Probability Foundations for Electrical Engineers
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Lecture – 12
Part 1
Example of Joint PMF

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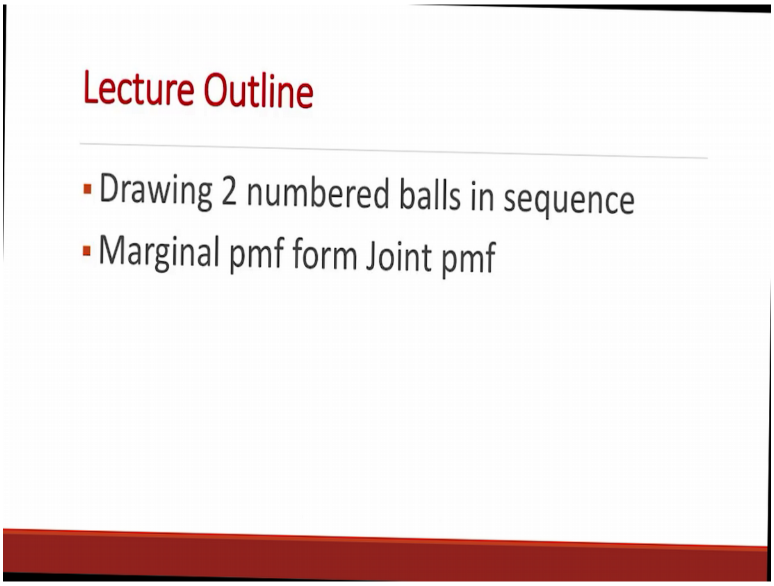

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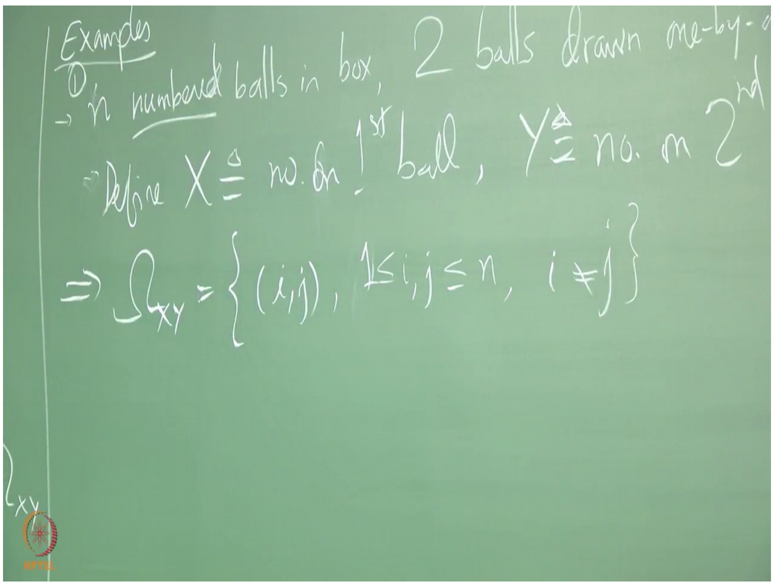

So, supposing you have a, let me see what notation did I use here. I guess I can stick with capital or let us me say, and supposing that n number let me say this is 1, 2 balls, right.

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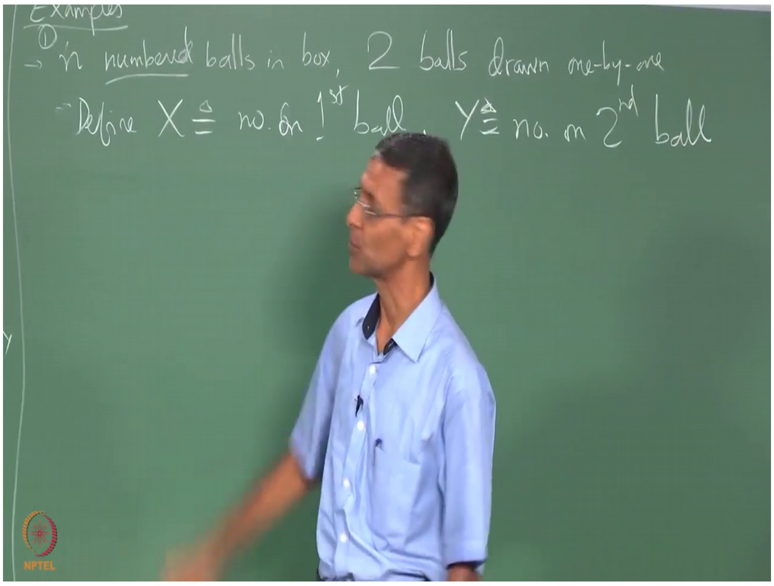

