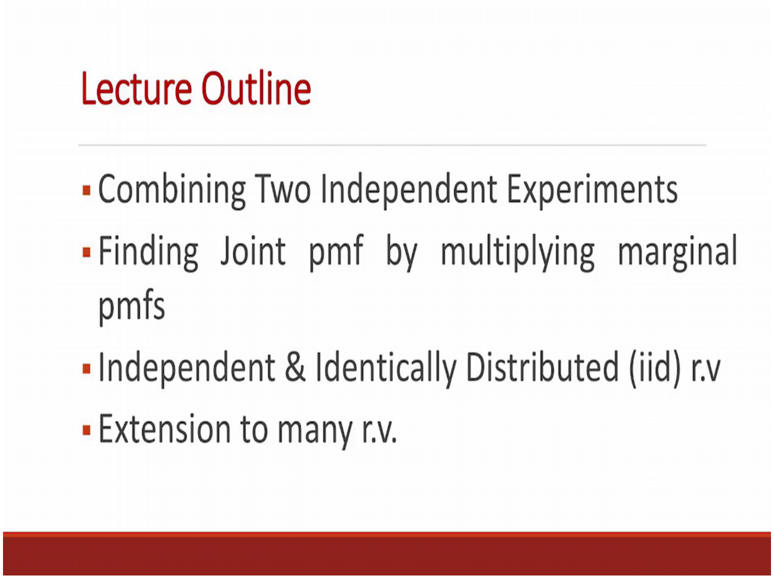

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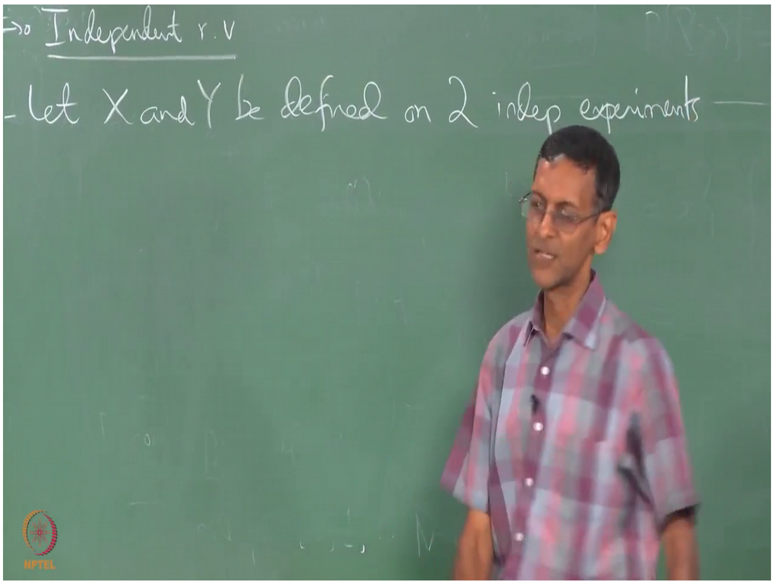

The next important topic is independent random variables, I want to conceptually consider 2 different situations here, the one is the most straight forward construction where you know that you have 2 independent experiments right. X is defined on that is when you start off you are already coming with this information that you have, 2 independent experiments X is defined on 1 and Y is defined on the other any 2 random variables that define that way right, no restriction on what X and Y should be. But they are this could be I do not think I need to explain this any further right, you can even consider 2 independent tosses of a coin or 2 independent rolls of a die as 2 different experiment in this for this case right. How do I in this case can I construct a joint PMF of X and Y answer is yes obviously, right.

