Probability Foundations for Electrical Engineers
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Lecture – 02
Set Operations

So, yesterday right, we began with a concept of sample space.

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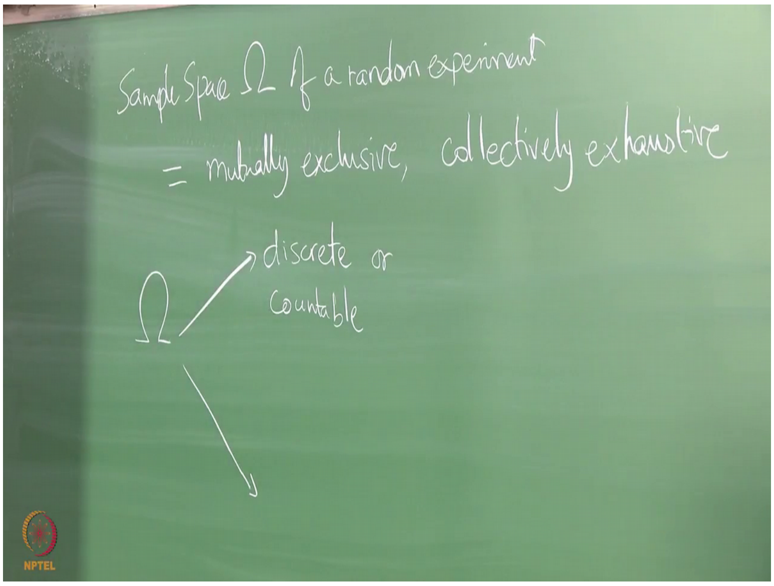

Let me just. What is this? In other words just some more words on it right, it is a. There is a very you know nice expression which I found mutually exclusive, and collectively exhaustive. These are the terms that characterise this omega. So, it should be mutually exclusive; obviously, right. One outcome should exclude any other outcome right. And the omega should be a collectively exhaustive collection, the word collection appearing twice in that sentence, but right, but I. Let me just put the adjectives and leave it at that. So, these are the attributes of the sample space omega right. And some very important properties of omega at this level itself we right so that, we will should look at classification right. Omega can be discrete or countable. So, in this course right the words discrete and countable will be synonyms. So, what does it mean, discrete or countable; that means, i e omega is of the form omega small omega 1 omega 2 omega n.

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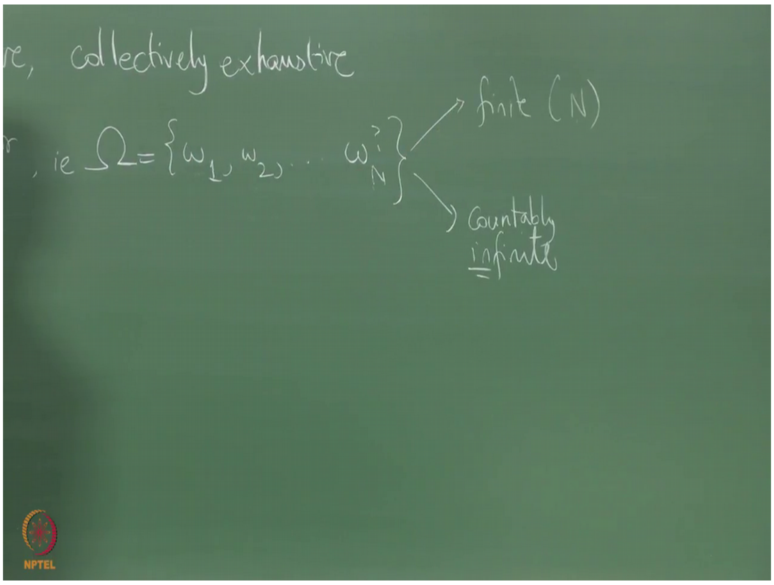

Now, do we have a upper limit like this omega n right. So, it turns out that, in some cases we do in some other cases we may not right. So, we have a another classification here, finite, well there is an N, and we have the case of countably infinite, right? We have all of these possibilities; I mean though these possibilities in the case of the discrete omega right. The finite case is this most right, the situation that most of you are familiar with right. So, all popular probability experiments fall in the finite n category and it also; obviously, include compound experiments which we will talking a lot about right; such as rolling of 2 die, or each trail itself consisting of a pair of coin tosses or a pair of die rolls and so on right.

