Probability Foundations for Electrical Engineers
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Lecture - 16
Discrete R.V

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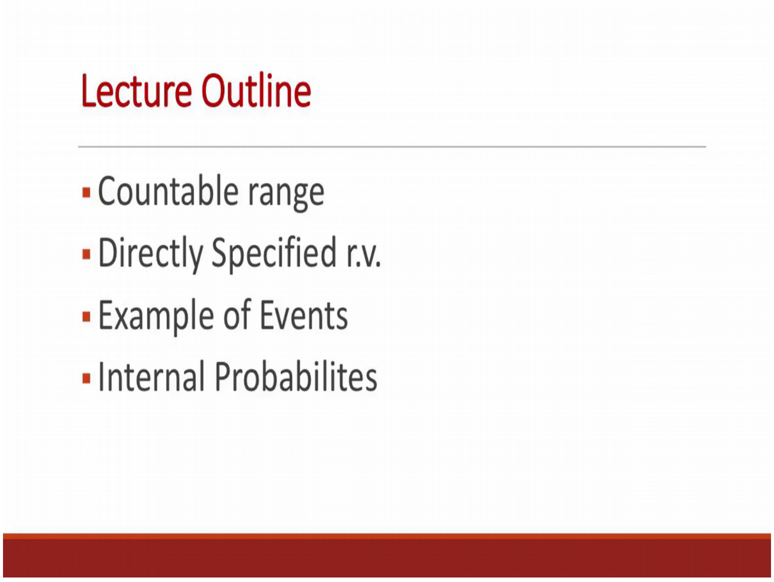

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Has roughly speaking right the omega X the range space of that right discrete now a random variable X has a countable or discrete omega X. Now this can happen let me say it very clearly right it can also happen in a continuous for omega, we will see how right it is very simple in fact. All you have to do is quantize continuous range and you can get a countable right what do I mean by quantize, supposing you have will come to this later as I said, but just to make sure that you get the idea that this is a general concept and can be applied in all experiments whether discrete or not right. If the a in a spinning pointer when their output can be any real number between let us say 0 and 1 normalized right, you can say if it is between 0 and 0.1 I am going to output 1. If it is between 0.1 and 0.5 I am going output 2 like that right that is a process of quantization

So, you can go for you can always go from uncountable to countable, but you can not go the other way right. So, that the idea is that when you have a countable range countable omega X right you have a discrete countable. So, the probability that X the probabilities the points are only what the points are omega is other by right other points in are in the on the real line do not have any probability right they have probability 0. So, that all the probability lives in this set is countable set, and that is why you use the term discrete random variable for this ok. And the pmf is specifically specified by let me write it out to here again.

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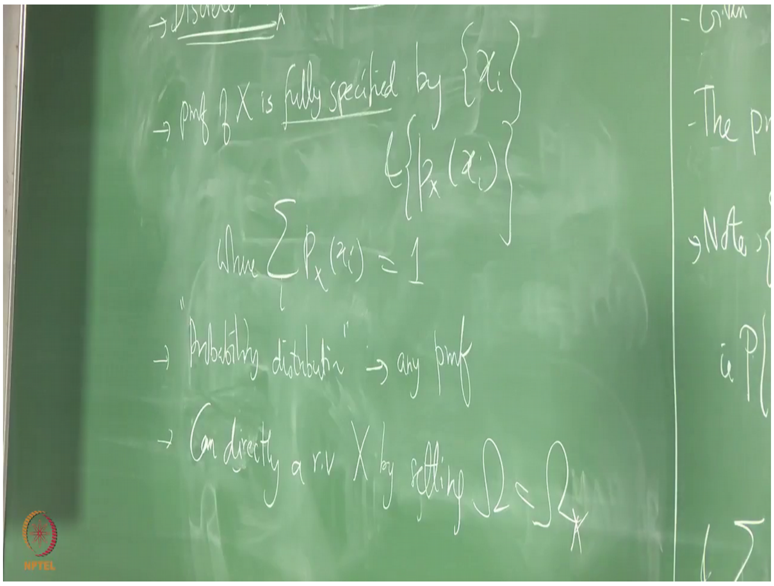

Pmf of X by let us write it like this, both of these are countable sets of numbers right, and as so the only requirement is that this has to be 1; that is this. So, you can now ask the question well why do I need a note, do we do we always have to have a more complicated experiment right and define a mapping etc answer is really no right, you can just say I can define my experiment itself to be just nothing but X, that is also perfectly ok, that means what my omega itself is just omega x my omega I is just the x i.

