Probability Foundations for Electrical Engineers
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Lecture – 08

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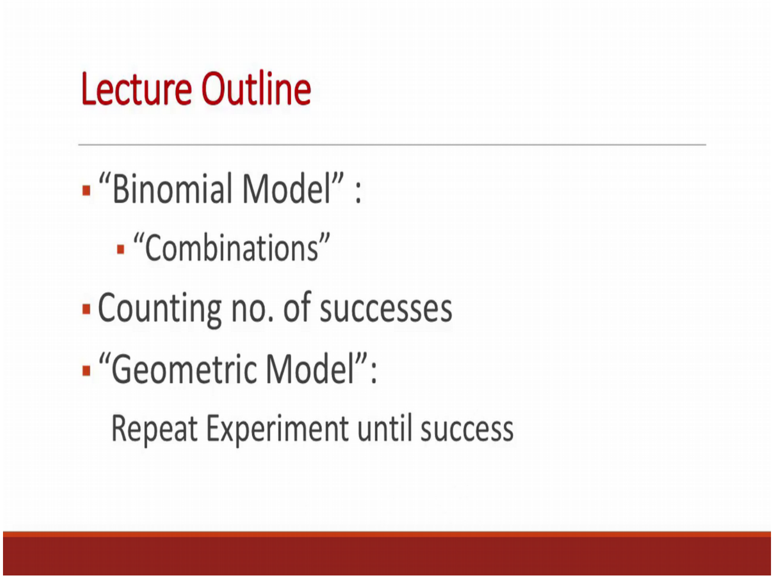

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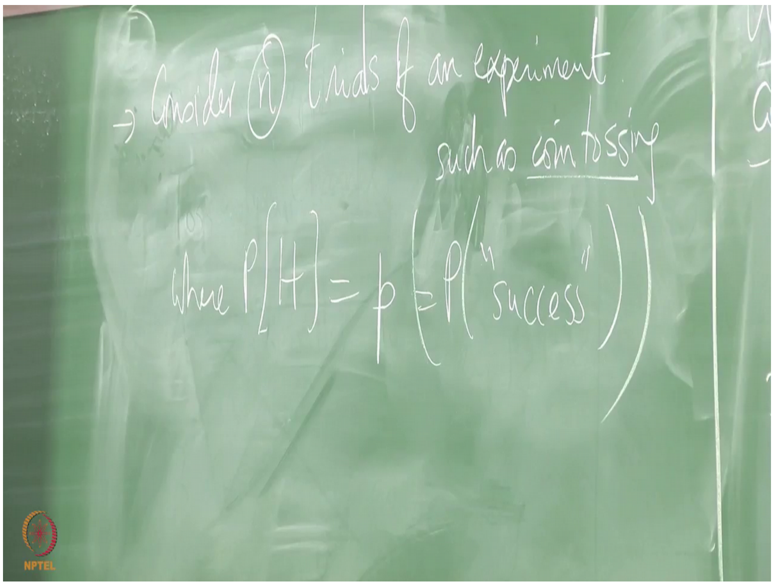

Now, this experiment where an event a occurs or does not occur right or you have each right; right by the experiment only tells you suck you are only concerned with right success or failure, right.

When I say x by let me say eg coin such as coin tossing right addressed to say you are only interested in success or failure or head or tail right where let us say the probability of head is P which is all probability of let us say success some on each trial on any one trial right probability is succeeding is this P right; obviously, probability failing is one minus P, right, this is a very standard situation supposing you consider n trials right what is the probability that you get exactly k successes right.

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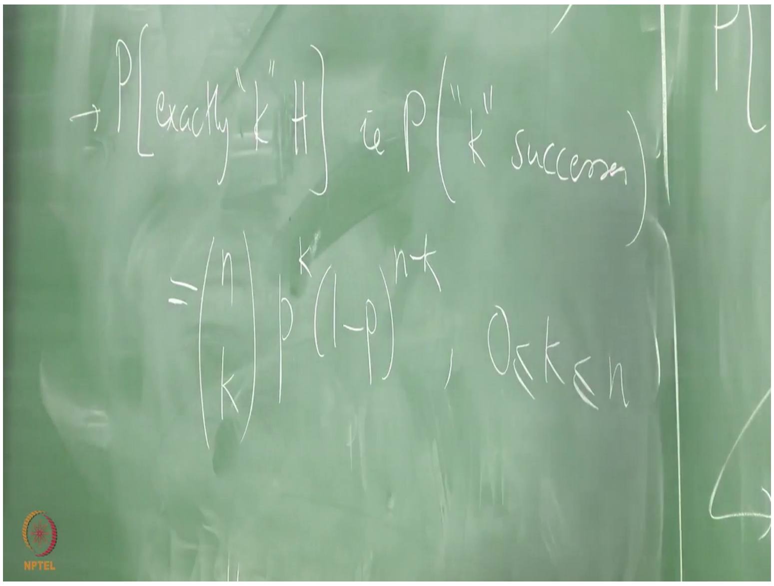