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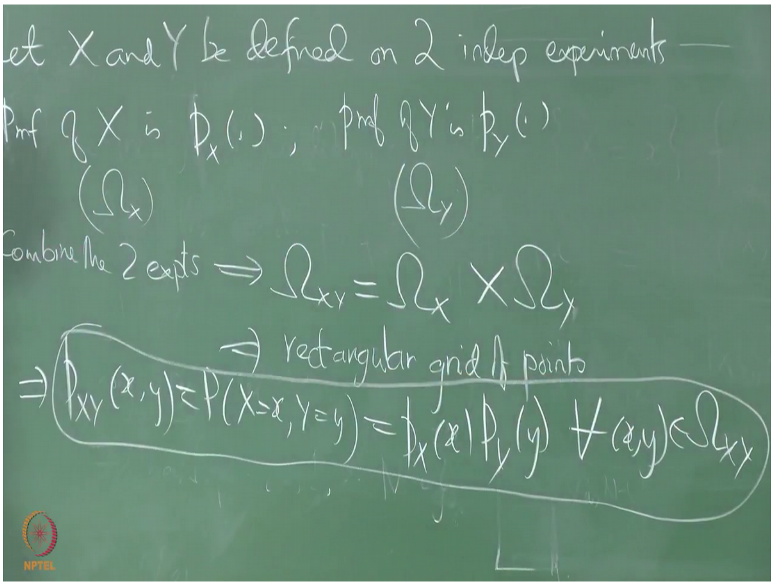

So, I we have PMF of x of course, this is this individual PMF this is p x and for y it is p y we have put dots here of course, I have omega x. So, I have an omega x here and I have omega y here also right, the set of values that x can vary over and y can vary over. So, now if I consider like mind experiment right which is doing both experiments together, I get the combined space omega x y is what? By combining the 2 experiments we get that gives you omega x y must be what, all always omega x cross omega y because in this case any the 1 experiment does not in any way constraint the other one because they are independent, so omega x y must be omega x cross omega y.

So, you have a nice rectangular grid of points in omega x y, remember this is where we are earlier we had what did we call it omega 1 cross omega 2 or some, when I first introduced composite experiments last some many classes ago we had some similar notation right, the first time we talked about it we used omega 1 cross omega 2 right remember right or I think so right. But here we are specifically looking at just combining these 2 random variables that is all; we are not interested in combining the experiments at a more abstract level.

So, if you look jointly at the values taken by the pair x y, they must take values only in this omega x y and that is all we are concerned about here. So, what is the joint probability here just going back to I mean intuition what do you think? So, this joint probability p x comma p x y, this joint probability which is basically nothing but p x equal to x, remember this joint probability is always the intersection or joint event x equal to x y equal to y and now we are going to drop on the fact that the 2 experiments are independent. So, the event x equal to x must be independent of the event y equal to y, because the experiments are independent for every small x and every small y.

So, this is most right fundamental equality that characterizes independent random variables right that is here we are actually going from the marginal to what? To the joint.

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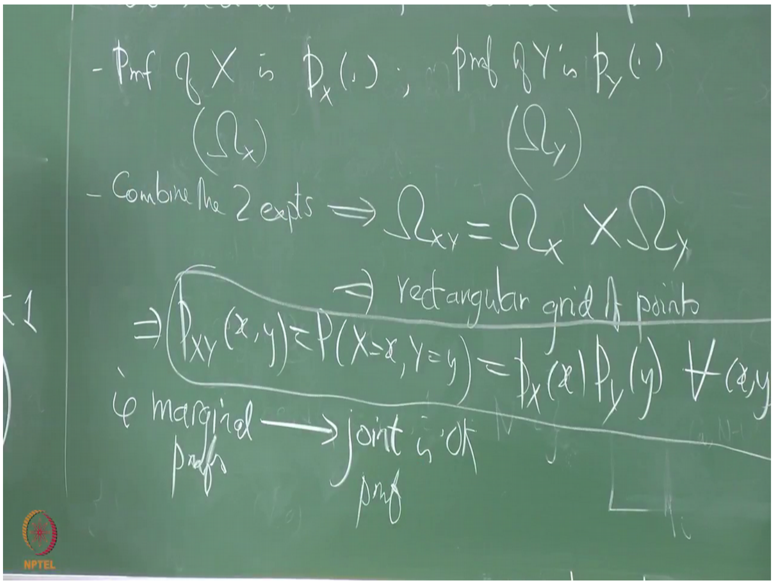

So, in the independent case you can always you can go from, you can always construct the joint as a pro act to the marginal, this is durable it is its fine the marginal PMF the joint PMF. So, why do I get this once again because this equality comes because x equal to x.