So, those are what are called directly specified random variables, but that that the disadvantages for that experiment you can only define that I mean you cannot define a more meaningful another random variable that is the only thing, but that is I mean when we want to just specify the proper sorry when we want to understand the properties of a particular distribution that is the term we are going to use for this kind of thing, this kind of information.

So, the probability distribution is how well let me write it down right the term probability distribution, what does it refer to, it refers to is any pmf right, it is a formal way of referring to any pmf not just any pmf, it will also refer to distribution of probability over a continuous interval that is pdf that will come later right, but for now any pmf is basically a probability distribution right you think of if you want to say formally and say how is it probability distributed.

So, it refers so directly specified random variable is just specified using a probability distribution right. So, I have directly specified random variables, or let me write it like this we can directly specify a random variable X by setting omega equal to omega X, and p of omega I to be. So, what do we need to directly specify a random variable X all you need is a distribution right you need the points x i and you need these probabilities. Once you have that you do not need any more anything more is it specified random variable. So, that is what is done in more theoretic I mean a simple problems right you do not necessarily have to say I have this random experiment I am doing this complicated thing, I am doing this measurement on this point I am getting this is how I am getting X out all of that is right completely circumvented, you just say let the random variable X be this, that means it is directly specified.

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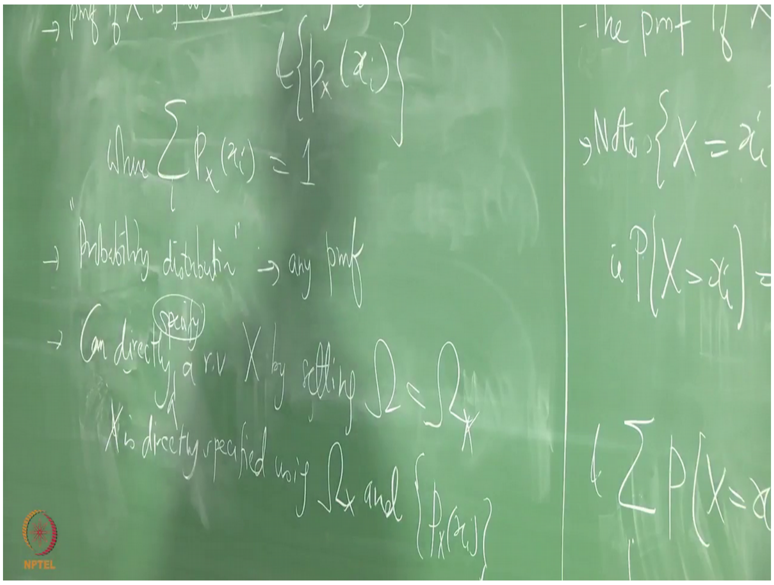

So, the terms directly specified and directly specify for what they were termed over here x is said to be directly specified right. Using this omega X and or in terms of omega X and the P x of x i, that is basically using these two things. So, this set of x i is exactly omega X. So, you are directly specifying and this is how right that term terminologies have come into popular usage like binomial distribution, Poisson distribution and all that right, where you are sort of diverging the random experiment as such you are just simply looking at these numbers ok.

Now what else can I do instead I can clearly look at events of the form what I know what is what kind of events can I look at I can clearly look at.

