Probability Foundations for Electrical Engineers
Prof. Aravind R
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 11
Part 2

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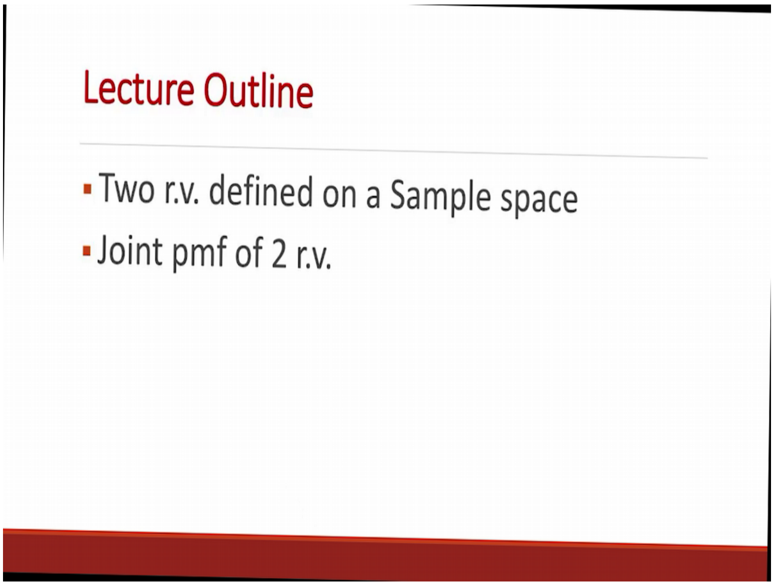

So, let me now jump to the case of or go to the case of 2 discrete random variables, right.

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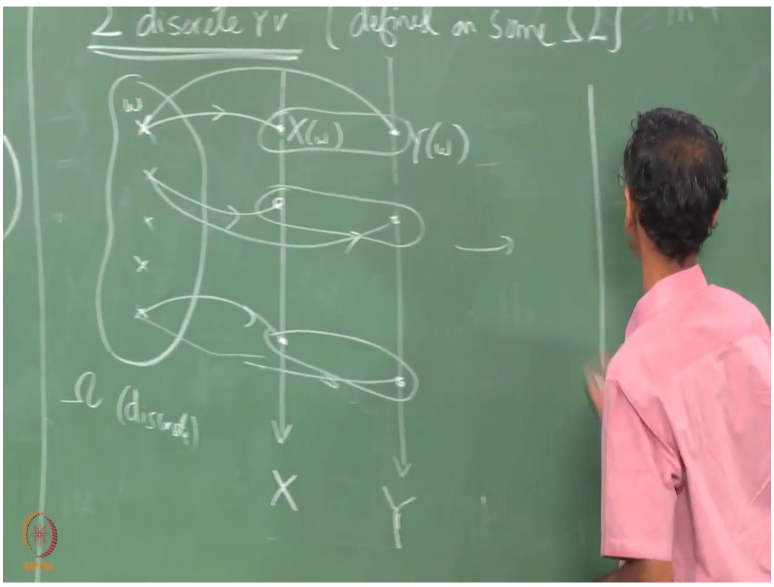

And it is always better to think start with some omega. So, it is always even though you can say well later on you can sort of dispense with this omega in some sense, and you go do a direct specification in which case, right you do not talk of an underlying experiment, but for the time being let the omega come into the picture because it is very important right now. So, we are interested in of course, we are not jumping into the continuous case right now it is still going to stick to discrete, right.

We are interested in the joint behavior of 2 discrete random variables, that is very important right.

What is the probability that X will take some value and Y will take some other value at the same time jointly, as a joint event that is what we want. And it turns out this joint event is the key to understanding the joint statistics of 2 discrete random variables. You do not look at X and Y in isolation, you look at them together. And so, the specification will need something more much stronger than just a simple univariate pmf, right. Whatever we have talked about so far, the pmf's are all univariate. There they just depend on one random variable.

