Probability Foundations for Electrical Engineers
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Lecture - 07

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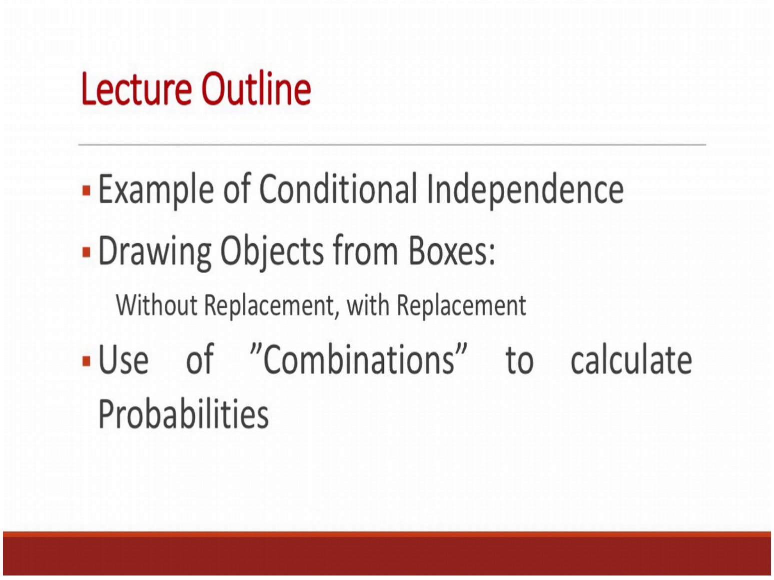

Another example of conditional independence comes when you write.

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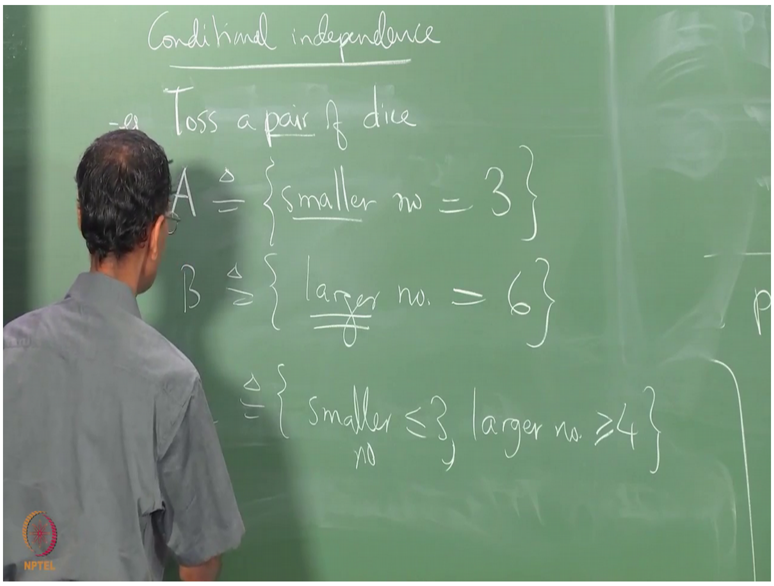

Let us say you toss a pair of dice right pair, now right at this I am not going to work out in great detail. I am going to let you do it yourself, let us say define A define this event a as the event that was the smaller number is exactly equal to 3. And then B as define it as a larger number is exactly equal to 6 and then C as the event that the smaller is less than or equal to 3 and the larger is greater than equal to 4 this comma always by the way is going to mean and from now on. The smaller number is less than 3 the larger number is greater than equal to 4.

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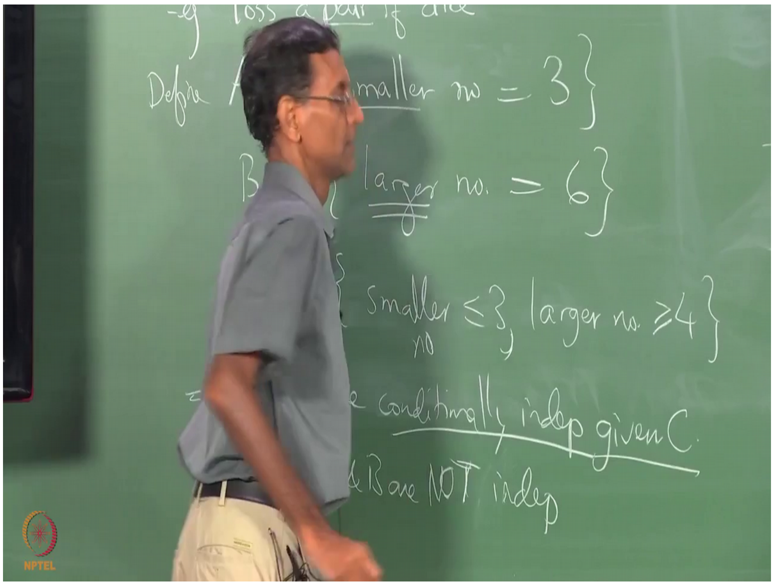

Now, I am not going to I do not know let us see how much time we have to widener, do not want to spend more than 2 3 minutes on this right. So, it turns out that A and B are conditionally independent given C. How do I prove this? I have to calculate P of a B given C p, of a given C P of B given C and do the multiplication.