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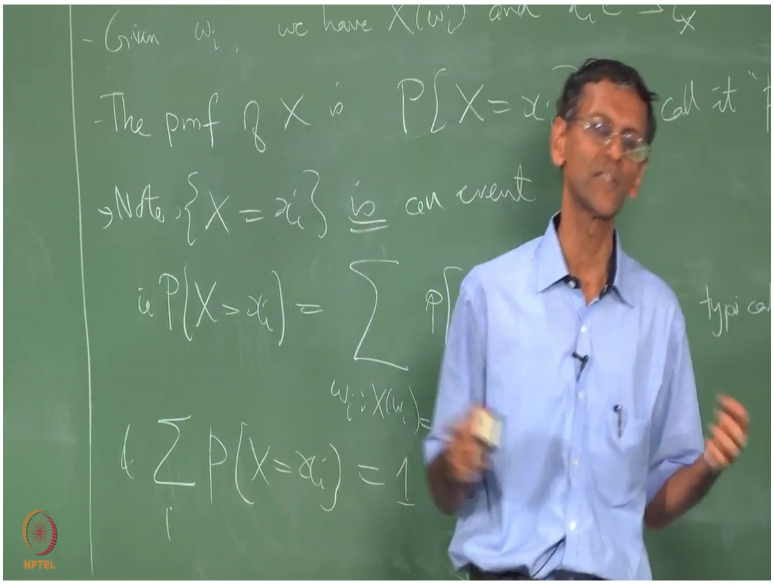

Events like x equal to x i, or I can look at x equal to x i, or x equal to x j, x i is not equal to x j note that x equal to x i is disjoined from x equal to x j.

So, for any random where any discrete random variable or any random variable for that matter right if the x i right, you have this fundamental disjointness right or this mutual exclusivity between x equal to x i and x equal to x j.

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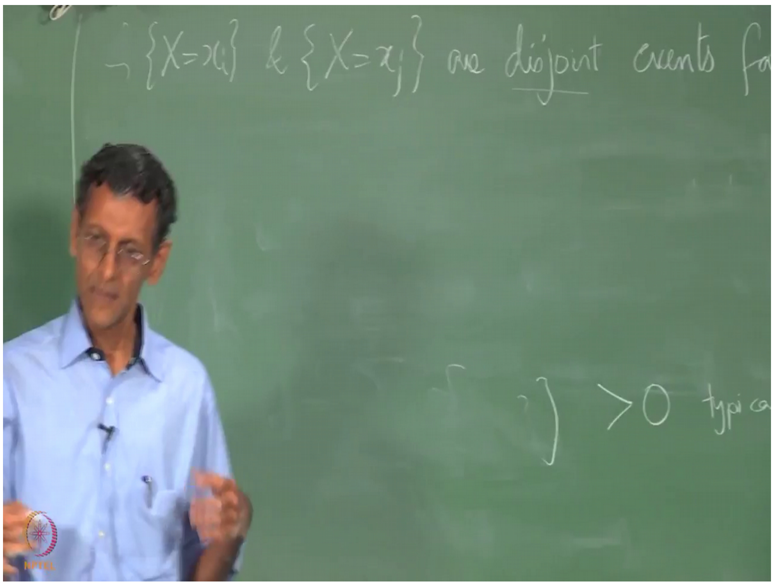

Are disjoint events, or mutually exclusive I think in the beginning I said it, let us stay strict to me, but sometimes we also keep using disjoint right it is not a; I guess it is not a problem you can think you can switch back and forth; so for x i not equal to x j.

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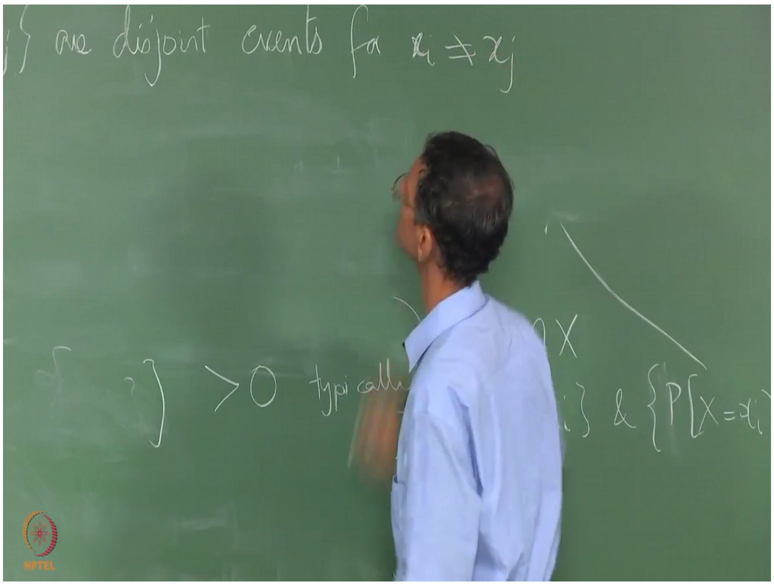

So, they can never be they can ever have any intersections it is just not possible right. Why do I say this because I can now look at right I can look at a union event I can say what is the probability that x takes the value x i or x j right. So, we can ask for.