So, this is going to take us to bivariate to where you know a pmf with this joint to a bivariate pmf. So, how do these arise? I mean clearly, we will look at some many examples, but just let me do an abstract thing, right. Supposing I have some omega here, I have let us say it is a discrete omega. Just for simplicity, right. I am the same idea will apply even when omega is continuous, but to so the basic thing sinks in right. So, I have 2 axis here that define X and Y. So, these are the names of my random variables X and Y right.

I have I have already said in some previous lecture, that you can define more than one random variable on his on a sample space, right. I am pretty sure I said it, right. If I did not if it is not if the if there is the significance of that did not sink in then, let it sink in now, right. It is clearly possible to define many random variables depending on the experiment. I think I did say it yesterday, right. If I pick a person at random there are many empty numbers there a person carries around and each of the means a separate random variable.

So, we are only interested now in the case of a discrete sample space omega and 2 2 random variables X and Y. So, this is a point omegas. Consider any sample point small omega n capital omega, right by the mapping, right. X mapping it maps to this is X of omega and this is Y of omega, right. This is by definition, right. We define 2 random variables and clearly both X of omega and Y of omega must be defined and both of these are numbers.

So, this is a number axis, this is this is a real line, this is another real line. Now what you going to do is; look at the collection of all these pairs of points. What is that? And the commonality the fact that these 2 are not happening separately, but happening together, whenever you observe this sample point small omega you are going to observe this value, this is going to be the output for X and this is going to be output for Y. You have no choice in that. If you pick a person at random, you cannot say I am going to pick the height of this person or the weight of that person, that that is completely wrong.

You have to go with that omega, if you look at joint behavior. Let this so, let this sink in again very, very crucial, right. Without this understanding your whole and you cannot understand how 2 random variables behave right. In fact, is that you are looking at jointly what values they take. Not just separately. If you I said you can also close your eyes to Y and only look at X that is back to the univariate situation we have been talking about talking about all this while. But you need to make the conceptual jump now to at 2.

So, this is, right. One joint pair this is another joint pair, which I am just leaving and then you can have one more here. So, if you collect all these for the discrete case. You come if you collect all these all these pairs of points and point and put them on the plane. What will you get? You will get a countable collection of points in the X Y plane.

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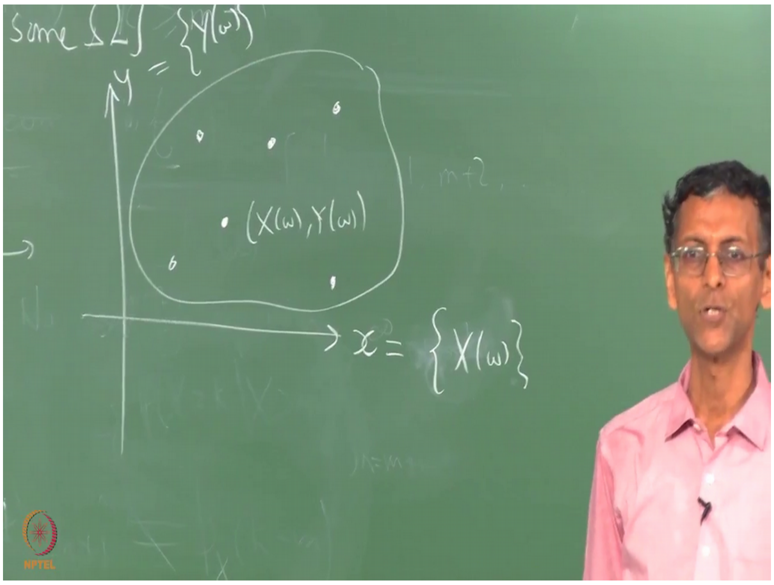

So, this is small X and Y. So, this will be what possible values of, or let me just write this as the collection of X of omega, right. And this will be, and because I am viewing it as countable sets, I can use this this braces notation right.