Or k head, this is heads; H stands for heads right, k heads or i e probability of k successes right, what is this? It turns out;

Student: (Refer Time: 02:02).

If you look at any one pattern of success failure it is going to be P power k 1 1 minus P power n minus k where as only one pattern like up one specific string of 1 0 1 like that if you will say one is success and 0 is failure right any one specific string such string is going to have this probability P to the power of k one minus P power n minus k.

This is right how many such strings are there? There are n choose case of strings and each string is exclusive of any other string. So, you are perfectly justified in adding up all these probabilities right they all have the same probability they are all exclusive of each other again you have right lets combinatorial coefficient coming here is n choose k if you do it for some small value of n and k use you will understand those who you are using it for the first time why this is n choose k right because that is the number of positions that you have for putting the k successes in the in the overall string of n right; n trials that you have.

So, each trial you mark as either success or failure right how many ways can you choose k out of n its n choose k if you have for example, for choose you know 2; 2, you will interested in 2 successes or four trials its four choose 2 which is 6 and you know how to calculate count those 6, right, it is if you how many ways can you distribute 2 0s in4.66 ways, etcetera, etcetera, right.

So, I am not going to elaborate this any further right you people have studied combinatorics already right. So, I just want to point out that these are 2 distinct places where combinatorics appears in improve, right at this level in the in the first course right this leads to what is called the binomial distribution and this leads to what is called what the hyper geometric distribution these names again are somewhat binomial, it is we can appreciate because this is a binomial coefficient right, but these this that other term we just mentioned that is not very important, but the thing is that there is another probability model which is built around this right.

So, these are the 2 lenka if you continue with this repeating an experiment right till you get success of it to you know. So, that is also since we have a few more minutes today right let us look at this geometric model you should also we should also look at that right. So, this is the binomial model.

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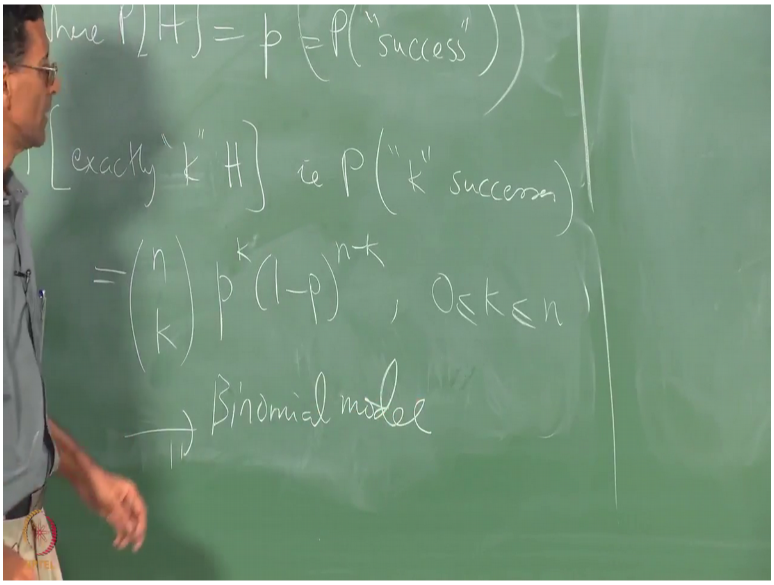

Right and once again I am right I want to emphasize that the term binomial coefficients is applied to these guys for obvious reasons right.

Now, let us look at the geometric model.