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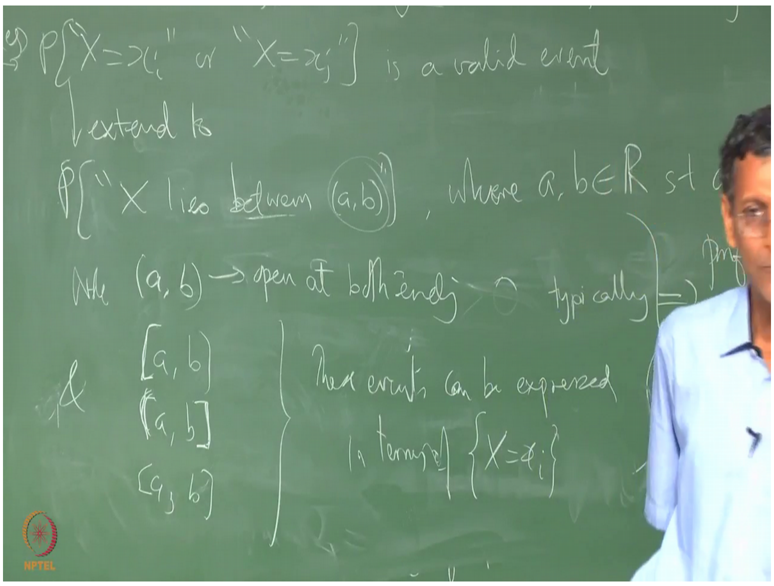

So, therefore P X equal to so I can say x. So, this so e g this is a valid event. So, this could be some English right, we are just writing it mathematically in some pseudo mathematics is not very rigorous maths in some sense right. What is the value of the probability that or what is the probability the event that x either takes the value x i, or it takes value.

Now you can even go bit beyond this I can say what is the probability that the random variable x lies between two numbers like a and b right I can do that also, even though I am not formally defining the properties of intervals I can break down that interval into decompose I develop the points that lie inside the develop point that lie outside the interval right. So, I extend this to, now you have to be very careful you say between a and b, do you say open or do you say closed.

So, you have to be very careful here because we are talking about discrete variables and so a b here is your open interval open at both ends etc right, we have seen this kind of thing right all of this will become necessary right and. So, you have to be careful about defining that square bracket for the inclusion and the curly bracket for exclusion that. So, you have to say which one you are talking about here right. So, you have four possibilities given two points a and b you can either exclude both or include both or include either one.

So, your question could be; what is the probability that it takes values between 1 and 3, but not, but takes value 1, but not the value 3 some things like that right. So, that would be you know you including 1 and not including any of those are also possible. So, these will be. So, these kinds of things interval probabilities can also be broken down into events like this. So, all of these events these events can be you right can be expressed how in terms of x equal to x i, all you have to do is look at all the x equal to x i is that fall in that interval whether you know counting both ends or not as a case maybe.

So, even though we are strictly not right, we are not including intervals in the list of events right now because, we are saying well I know remember a and b can be totally outside omega X right in which case there is no sense in I mean there would not be any difference whether you include them or not right. So, these interval probabilities will become very basic to study of continuous random variables at you some of you have seen that will come later, but even here if you discrete random variables also you can talk of intervals that is all I want to say right now and a and b it can be any two real numbers.