So, they have numbers, when I say n numbered balls it is automatic that the numbering is from one to n, right. Do not ask what numbers are there. One by one which is the same as without replacement you draw one by one. Now this is the experiment, simple. You are drawing them at random. Just putting your hand in there and pulling out 2 balls.

So, what are the 2 automatically definable random variables X for example, right. Number of the first ball and Y is a number of the second ball. Y by definition is a number the second ball. This is enough to ask, or you know apply all this all and all this rotation to this problem, right. This information is sufficient to be able to do it. So, first of all what are the, right. The experiment is not complete until you have drawn the second ball. So, obviously, at every drawing you have a value of capital X you have a value of capital Y right.

So, what is this omega X Y now? If I want us to write it mathematically, how would I write it is the it is the sum of all numbers I mean integers natural numbers i j for 0 less than I sorry, 1 less than i comma j less than n, but I cannot you can say it like this, right. I obviously, cannot be equal to j in this, why? Because once you have drawn number i, I cannot, I do not have that number again to draw again right. So, j cannot be equal to i.

So, what is the probability of this? And it turns out that it does not matter which what the numbers are. The P X Y before that let me also look at omega X and omega i, when you can also look at the random variables individually. So, what are omega X and omega i note omega from our usual I mean the old theory which applies completely in (Refer Time: 03:11).

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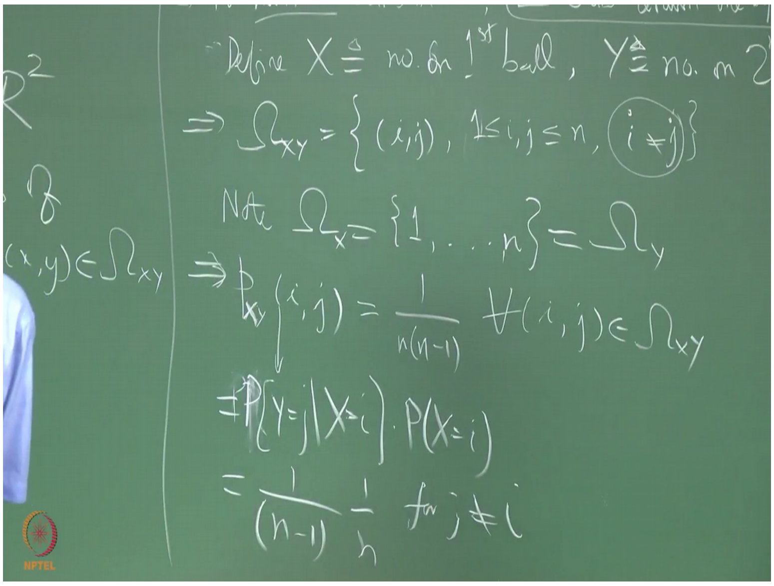

So, omega X will clearly be so, for both omega X and Y, you can get all the numbers from one to n. Note that this is not sufficient to tell you exact the nuances of this, right. This will not tell you that you cannot get i not equal to j.