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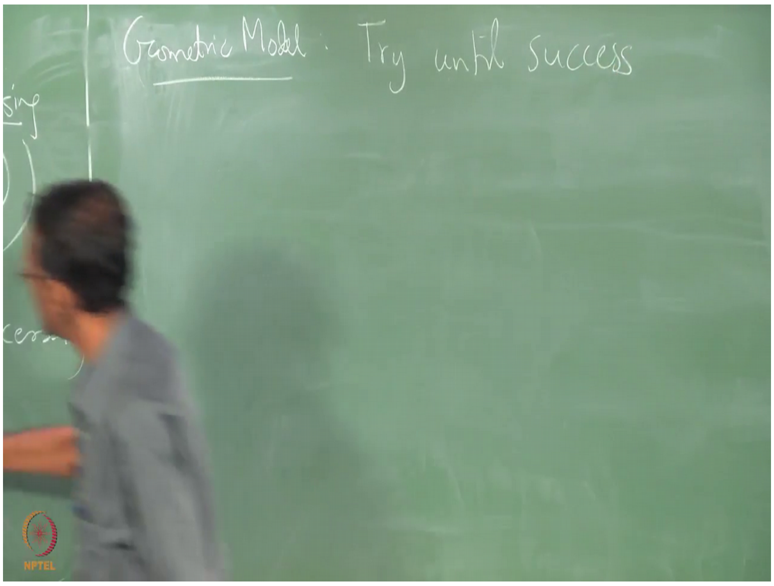

Which is what; which is do until you succeed right, try till you succeed; the same instead of fixing the total number of trials right we are going to keep repeating till we till we succeeded and count the total number of trials right. So, when is the right first success or first head going to happen? So, here your omega try until. So, here again independence of the trials is seen in very much used here right other the product of these terms is crucially dependent on the independence assumption right each it right is independent of any other trial. So, the probability of one 0 or success failures P in that P into one minus P that is; obviously, built in. So, the same concept we are going to try we are going to use here.

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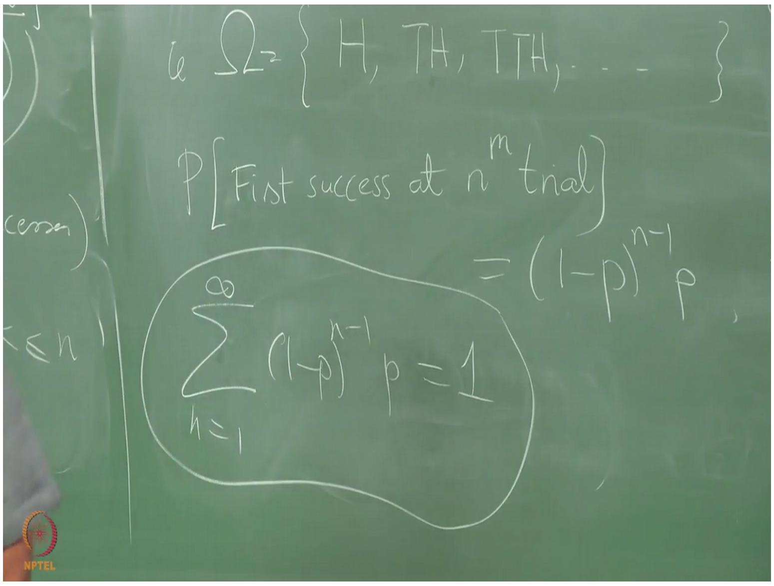

So, i e your omega the capital omega consists of strings like this either you may be; I will use; I will use H head tail; you can get head to the first toss itself or you can get T H stock.

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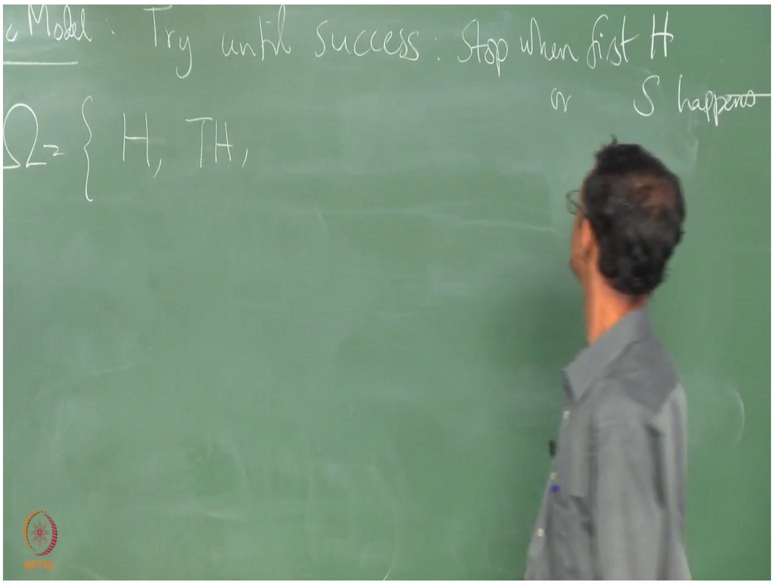