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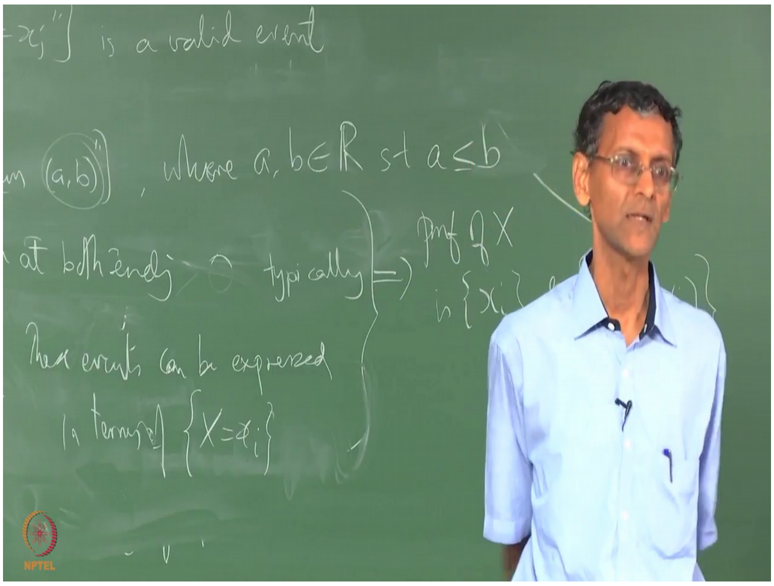

Where any two real numbers such that let us say a is less than equal to b. Of course does not make sense for it to be equal because I mean there is no interval if you say right a equals b where I can say right between 0.5, 3.5 or something in the in the next x integer values right and that would be a valid event because I would interpret it whether express is right word or not I do not know, but you can interpret it in terms of events for which makes sense like this

So, whether or not a random variable is discrete later on we will worry about continuous you can always talk about intervals. And without violating the maths right discreet you are always breaking it you know ultimately any of these things you are going to write it down write it in this form, only because you have only there what do you have you only have the p of X equal to x i to work with. So, any question regarding probabilities of x you have to break it down or get it to that form.

So, you can only have unions of x equal to x i if you right that is what right, but that does not stop you from asking for the probability of an interval, but when you say interval you have to be careful right because it is discrete this all of these can have different probabilities, because including if a or b lies in x omega x including or excluding them will have will make a difference. So, just be careful when you use it to us that is all. So, we will stop here right and pick it up tomorrow.

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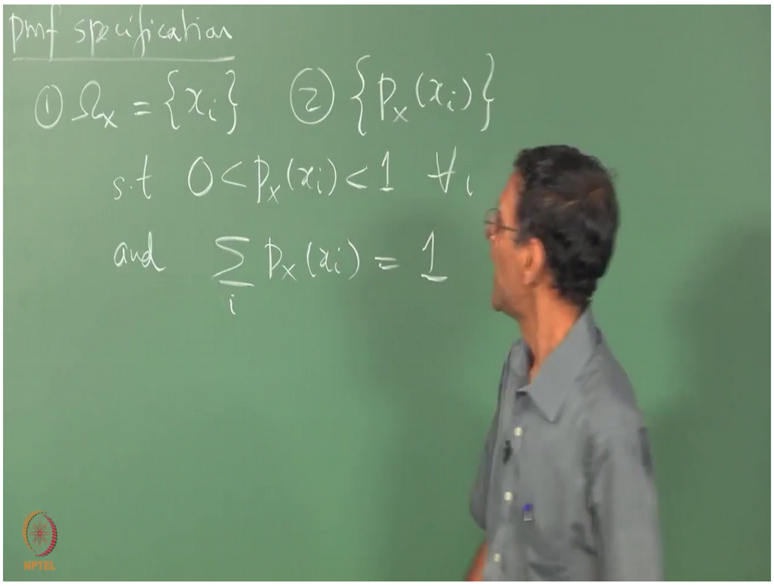

So, yesterday we were looking at just basically defining a pmf, or specifying a pmf, important to note that all we require are these two countable sets of numbers right one is the set of values that the random variable x can take or any and need not be just omega X right you could have some omega Y or you know there is no requirement that the label you give that has to be the same right, you can have multiple random variables with the same specification that is one very important extension that we will see right.