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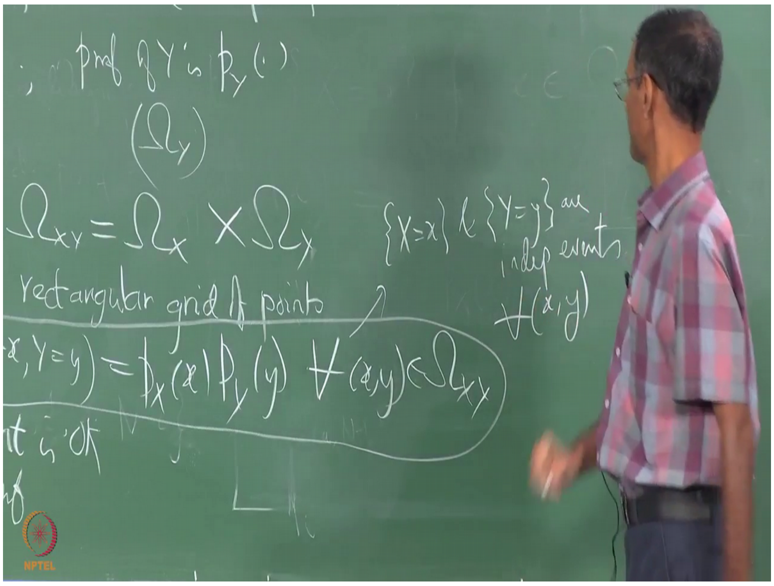

Again I write it because this is important are independent events, not just for some x and some y for all points x y, like in omega x y of course which is obvious; this is the only this is the most popular or the most commonly encountered situation right, we always combine independent random variables to form bigger spaces or independent experiments to form a bigger composite experiment. Supposing you find that you define 2 random variables x and y such that, your omega x y first of all is a nice grid and secondly you find that the joint PMF you calculated and you find that it is in fact, the product of the marginal this is the situation that we had first when I talked about independent of events right magically.

Some p a b happened to be p a times p a b like that you can also get p x y this equality can hold even without explicitly being forced, in other words you have the same mathematics whether by whatever means, whether you are forcing it or it just happens by itself. In either case it does not matter x and y are said to be independent random variables right. So, either you are independent by construction or they are independent somehow by. So, you do not care how they become independent.

So Sometimes it cares it turns out that x and y can be independent just by accident, just like the in independence of events a and b.

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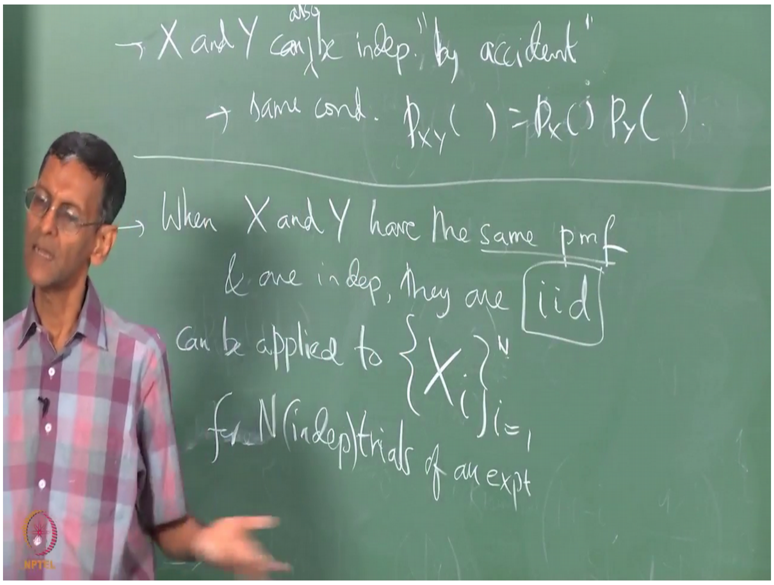

So we will look at some examples of this in the homework in problem set 3, instead if you do not like the word accident you can say by coincident or whatever you can use a different way to say it. So, this is independence by construction I can add it here.