So, as long as total number of possibilities or total number of elements in the capital omega, which is subspace of that n is a finite integer then right, the sample space is both discrete and finite. On the other hand you can. The samples case space can be discrete, but it can be countably infinite. Now what would be good, what is an example of this? Have you people encountered countably infinite sample spaces? I am sure this is not the first time you are seeing this. What would be an example?

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Yes that is most, standard example is of a countable, of a sample space who is countably finite, is this flipping coin, until you get heads. Now you might wonder why do we need to consider n going to infinity, because not all coins have probability of getting heads is equal to half right.

You can have that probability of getting heads to be some absurdly small number right, in which case right you might say well I have to flip the coin forever; that means, basically you are repeating that experiment right, that many times till you get a successful result right, which is in this case is ahead. And if you call the number of trails that you need to keep doing it, till you get heads you may now want to put a limit on the number of heads, that you have to.

You may be willing to toss the coin infinite number of times till you get the heads right. So, in such cases right, we model the omega, the sample space capital omega is, having a infinite number of a countably infinite number of outcomes right. Where you know you still have omega 1 omega 2 like a small omega, which are the outcomes that the outcome number 1 would be head in the first toss, outcome number 2 would be head, omega 2 would be head in the second toss that is and so on right, going up to all the way up to infinity right.

So, this is one right possibility. The other possibility is that omega itself is uncountable and right. This is very important subtle difference, which needs to be understood right upfront right. Do we have examples of uncountable omega?

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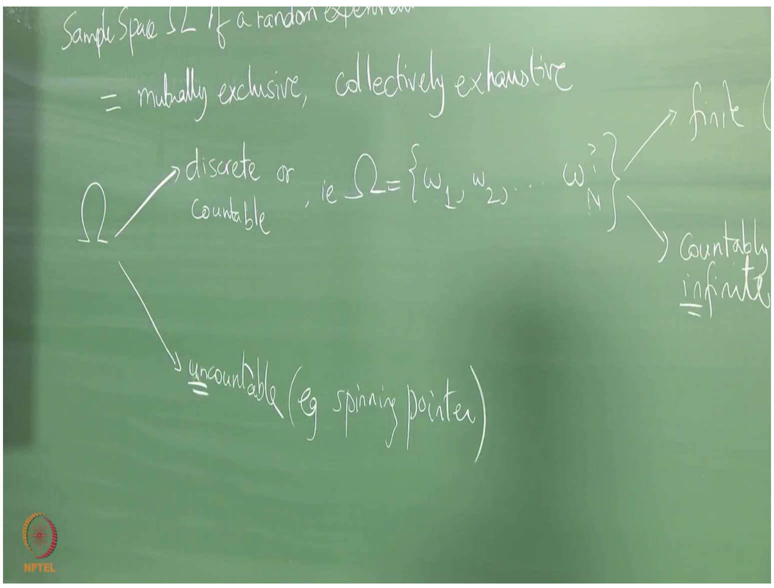

Nobody has an answer to that question it looks like right. This is what happens when; for example, you have a spinning pointer right. You spin the pointer, and it eventually comes to rest somewhere right, and you measure the angle that the pointer comes to rest against some reference axis right, some theta. How precisely do you want to measure that angle. You might want to measure it to infinite degree of procedure potentially right.

So, what then would be the allowed set of angles that you are willing to consider in capital omega, because any real number between 0 0 and 2 pi, or normalise at 2 pi 1 and any real number between 0 0 and 1; whatever 2 pi 1 does not matter, but you get the point right. How many can you call the number of numbers is a problem to say that way, but I cannot help right, can you count that said. No its not possible to count it right. One of the fundamental results in mathematics is that, the number of points in any segment of the real line be 0 to 1 or 0 to pi is uncountable. Is the order of infinity much greater than or right, much bigger than the set of integers right.