When first H or S success happens there is no reason why you should not you should stop in the first stage happens of course, here you are not stopping right in general here you are going all the way till end and you are stopping at the ends I will know matter what the n; you could you; it is possible that you know you may not get even a single success in n trials here, here what; this is I think we already hinted at this experiment right in the first class first 2 classes when you talk about how a sample space could contain a countable infinity of points, right. So, there is no upper bound on how many times you might want to keep trying.

Whereas here this capital omega has exactly how many points it has exactly 2 power n points there is it not. So, here the compound space omega has 2 power n points here it is not really a compound space because it is not constructed as the Cartesian; the product of elementary spaces right nonetheless right you can look at the probability of this first success are occurring at the kth trial or let us say the nth trial; this n is not the same as that n, but since I started writing n as. So, I stick to it what is this turns out that this is one minus P to the power n minus 1 into P right for n greater than equal to 1.

So, for this space we are going to attach right to this H we attach the probability P to this we attach a probability one minus P times P for this one minus P whole square times P and you can clearly see that the summation of n equal to one to infinity of this one minus P power n minus 1 this is the standard geometric summation and this will add up to one.

So, it is a valid distribution valid probability assignment right please check this out also it is a very trivial summation I hope all of you can do you remember how to do your geometric summations because it is something we should need again, right.

Student: You need the first success at the nth trial of the (Refer Time: 09:11).

It is the first success at the nth trail; could be first trial itself, second trial, third trial like that.

Student: What is a m 1 (Refer Time: 09:24).

N.

Student: M m (Refer Time: 09:27).

I do not think I wrote an m, did I?

Student: (Refer Time: 09:32).

So the gong is sounding. So, I will stop here.

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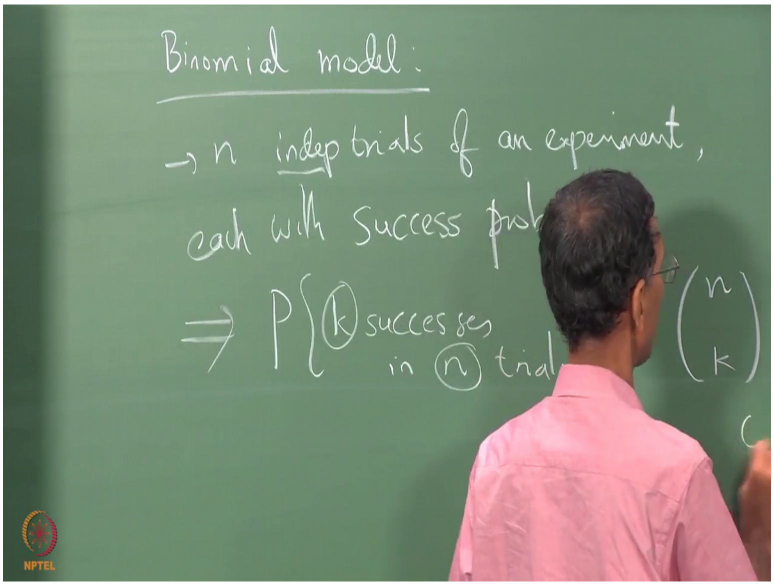

Let me review those binomial model again. So, you have n independent trials. So, in instead of calling a head or tail, we will call it success right each trial has success probability equal to P right; that means, that.