In fact, if you did it with replacement you threw the ball back, then you would still get the same omega X on omega Y, but omega X Y will be every possible pair. You can even get i when you do with replacement. But I have said here clearly you are drawing it without replacement one by one. So, this that clearly tells you that i cannot be equal to equal to j. So, what is the probability for this case? Which implies that P X Y of i comma j, now I do not have to necessarily use X Y remember, right. The convention is the value is the random variables take are written in the order of the subscripts. So, if you write P X Y the first one is going to be for i X and the second one for Y. And this is Y, I am not that makes no sense to write P X X right. So, it is P X Y.

If you want to write P Y X that is fine, but make sure that you do the subscript I mean the arguments appropriately. So, it turns out that P X Y ij is 1 by n into n minus 1; for all ij, all pairs ij in omega X Y in here. How do I know this? It is obvious, right or not so obvious? Just we have done this earlier right. So, you draw the first ball what is and conditioned on that, I supposing what is the probability of X equal to I 1 by n. So, given that X equal to I what is the probability Y equal to j, for j not equal I will be 1 by n minus 1. So, probability of so, this is basically you think of this as probability of this is equal to probability of capital P of Y equal to j given X equal to i into probability of X equal to i, right, which is 1 by n minus 1 into 1 by n for j not equal to i. And a 0 for j equals i if you want to insist on setting j equal to i if you want.

In fact, it is somewhat inconvenient to leave out these diagonal entries in that if you write it as a table. So, sometimes they expand omega X Y to include those write some another countable collection, and then give at a 0 probabilities to those points. It is perfectly alright this right. So, you could say in some other specification, you could say you could move this part to look here. Instead of saying for all ij you could say P X Y ij is 1 by n minus as long as I is not equal to j, and equal to 0 for I equal to j. You could say that also.

So, you should be flexible. Do not insist that this has to be part of this Just writing it like that so convenience out here. So, everyone is agreed that it is 1 by n into n minus 1 yes this is correct. So, do these guys add up to one, they must they will go look at look at how many increased you have in that in that square, you do not have the diagonal entries, right. You have n into n minus 1 off diagonal entry. So, each of them has 1 by n. So, definitely adds up to 1 right. So, if I easy for example, if I take n equal to 3 or something I will get 0 0 0 on the diagonals.

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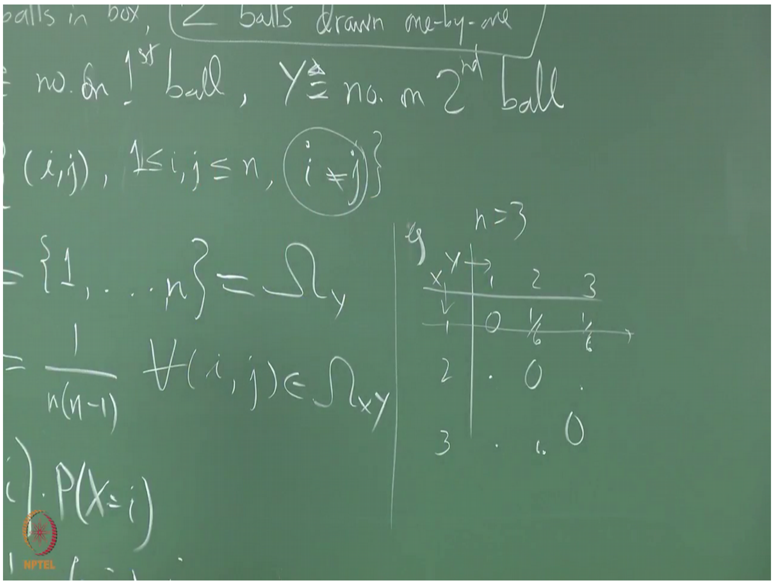

So, this will be Y, this will be X 1, X can take the value is 1 2 3, this X can take Y can take values 1 2 3, what would be all these points out here? Everything will be what will be? What will be the probabilities here?

So, here I am going to put probabilities 1 by 1 by 6, right. And you have exactly 1 2 3 4 5 6 possible 6 positions. So, this is just to make sure that, right. There is absolutely no ambiguity in understanding this very important concept of the joint PMF itself. Note that I given some experiment like this, we can define a these are not the only 2, I can define any you know, whatever random variables I want I can define X to be first number, draw the first number. I can define Y to be the bigger of the 2. Why should some in some way in all the second one, right? That is all. Because the experiment consists on drawing 2 (Refer Time: 08:11) well, you can say if you define let us assume alright.