So, in such cases right when an experiment is designed to model that situation, the number of points in capital omega; obviously, going to be uncountable right, and this is when things get very, little more interesting, I should not say right. Our standard tools that we use for the discrete situation, some we have to be tweet right. They have to be modified to be made applicable to this situation to the uncountable situation right. So, let me write here e g spinning pointer. The other example that you find in books is the dart board right, which again will be coming to in this course. You throw a dart at a board and then you right you measure the coordinates of the heating point right. Again or if you do not like the 2 d version of it the 1 d version you, let us say you are constrained to or look at the edges the x coordinate of it, separately, that will be your single real number right. You do not want to quantize that coordinate to just a finite or countable number of possibilities. You want to allow an uncountable number of possibilities right.

So, in other words any portion of the real line right, if omega is that portion of the real line or in some cases the entire real line itself right; such as a noise voltage or something where you are willing to allow right. The entire collection of real numbers, although it might sound strange to all of minus infinity to infinity right, but still mathematically in some cases more convenient, not to constrain right. The voltage that you might measure, even though physically its impossible to exceed a certain magnitude right.

So, in all such cases right, we invoke the uncountable omega right, and tools are somewhat different to deal with this situation right. For the first part of this course, for the first three weeks at least right we will be focusing only on the discrete with a countable version right.

We will postpone our treatment of uncountable omega to a slightly later date right, but some of the things that we talked about in terms of properties of sets and so on are the same in both right. So, basically this, I mean most of what we going to talk about will also apply here right, the stuff that will not apply I will point out separately. So, do not think that whatever I am going to talk about, which is with specific you know relevance to or importance to this countable case is not at all applicable; that is not true right.

So, in some sense this is all of this and more in a conceptual manner. So, we will have to wait for that right. Anyway coming back to some terminology right.

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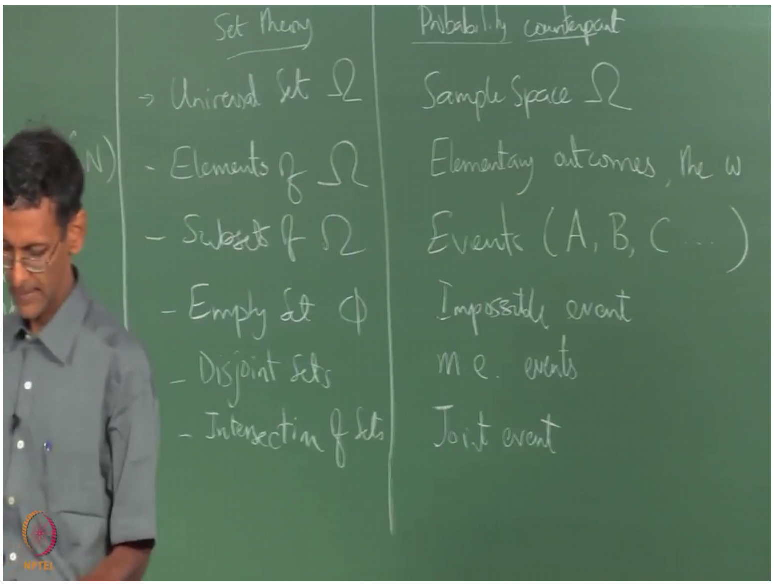

So, we have here set theory metrology and probability theory right, the probability counterpart. So, set theory plays a very important role in understanding probability right, because everything is defined in terms of sets. A probabilities are assigned to sets right. So, let us go through this one by one. So, we have of course, the sample space, I will start with this, the sample space. What is the set theory counterpart or sample space omega? It is; obviously, the universal set. Now note that the universal set is, the name up is a specific particular experiment right. Obviously, it does not encompass everything in the universe right, it just a terminology right. All of this is with reference to some particular experiment, then the elements of omega. What are these? The probability, the counterpart of elements are basically the elementary outcomes. This is what you observe on a track, or every trial of the experiment right, you observe one of these.