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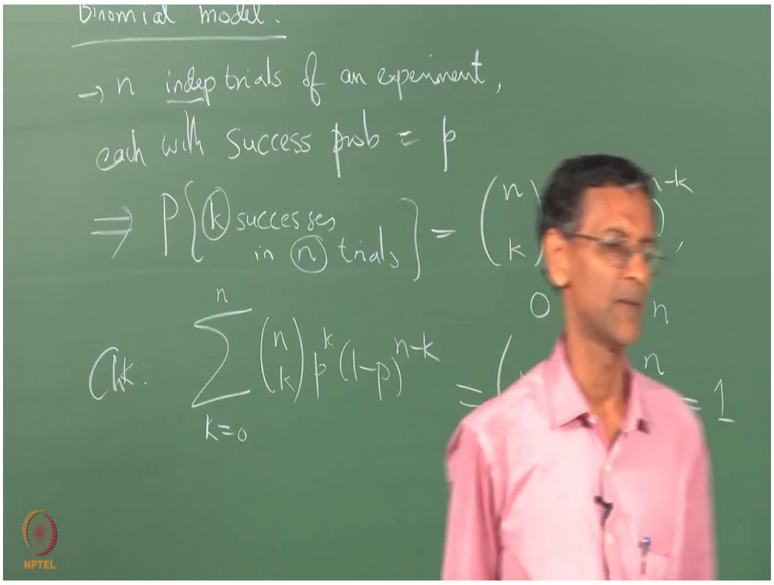

Probability of k successes in trials and this expression n choose k P power k one minus P power n minus k is valid for all k in this range right 1 to n, 0 to n right and you how do we show that the sum k equal to 0 to n, this also we want to be equal to one right what is this actually equal to why.

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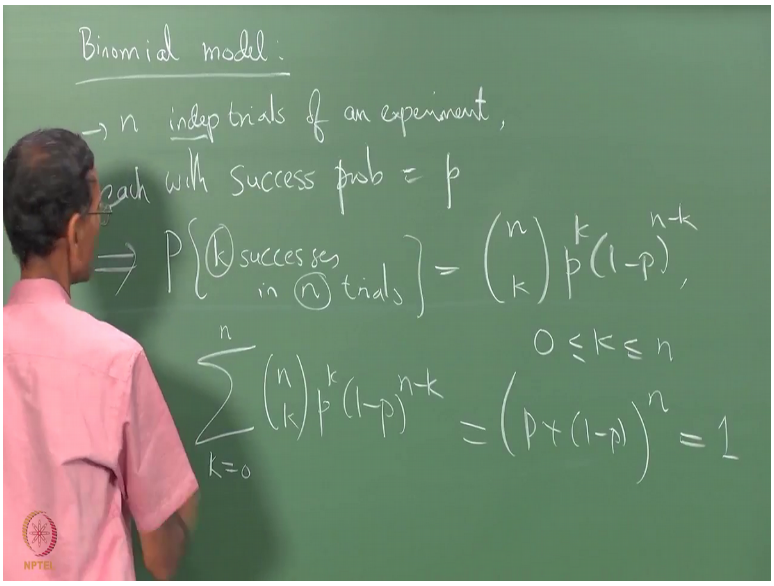

Because this is P plus this. So, this is a check.

So, these kinds of distributions of probabilities we will encounter a lot write down the course by maybe right we will give them formal names right and. So, on and. So, forth this is our kind of first exposure to them right. So, what are we doing we have a bunch of numbers here right just like probability distribution or any other probability you know the way we apportioned in 2 different sample points right except that we have nice a nice formula in this case right. So, and; obviously, these all these are not equal; is it not; they are all can be very profoundly different from each other even for P equal to half we take P; P equals half for example, you get maximum around n by 2 right we will see all of these things little later.

So, this is the binomial model and it uses these binomial coefficients right. So, the n choose k.