So, the collection of X of omega. So, if I therefore, put a point here, this is going to be I have the coordinates X of omega Y of omega. Now let us say I do it for all possible points. So, what will I get I will get some grid. Some collection of points which in many cases falls on a nice grid some it is your grid may be right. So, this collection of points, it is the fundamental, starting point for studying the joint behavior of X and Y. Note what we cannot look at just the point the values of X alone, or the values of Y alone. You have to look at and you cannot associate this X coordinate with the Y coordinate of this point.

So, that is why it is better in the beginning to not draw it as a regular grid, but to randomize it enough. So, that, right. You realize that this point is uniquely specified by this value that X takes and that the and the value that Y takes. This is clear to everybody? Has anyone does anyone have any questions? Remember, once again last time I will say. You cannot combine arbitrarily combine the X coordinate of one point, and the Y coordinate of another point. That makes that completely violates the whole mathematical structure of what we are trying to do. So, this is the set omega what we are going to call as omega X Y.

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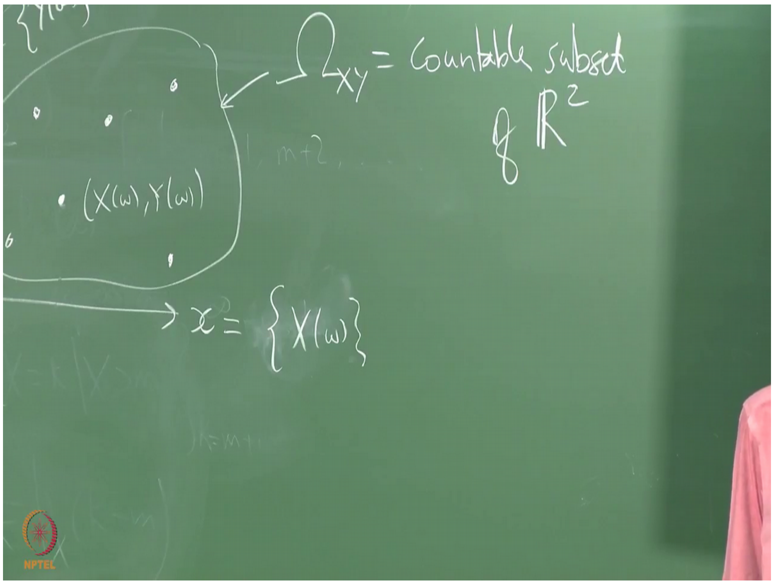

This is a countable subset of the plane and the plane we write as R square.

The line is R power one or just R, right. The plane is R square. What is omega X Y? Omega X Y is the cap is a set of all possible values that X and Y can take jointly. So, how did I write it here? Let me since I have already written it and brought it here, I do not want to I just wanted you know for the sake of this just. So, just write it as X, does not matter. So, maybe I would not write it here, because it is not possible that it might get not come out. So, well in the video, omega X Y is essentially defined to be X omega Y omega, for all omega small omega and capital omega. Just think of it like this.

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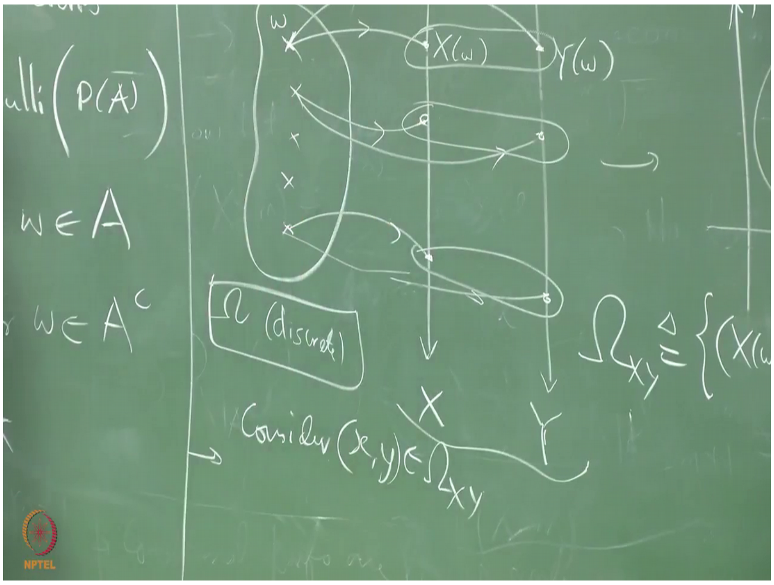