So, and what about I can define X to be the sum of the 2 and Y to be the difference of the 2. I can do all we know anything I want they will still be random variables right. So, we will generalize this a little bit. So, in other words there is no restriction that you have to define it in the simplest way. Although, it makes some sense to be defined in the simplest way possible, and then think of the other random variables has derived from these 2 by some algebraic operation. For example, if I say X is the square root on the first ball and Y is the third power of the second ball, you can obtain that by transformation. I mean all this we will study, formally little later.

But for right now let me take the simplest possible definition which is this and work it out only for that which I have done here. Then I want to define what are called marginal PMFs. Given this joint PMF can I define can I obtain from it marginal PMF. What do I mean by marginal? That is if I if I go like this and add what do I get the probability of? I get the probability that X is equal to 1, if I add horizontally. If I add vertically, I get the probability Y equal to whatever 1 2 or 3. So, calculate marginal PMF, this knowing this P X Y can tell you both P X as well as P Y.

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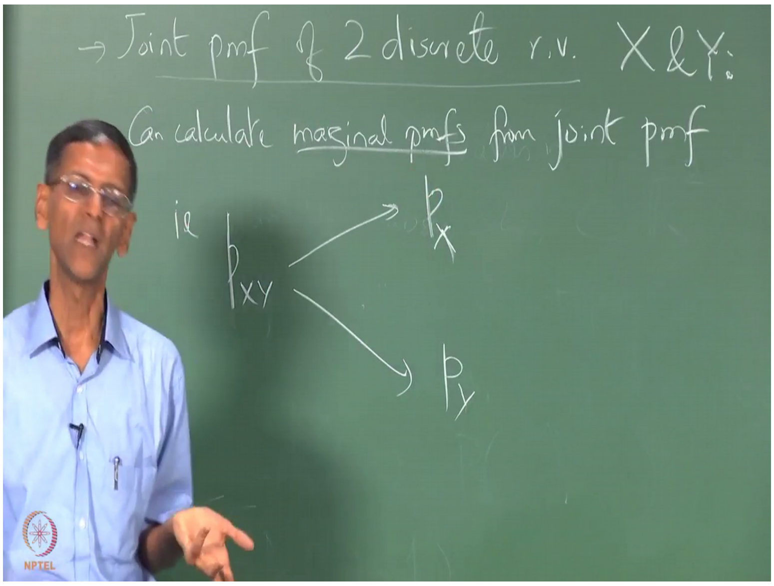

So, what is this P X? This P X of X. So, before that so, again I mean the first to do this the first step is to identify clearly omega X and omega Y.

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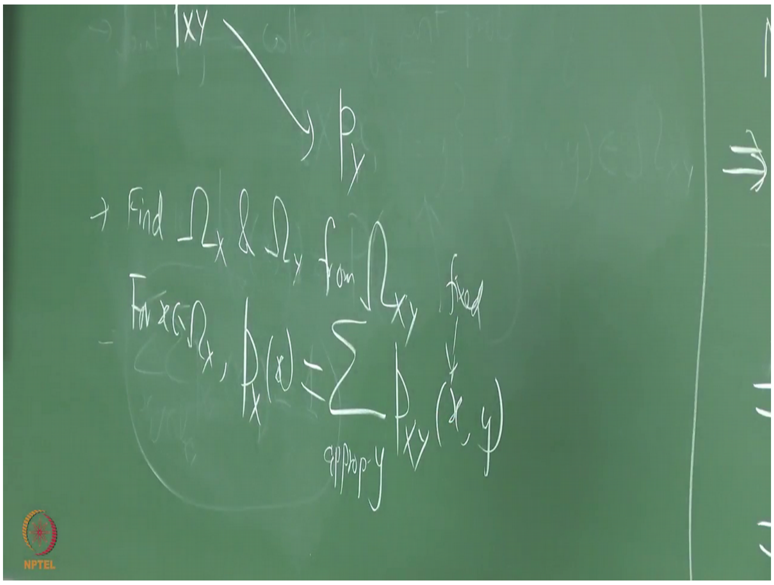