So, mathematically of course, the same notation is going to be used in both sides of this right diagram, just terminology that is different right. So, here you have the omega i or I will just, I am not necessarily going to put an i subscript. i subscript is used only for the countable case right. And since we said this whatever, this is going to be applicable to all experiments right. So, we just call it omega right, the small omega which refers to any outcome, any elementary, or any element of capital omega which can occur on the experiment right. Then you have the subsets of omega. These broadly corresponds to the events right including omega itself right. So, omega itself is called the sure event, a certain event, because; obviously, right in nay experiments, something in omega capital omega has to occur. So, how do you say that a particular event has happened on a trail of experiment. If is right, an event which is, you know denoted A B C etcetera right, and event A for example, is supposed to occur right, if the omega that occurs in that trail belongs to A, then you say A has occurred right.

So, it is quite possible for more than one event to occur on any trail right. So, it is a outcomes which are mutually exclusive, not the events in general. Although you can have mutually exclusive events also right; that is very important, but remember by definition the outcomes are exclusive. One outcome will always exclude anything else. So, subsets are, they even said we work right. So, empty set phi is the impossible event right, then disjoints sets.

So, these are sets which are mutually exclusive, which is not right the case for all sets obviously, but some sets turn out to be mutually exclusive right. For example, getting odd number or even number, in a die tossing experiment; obviously, you cannot both the same time right. So, here in set theory you have a terminology here, in probability theory they are mutually exclusive which I am, which we will abbreviate to m e right, mutually exclusive events right. It is very painful to write it out all the time, so we will need acronyms for, things that we need use a lot, mutually exclusive events then intersection of sets.

So, if two sets A and B right intersect; that means, that they can both occur together, they can right. So, if both sets occur, A and B occur on a certain trial then you say that, the joint event has occurred right. So, the intersection always refers to a joint event right. So, these are the most important, most commonly occurring. Now let me see if I can add the union has a same terminology in both set theory, and the in probability also right.

So, in fact, it turns out that. So, we can write that also; union is basically the same. So, this is the union event. So, here we do not usually keep talking about a set, we talk about events right. So, just simply substitute, in many cases we can get the probability counterpart by just substituting event for set right, except in this particular cases to term joint is very important to understand right, because it will come up 1 million times in this course right. So, better get used to that terminology right. So, this keep this somewhere. So, that right, its always available to you if you have. And I am going to assume of course, an addition to basic calculus for this course, if you are also convergent set the basic set theory right. By now I think set theory is made its way fairly to a fairly low level in the curriculum right. So, we cannot, we do not have time to spend too much time on that right.

Whatever set theory we are going to use, is going to be very intuitive and basic right, not the more advanced stuff right, which is only taught in some mathematics class. And this is not, I repeat this is not a mathematics course, this is a applied course where we are going to only use a level of maths that you need for getting the results we want right. So, let me, let quickly right, go through some set operations that we will need right. We have already mentioned union intersection right. So, let me keep this with me. So, that I quickly do it. So, we have.

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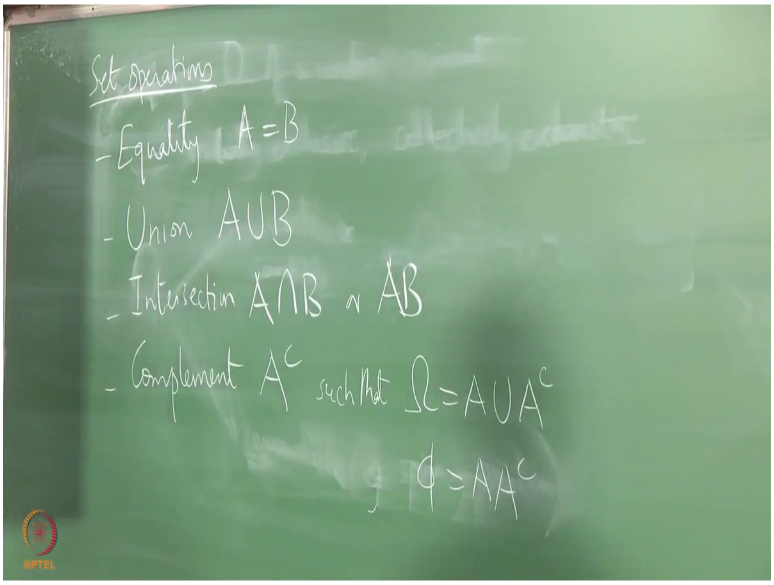