This is a qualifier, colon. Some people use a small vertical line here, but since we have been using vertical lines for conditional probabilities I am using a colon. Colon is also used as often as a vertical line here. So, you look at vary this omega a small omega for all points you look at jointly what happens to X and Y. So, this is starting point for this joint specification and when the sample space is this when capital omega sorry, which I have written here. When the underlying sample space is discrete there is no way that this omega X Y is going to be anything other than countable. It has to be countable, right. How many points supposing I have n points here? I can only get n totally n points here. I cannot get more. I can get less, but I cannot get more.

If I have a countable infinity of points here, I may be able to get a countable infinity that is a different issue, right. But basically, there is you can say one to one corresponding I mean maximally one to one mapping between these omegas and these points. So, omega X Y has to be countable. So, what about the probability now right. So, supposing this is the point you know instead of writing capital X omega Y omega, we need to have some more notation for the point.

So now supposing you consider some X Y a point X Y in in that set, right. You consider this point X Y in that set. So, then you look at the joint probability that that of X taking the value X and Y taking the value Y, right jointly. So, that is the if you repeat that over all these points X Y you get the joint pmf of X and Y right.

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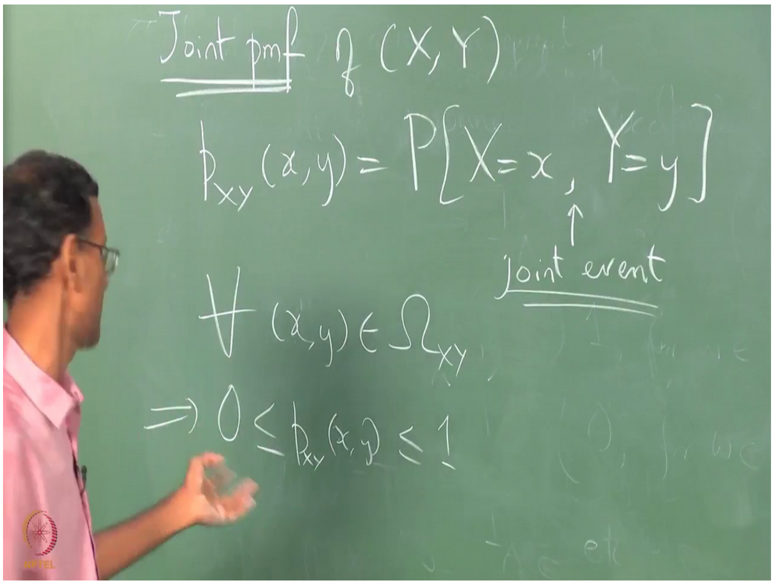

And you indicate the 2 random variables with a comma, right. This is the probability that X since the joint event, right. And you repeat it for all you do this if you write out these probabilities for all X Y in the countable subset omega X Y. If you do that you specify the joint pmf of X and Y. So, what properties do you have now? So, that which implies that the this p x y has to be between although you can say why should it be 0; obviously, I mean we are let us not split has over it let us say it can take you know you can go up to 0 and you know up to i equal 1 also does not make any sense, but does not matter we will just put it here. So, but this and this summation over all and I want to write it as a single summation now.