So, the step is find omega X and omega X Y, omega Y sorry omega Y from omega X Y, this there is no magic formula for this you have to go you have to look at the structure of the omega X Y and determine, right. All the points that all the values that X can take. So, from this it should be clear, right. What this is and what this is. And then this step is, right. In many cases many times it is not emphasized, but it is very important because even before you find the probabilities you have to look at the possibilities, right. What are the possible values of X? Then for each n for X in omega X P X of X is going to be what obtained by summation over appropriate Y, appropriate values of Y P X Y here X is fixed in this summation X is fixed you are only, right. Taking the summation over Y and you are repeating this for all X in omega X.

You freeze this X, and then look at all those points if I have something like this if I freeze some value of X I may have some 3 points here right.

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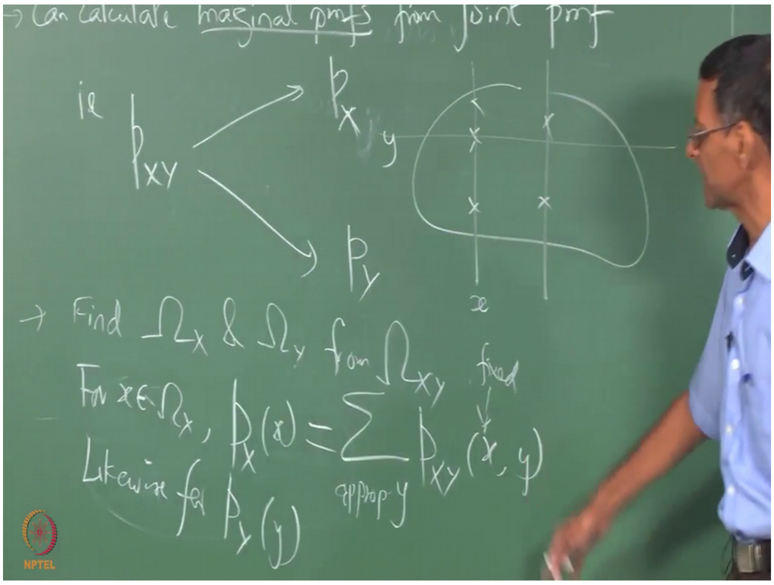

So, this could be X Y 1 X Y 2 X Y 3. I have to add all those probabilities and get this if I go to a different value of X. I can get some whatever number of points. If it turns out there is no grid at all, then there may be only a one to one mapping between this and this. It may happen that all these points are so funnily located that there is no, right nothing no nothing in common, right no coordinates in common.

Similarly, P X of Y P sorry P Y of Y, if your going like this Y you find by adding all these probabilities you will get P Y of Y. Why are they called marginal prima's or marginal probabilities? Because in this table where would you write them you would write them on the margins here and here.

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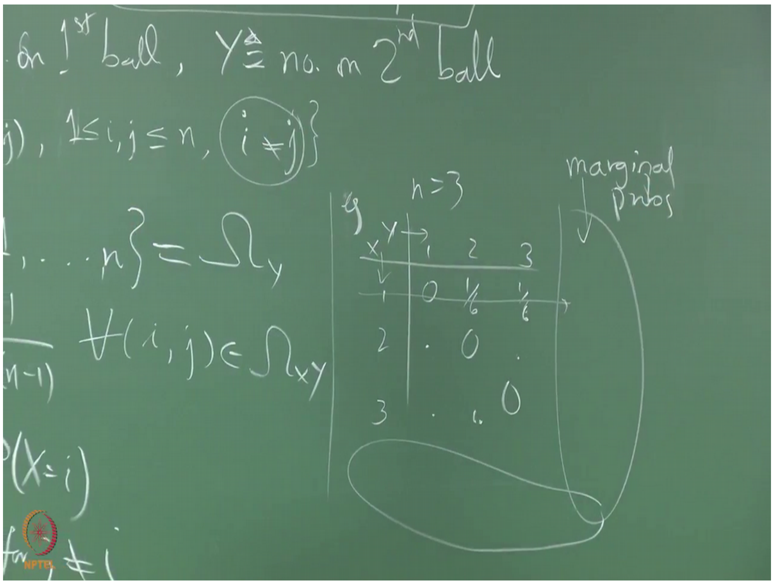