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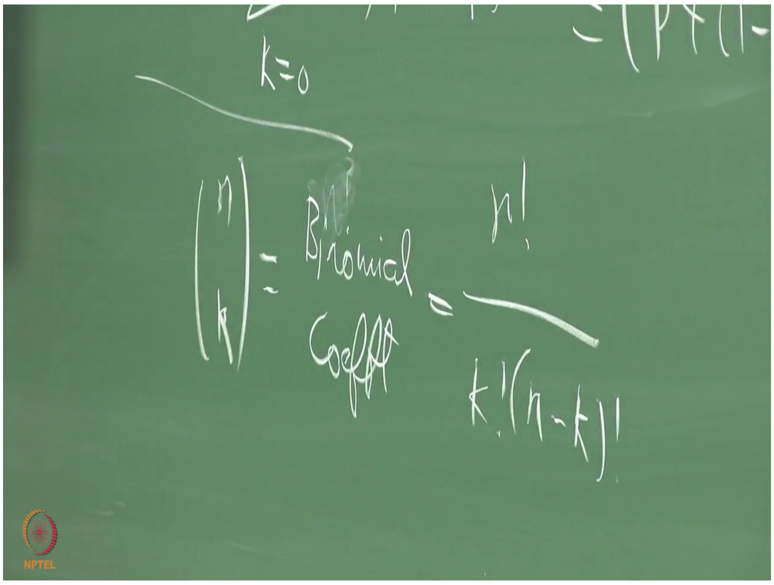

Like again for the sake of the camera, I am writing this as this is n factorial this is let me write this is binomial coefficient right and I assume that all of you can evaluate this for some right or write expressions for it for some values of n and k right if you have small values of k you know how to simplify this. So, let me not spend too much time on this right now the reason why I introduced this binomial model was to show you how right these binomial these binomial coefficients are the combinatorial coefficients keep coming up in this business right there at least 2 different places that we have seen it crop seen them crop up right one is their choosing of m objects out of n or k objects out of n right the number of combinations there will again involve n choose k and this is another place right where in all of these n choose k combinations you have a common term here that is how that is why right this P power k again remember times one minus P power n minus k is the num is the probability of any particular sequence right which has k successes and in n trials ok.

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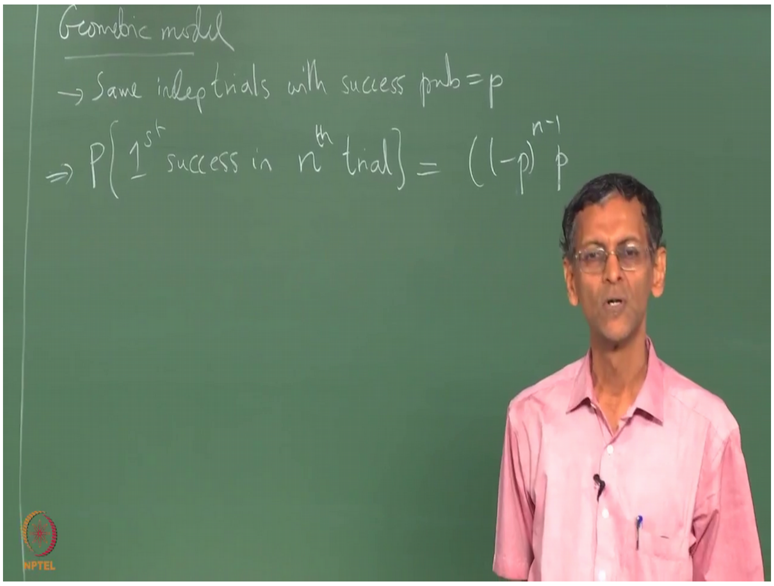

Now, the geometric model which is very linked to this right because the starting point for the geometric model the binomial is basically independent trail of an experiment right. So, each mean here of course, it refers to is the independent trial right. So, similarly you have the same let us say independent trials. So, here what is what did we write yesterday probability of the first success in what the kth trial or the nth trial let me what did I use yesterday, I want to stick with that notation.

Student: N.

Nth trial.

Student: N.

How was this; this was 1 minus P n power n minus 1 times P. So, this is even more easily understood than that because this is straight away right seen to be the product of all those prob basic trail, I mean in individual trials being a fail, fail, fail and then finally, ending with a success right.