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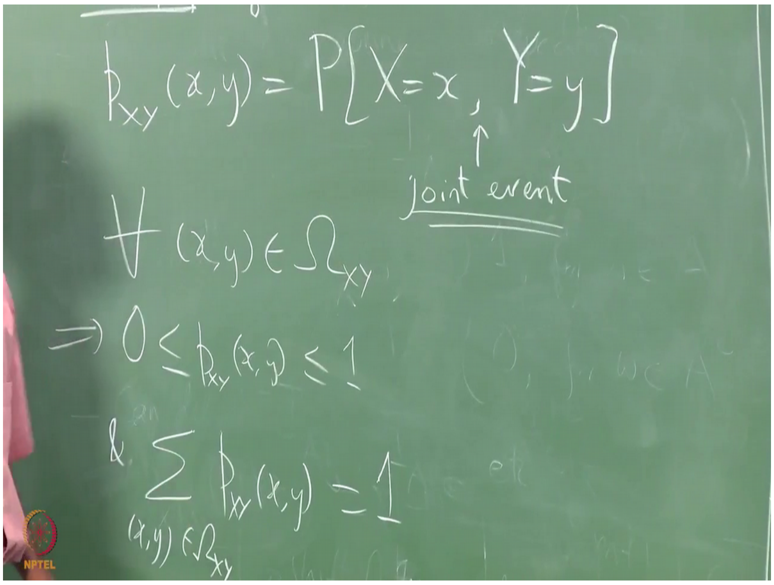

Not double summation if I add it over all points omega of small X Y, I will get what I should get? I should get 1.

Now, otherwise if I go here and look at the probability of this point, plus probability of this, plus probability of this, they all must add up to one they all have to be non-negative they I have to add up to one. So, these (Refer Time: 13:59) basic rules with which we operate. What and again the meaning of this p x y, x comma y note the notation. You write the random variables subscripts in some order, right. And I am going to dispense with the comma in between some people have put a comma here I am not going to put that comma, right. Just X Y, if I have 3 random variables, right. I can write them as X Y z or X one X 2 X 3, whatever without commas. Then I have the same number of arguments as I have random variables. And the first argument is the value taken by the first random variable the second 1 by the second random variable then subscript and so on. That is the convention that we use, right.

And this again is a joint event always. This event is independ a sorry, is exclusive of any other point, right. X Y X 1 Y 1 is obvious automatically exclusive of X 2 Y 2 or X 1 Y 2. Once you mean that X one Y 2 is also in the set. So, if you have a regular grid; obviously, you will write you will you will you can draw a vertical line and or a horizontal line you will find more than one point on that on any given vertical or horizontal line, right. In supposing it is an integer grid as we will see do an example I am running out of time. So, pick it pick up the example in tuesday's class right, but for now X 1 Y 1 is unique by itself or is a is (Refer Time: 15:42) by, right. It is exclusive of X 1 Y 2 or Y 1 X 2 Y one or any other point, right. And we are interested in this joint probability always write the starting point is.

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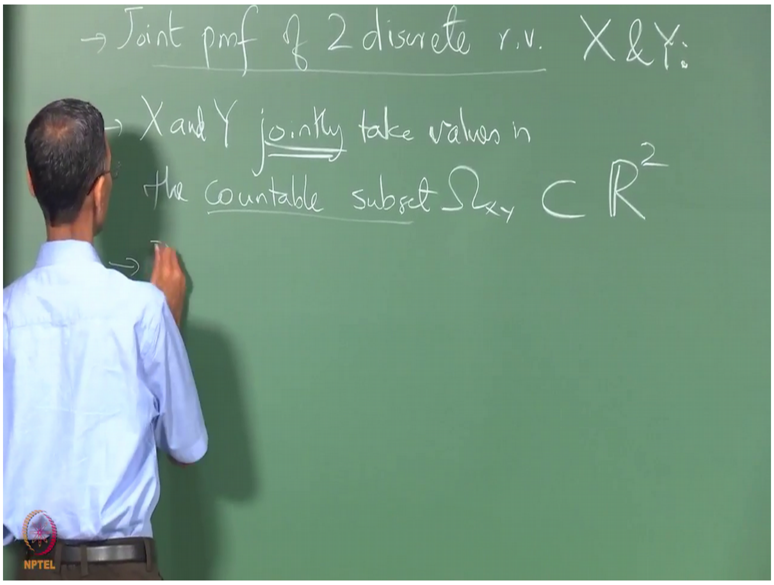