Let me start with equality; of course, in mentioned this yesterday right. Two sets are equal if and only if they contain the same number of elements, the same elements not number I am sorry right. So, any two sets can be tested to see they are equal or not right. This will come up in a certain way in many situations right. Let me write them out first, and then let us go through this 1 by 1 union. Now what we talked about equality union is right again very intuitive right. A has some, A consists of some collection of points right and sample space B consists of, some may be a different set of points right. So, A union B is; obviously, a superset of A and B which consists of all the points, which belong to wither A or B right, all of you know that. And intersection is the only those points which are common to both A and B right. So, all of this again and when do you say that A B is phi.

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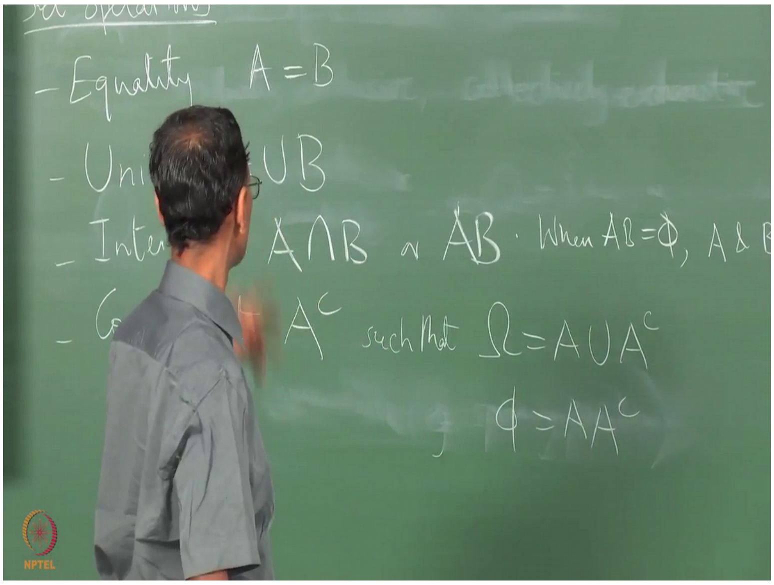

So, when A B equal to phi, then A and B are said to be mutually exclusive. So, they right. So, we will not use the word disjoint right. We will use the word, as I have already said that mutually exclusive. Then complement right, whatever is not in A, if you define A whatever is in omega, but not in everything, is also with respect to omega remember, capital omega. So, clearly A A complement has to be the empty set phi, and the union A A complement, A A union A complement is phi itself.

Sorry omega itself right. So, note that right, you can extend this to more than two sets also, if you have A B and C, you can as we will do that, but before that let me point out this notation here A B right. We find that intersections occur so frequently right that we do not want to keep that writing the inverted u all the time. So, right we drop it, and just write A B, which is unambiguously the intersection only right whereas, for the union we will keep the u right. So, what it means is this U, cannot be used as the name of a set right. So, one letter gone there, but that does not matter right.

If we need lots of sets, we can always index them A 1, A 2, A 3, A. So, on right, we never going to run out of letters to represent sets right. Let combination letters and numbers. Anyway, so what if you add a third set C. This is pretty much what we can do it just to say you know one set or two sets. What we can do if we have three sets, you can talk about combining unions and intersections right. So, let me, maybe I will go here for that with A B C.