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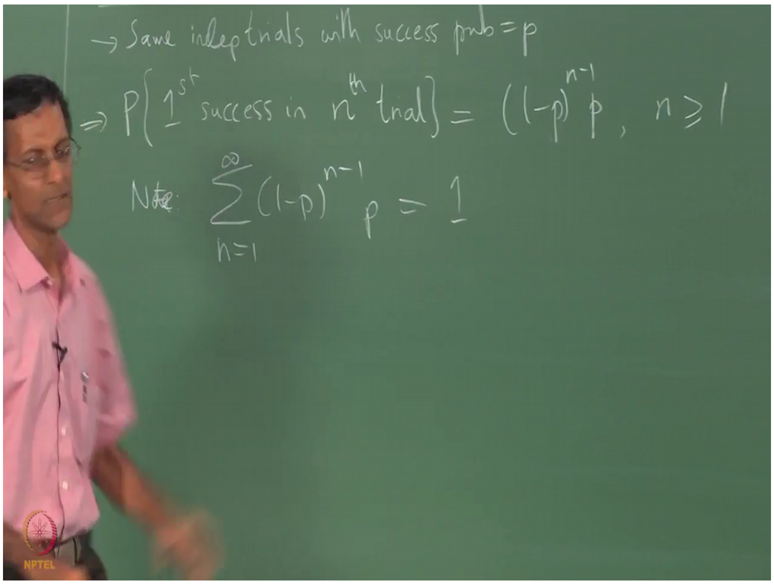

So, this is true for n greater than equal to one and in this case right you have a countable infinity of points theoretically in the sample space right whereas, here you had only 2 power n points. So, here your; we again I just want to say this for completeness, this is a simple geometric series because I mean this is valid because P is a number between 0 and 1 right.

So, you can add up the series simply for that reason P and 1 minus P are both between 0 and 1. Now let us see how there are certain actually this one common thing between these 2 right up mean basically there; obviously, has to be some commonality between the 2 models right and that is the answer to this question what is the probability there is no success in the first m try some I want to use a different number right by different.

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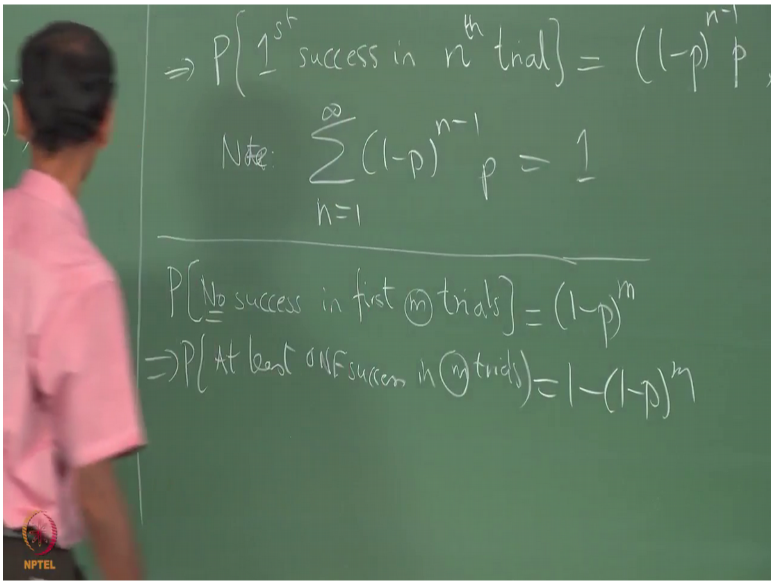

No success in the first m trial.

So, you can calculate this according to both models right and you will you should get what should you get.

Student: (Refer Time: 16:24).

1 minus P to the power.

Student: M.

M right I put m here; I do not know what I want to keep re using n right. Now why did we; how is it they no obtainable from both models, they have to give the same answer right otherwise the 2 models will not be consistent. So, how did we get this from here of course, this is the problem this is got by putting k equal to 0 and n equal to m right m choose 0 is 1 always right and if you chew if you put k equal to k equal to 0. This will dropout and you will get 1 minus P whole power m from here from here what happens well you will get no success at all its like 1 minus P 1 minus P; this is basically exactly the same thing right.

So, no matter which way you do it you have to get there is only one way of writing this right no success in the first m trials is clearly 1 minus P power m right the other nice thing about this geometric model is that its memory less right and making sure that I do not miss saying this because it is a very very very important point right what do we mean by memory less model one before that let me say one more thing what is the complement of this event no success in the first m trials what is the complement of that in voids at least one success right, (Refer Time: 18:02) write; that right at least one success in m trials right and this can only be obtained from here because in some sense because here right here you are saying I am going to stop if there is one success, but here we are saying that we are going to go for m trials anyhow. So, this is actually obtained you know right obtainable from here right which is basically one minus the probability of no success. So, this is going to be what one minus 1 minus P power m right.