We have X and Y jointly taking values what set idea is they jointly take values in the countable set of the plane, right. R squared is the X Y plane. And omega X Y is a set in which they take they take values and the key point is omega X Y has to be countable, right. There only finite number of points in that sorry countable number of points in that set.

You cannot have for example, the whole unit square 0 1 for example, that would be straight away an uncountable number of points. So, alright. So, this is the omega X Y, and then you have it the joint. So, the joint pmf is basically joint pmf is a collection of joint probabilities.

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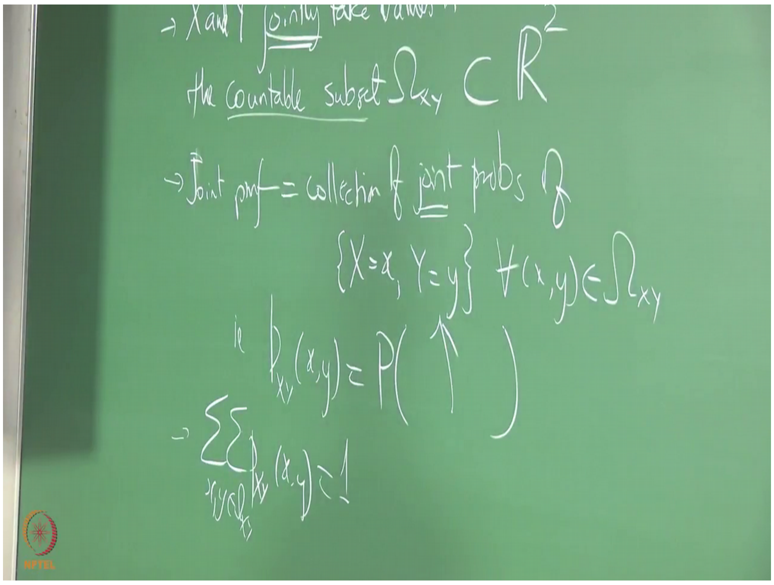

What these are all joint probabilities of the type of what events X equal to X, Y equal to Y and Y equal to Y; for all X Y pairs inside omega X Y. In other words I is the notation we write p X Y X comma Y is the probability of this event, any questions on this? This is very, very crucial definition right.

So, this is our atomic event now regarding these random variables X and Y, and looking at this joint probability X takes the value of X and Y taking the value Y. Remember, when you do the experiment or when you have one run you must get a value of X and you must get a value for Y. So, this kind of joint event is; obviously, what we should be looking at. So, if you go back to that picture which we drew we have a point small omega and capital omega, which is mapped to one point for X and different point for Y. And you are looking at when that point omega occurs that X and Y occur together.

So, that is the joint event that is that this is. Note also you can get multiple points in capital omega mapping to the same value X and Y that is entirely possible I see. So, I think the best thing to do is to look at some simple examples. Let me make sure that I yes, I think we are ready to look at examples no point in rehashing. So, if you do this, right. The joint pmf lay I have to complete their mathematical specification, I will do that and then you go to the examples. So, if you do this for all points X Y in omega X Y, then you have specified the joint pmf.

So, what are the properties of the joint pmf is basically a collection of joint probability. So, the double summation of p of X Y must be equal to where or single summation. I do not know maybe last week I wrote it is a single summation, but it is come sometimes, right. Written as a double summation is does not matter. So, this is for if you add it, right. If you add all these probabilities you must get 1. So, that is the most important I mean that this the key idea that you have not left anything out, no possibility with finite probability must be left out in the specification of the joint pmf; obviously, each of these probabilities must be between 0 and 1 also.