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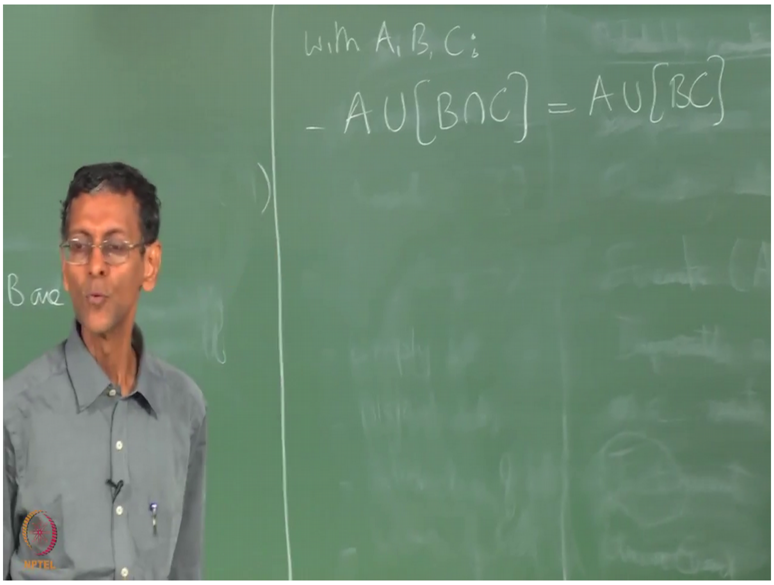

What can you do? You can do A union B intersection C. I will just writing it, writing it to show right, this C, which is same of course, in our notation as A union B C right. So, again in this connection let me also point to the classic way of understanding all of this. You can always draw Venn diagrams right to make things clear right.

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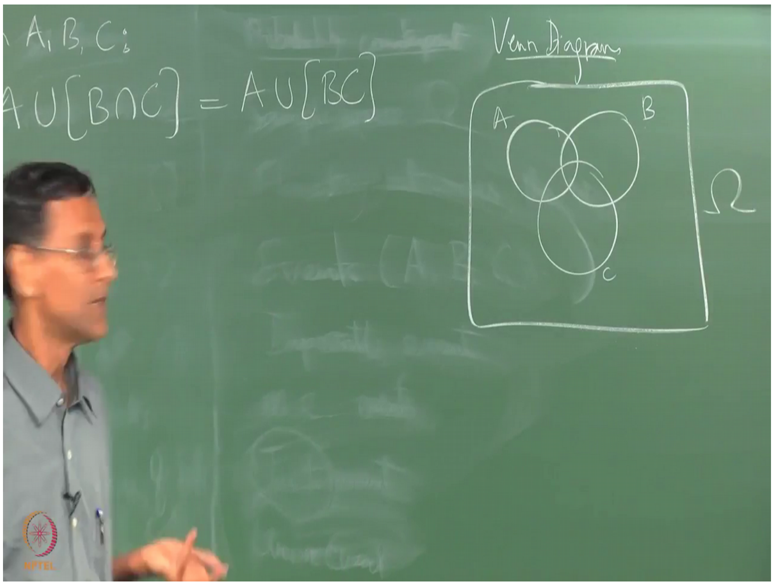

So, the Venn diagram which all of you must have drawn in the past. If I have this universal set, is big outside rectangle right, and then I have A B C like this on three arbitrary sets. Of course, write it I am not. I know if the set were discrete, it would have points here, but I am not showing those points right; that is a usual way in which we draw these sets right. It becomes very cluttered if you, you know if you insist that I the points be shown right. So, we showed without the points.

So, this A union B C right, is basically the union of A and intersection of B and C right, and by distributive law what is this equal? It equals A union B intersection A union C.

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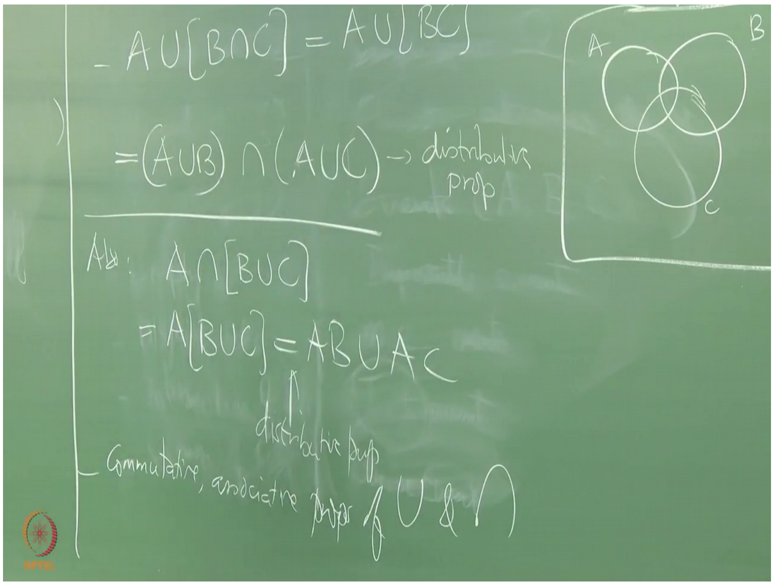