So, what happens here same math same condition namely p x y equal to p x p y, I am not writing the arguments because I have already written it out here, main thing is that condition should hold for all points in the in omega x y, the maths is the same how you get it is different. Let me put it here can also be, but it is right it is useful to note this right, that you can get it when it does not it is not. So, obvious anyway we will come this is best taken up in a problem in a homework problem and not here.

So, I am I will leave it I just make the statement and leave it like that. Now inhabitant random variables it turns out right at the one of the most useful things that we have because, we will see later on how you can combine more than 2 you can combine. In fact, a countable number of them you can combine some finite number of then you are have right then it. So, maybe I should talk about this.

So, when x and y have the same PMF and are independent, what do you call them? supposing x and y are independent random variables; that means, that essentially this is satisfied no matter how and it turns out p x right, the p the function p x is equal to the function p y what do you call them x and y, they are called iid. Iid this very important that is the word I am looking for iid stands for what independent and identically distributed and clearly if you have n identical repeat repetition of an experiment.

You can define if you say x i Some random variable on the i th experiment, then you have n independent iid random variables and these kinds of things are extremely helpful right, when you look at n coin tosses and so on. So, we can apply this to more than two. So, I can have a sequence, so Xi can be where right, let us say not where n independent trials of course, they are always independent I am just saying it right, usually you might we interchangeably use uppercase and as lowercase and I just wrote let is leave this is an n uppercase.

So, if you say you have the n independent of the same experiment; obviously, the Xi have to be identically distributed they cannot be and if it is measuring the same if each of the x is measure the same quantity at each trial, obviously you have iid. In this case you can talk of the joint PMF of all of them right. So, I can have a vector of Xi, I have a vector random variables Xi also have a vector observations right.

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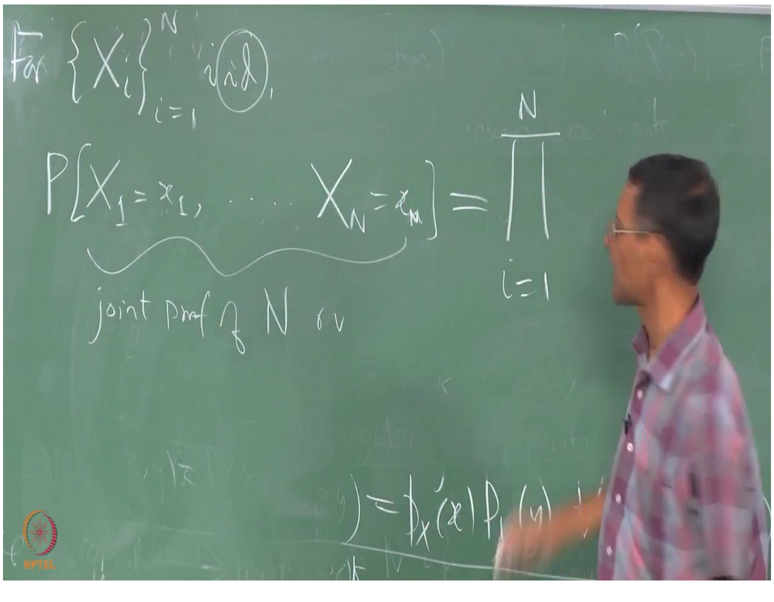

So, what is the most important equality we have here, for iid case that is when for this probability let me write this like this x 1 is sum x 1 dot, this joint this is the joint probability try doing joint PMF of n, this is a joint PMF n random variables n I should put. So, x 1 is x 1 through x n they are some random vector you pick as possible values right, that the whole vector random vector x can take.

So, what is this going to be equal to this is going to be how is this going to be? What this going to be equal to.

Student: Product.

This is a product of what. So, you put a pi notation here pi I equal to 1 to n of what?

Student: (Refer Time: 14:02).

I actually IID is not really required id is what is really required.