Anyway so, but this can also be obtained by from this model if you add up the probability of the strings right f fs ffs up you know go up to m you right, but let us there that would be a different kind of I mean is very similar its summation like this except the upper limit would not be infinity. So, I urge you to go and obtain this number this expression I mean this expression both from here and we have just obtained it from here anyway, but the memory less property of the geometric model come back to that.

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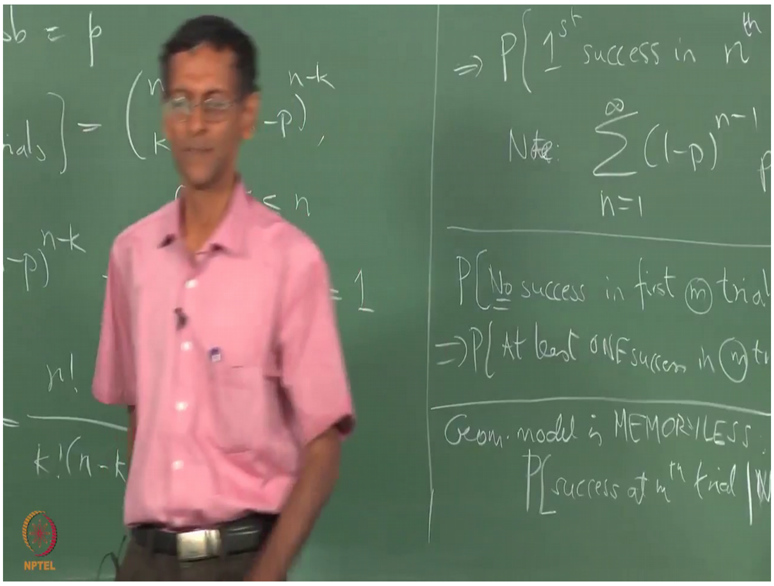

What do we what do we mean by this term memory less here that is you do not care right supposing you say that they have been m minus 1 failures m just the same m given the information that they have been m minus 1 failures what is the success at the mth trial, it is still P because the trails are assumed to be independent right probability of successes at nth trial given no success.

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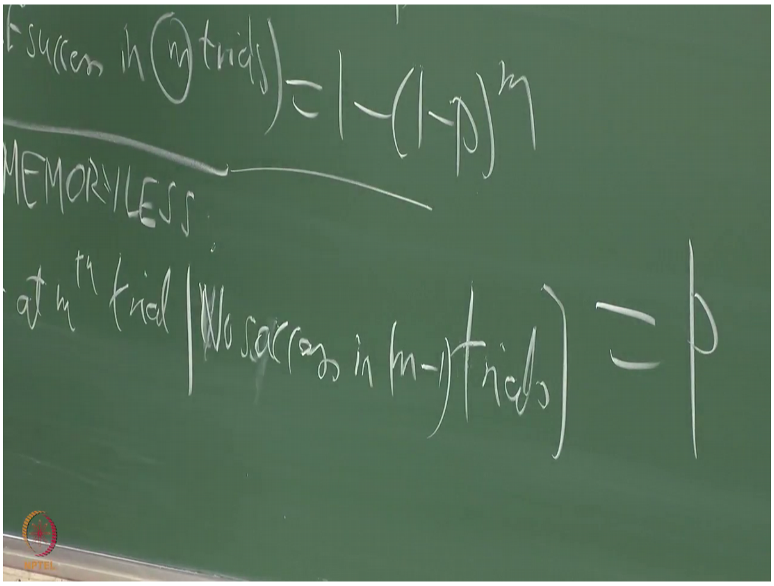

Or no success in m minus 1 trials is still equal to P right which is the probability of the success at the nth m trail by itself right. So, this again is a theme that we will pick up on later has profound implications for the so called geometric distribution that we will be studying it in great depth later on right anyway. So, I think for now I think this serves to just introduce the idea right of repeated trials right and these are the 2 most important distributions that come out of that right notice that, I am not using any language of random variables or anything to describe these because you must remember that probability by itself is basic right and you know what we do wont top of that the jargon we add on top of that is something else right, but as of now you can write these things even without using any fancy terms right.

So, do not think that as soon as soon as we talk about binomial model you have to bring in some other concepts it is not necessary, right.