So, this is one version of distributive law. Now I am right, I am going to ask you to verify this for yourself right. You can do these operations separately, and note that this set is equal to this set right. You can do the shading and so on and do it just entirely geometrically, as I said this course is not about rigorous proving or all of these results right. So, what is another distributive law that we can do with A B C. Another distributive law basically is, the intersection in front of A union. Here we had the union in front of intersection; we have also an intersection B union C, which is the same as A. So, this here and now right, which is looking like the distribution of multiplication over addition, which is right. You immediately you will say yes; A B union A C right, without disclosing your eyes, this is what you have been trying to say ever since you saw distribution of this kind right, from sixth standard or whatever.

So, this basically a distributive property, both these distributive properties will helpful right and useful. Then of course, you have the commutative associated properties of union and intersection, I am not going to. You know that those things are. So, A union B is always equal to B union A, then A union B union C you can do the union which were ordered right. So, let me just simply, just say commutative and distributive properties right, commutative associative properties right of union and intersection right. This is something I do not even want to write say in anymore details then than this right. And then we have as a last, may be collection of a things, that we need to keep in mind Adam De Morgans Laws.

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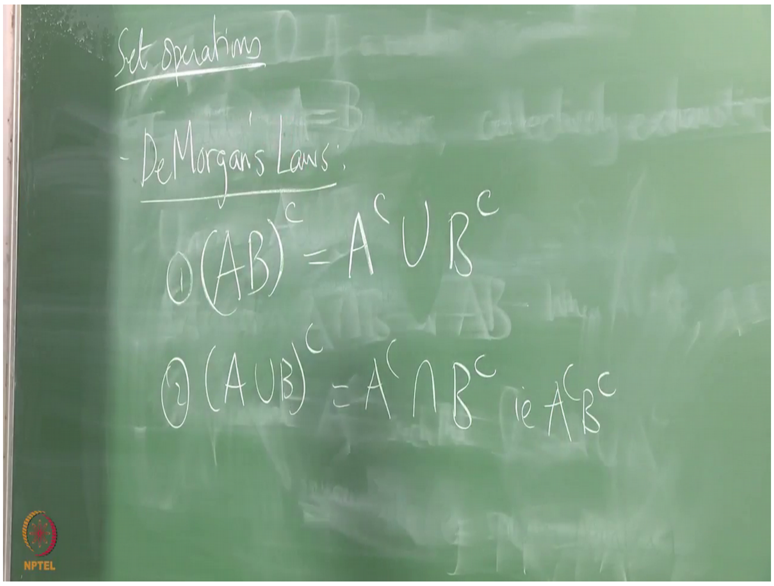

What are they? A B complement, the two versions A B complement is what. So, A B complement is A compliment union B complement right. The other one let me write it first and then we can look at both of them. The De Morgan's Law number 2 would be A A union B, the whole complement, this is what I wanted to say A complement intersection B complement right. If I make a small mistake please get up and say right, I will correct it. So, clearly here the union the intersection gets replaced by union, and note that right the union gets replaced by the intersection. So, this is A C B C right.

So, I am not going to spend any time on this, because you people are supposed to know this already right. So, with this much set theory we are pretty much ready to embark into the rest of the subject right. So, these are the things that you will be called upon to, use at any point in them, in the course. So, are we comfortable with all this? If you are not, please review this material from any number of books out there right. Remember we are not going to go beyond this, and we are going to stay at this level. It comes under what some people call the intuitive theory, as opposed to some non intuitive, the next level higher up set theory, which uses more complemented things like sets including themselves and so on right. We are getting into any of those controversies here right. So, here we will not encounter the set that includes itself right.