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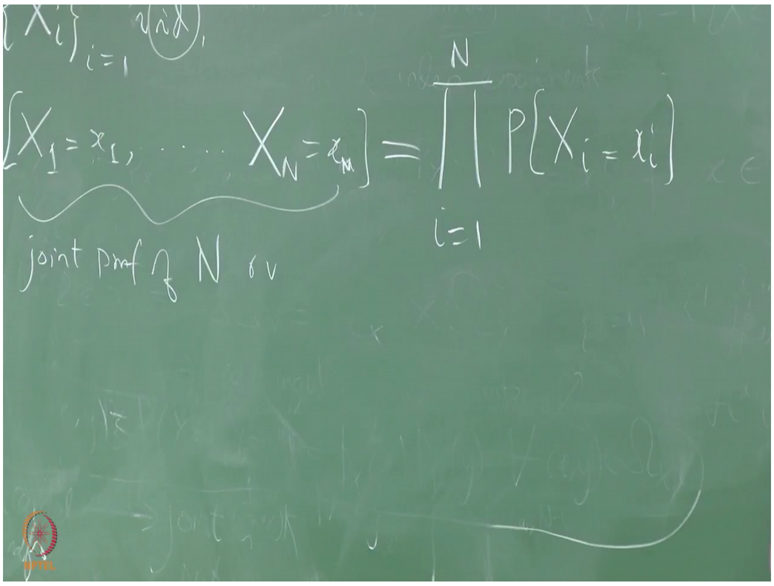

But this situation occurs quite a bit in the iid case there nothing wrong in writing iid, but what we are really as right using here is just the independence basically you have n independent random variables most commonly found in right. In the case of n trials in which case this; so each of them this is the Bernoulli random variable, for example measuring the problem or indicating the success or failure.

So, in that case this x 1 through x n would be any arbitrary n tuple or an n vector right, with zeroes and ones right and the probability of observing that which we have already done earlier without calling it a random vector or anything like that right, we just said 2 heads before 3 tails and so on. Now we are just saying; what is the probability of observing the random for 101011 whatever. So, you take it as a pro right it is going to be essentially nothing but a product of the individual probabilities, but this is just a formal way of writing this and obviously it extends to more than Bernoulli, it ended this condition applies to any connection of independent random variables Xi you can have die rules whatever right.

So the other important point is that if you know that all end and this is the collection of n independent random variable is any sub collection has to be also independent.

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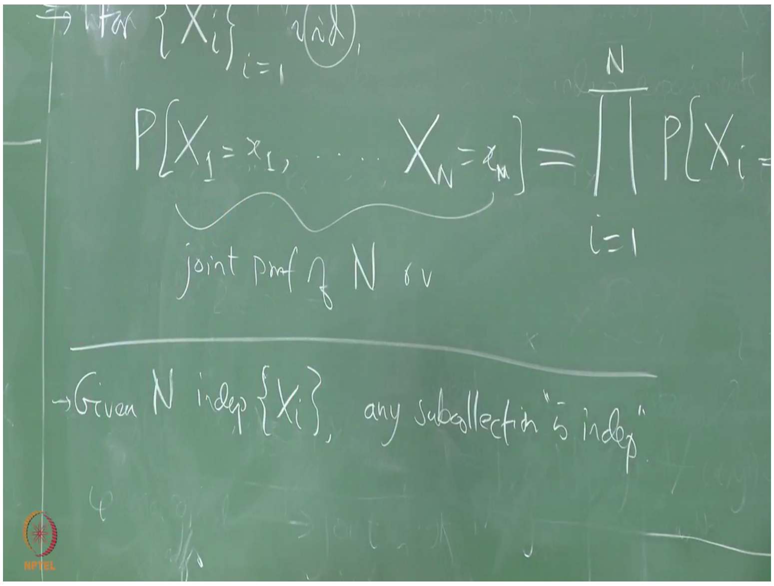

So, given n independent random variables not necessarily iid or when I say is independent, I mean it consists of any sub collection also contains independent random variables and every sub collection the point is any.