So, we have this bunch of real numbers right x i, I have not said that x i is r, but I written it out, but it is obvious right and then you have the probabilities which satisfy this right and once you have this you are ready to go you do not have to you do not require a real life experiment where these x numbers actually happen you know on the experiment and with these probabilities right. So, you are free to look at whatever model you want some people just it keep inventing models just to suit themselves right without any reference to any reality. This is where the subject actually departs from physics or any real experiment right you are you are you are you are completely divorced from any physical consideration whatever, anyway.

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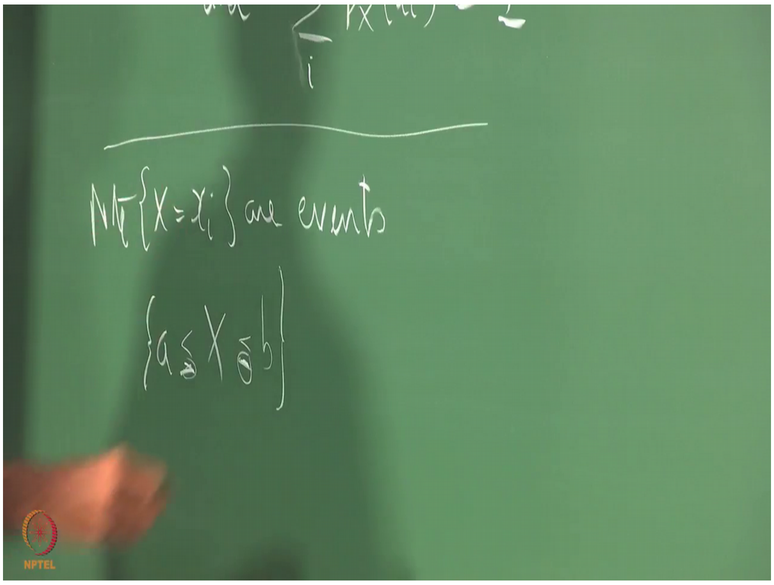

As I said yesterday x equal to x i, note right these are events right and any and the and what else we write say yesterday this a less than equal to x you may or may not put this the you know may or may not have the equals whatever right depending on this is also.

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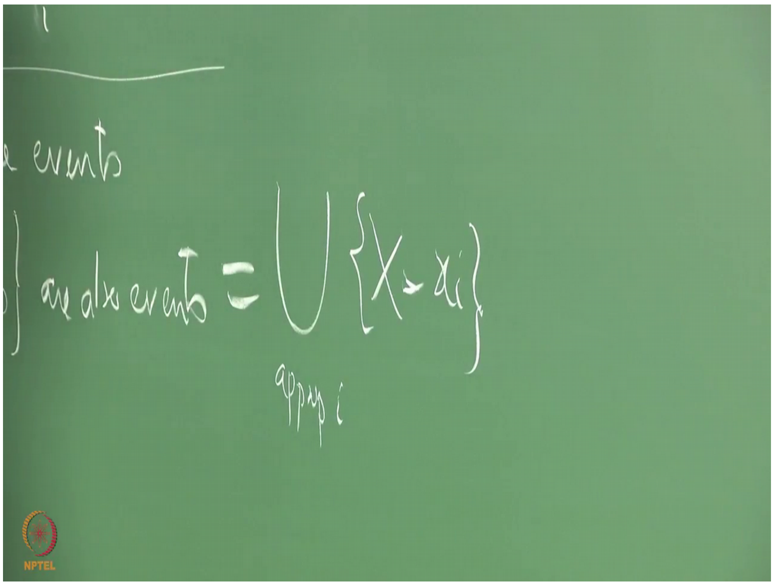

Are also events and these are events which are the union of x equal to x i for so whatever appropriate I again I am not being very rigorous about this, but this is a you know we do not need to be very rigorous right, just think of this as an event right that the random variable x takes values in some interval a b including or excluding the ends of the interval right and write it in this form. So, you do not have to create at this point right you it is the it is enough if our sigma field of events includes only x equal to x i, you do not have to make a separate exception for saying intervals are not this, in fact they are nothing but union of x equal to x i. So, just from a mathematical point of view also there is no issue in treating this as an event.