So, you write the marginal probabilities on the margins of the table, that is why that is a nice way of remembering, why they call that. So, clearly in this problem it is very simple P of X equal i has to be 1 by 3. And P of Y equal to I also has to be 1 by 3. So, it does not matter which one you draw X and Y have the same joint by say marginal PMF. And what is important is that you cannot get the in general you cannot get this joint PMF by looking at the marginal PMFs alone. You can only go from always you can only go from join to the marginal.

You cannot go in the opposite direction in general. There is a very important special case where you can, but in general you cannot. So, if I want to say this clearly.

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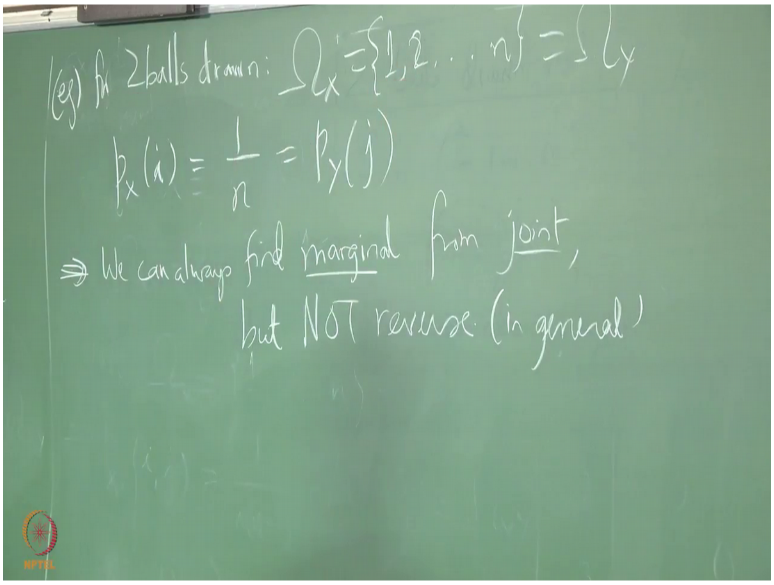

eg for 2 balls drawn omega X I think I am let me write it again 1 2 up to n and P X of i equal to 1 by n this is equal to omega Y. I have I have already written it here, does not matter that I write it again, this is equal to P Y of j if you will if you want. Sometimes it is nicer to stick to the original notation i j, right. Or X Y, just to keep your thinking straight. There is no reason why of course, I cannot use I here I could use I here there also, right. After all a dummy argument, but it is just better to if you start out with P X Y of i comma j is better to stick to j here. And certainly, it is convenient here to stick to Y.

So, likewise for I will write here, only thing is here you might want to if you if you have subscripts. So, if you have trouble when you review this material, you could put a subscript X naught if you want. But know people usually do not do it. So, we would not write it. I think let us not spend more time on this. So, it can always the what I want to say message is that; so, which implies we can always find marginal from joint, but not reverse. That is, you cannot get the joint for the marginal in general. I think this example is clear, right.

There is no way that somebody tells you that these are the 2, right marginal PMFs of X and Y to get that particular joint PMF, right 1 by n into n minus 1 you know you just cannot get it. Because this same joint PMF also comes when you do balls with replacement, but when you do balls with replacement, it turns out that the second drawing is completely independent of the first drawing and you get a totally different marginal sorry, totally different joint PMF. We will have to save that for a little bit, but anyway. So, you cannot reverse do the reverse calculation, right. And this has to be kept in mind.