So, I need to do all those calculations right, but they are not independent. But A and B are not independent conditionally independent given C, but they are not independent less than equal to 3 will have.

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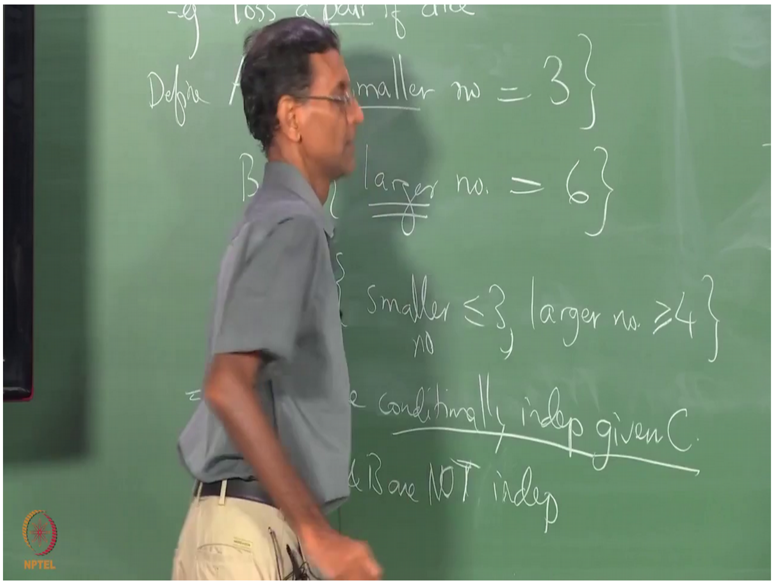

So, how do you had have to actually write out all combinations which would give you this and this is a compound space where and you are assuming whenever you see pair of dice at each, write that now the sample space is 36 points in each point is equally likely. That is the standard assumption the dice are independent each dye is independent of the other and that gives you the 1 by 36.

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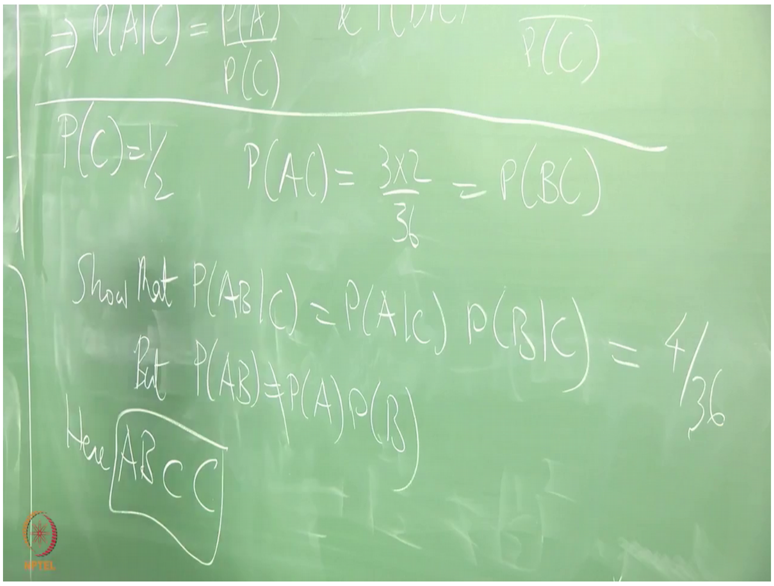

So, this P of C is half that is there any way there right, what I meant to say was P of s C will be equal to 3 times 2 by 36, this will be equal to I meant to say this in wanting to say this. I said that right the AC and BC have the same probability or you are right that this is not true, but what I meant to say was I if A is a subset of C and somebody asks you to find P of a given C, you can just easily find it as PA by PC that is basically or what I wanted to say here, but this is not true right you are absolutely and that in this particular case is not a subset of c. But I just want to point out that BC and ac have the same probability you need all of these things right and you have to show that I am not going to do it any more than this, show that P A B given C is what will be a given C multiplied by this. So, in this case it is exactly equal to 4 by 36, but P of a B is not equal to P of A into.

So, I just I am not going to spend more time, I do not have write any more time to spend on this; but this is what I write, I want you to show by yourselves you are perfectly capable of doing it. So, you would require all of these things right a in what I meant to say in fact, what is a subset of something I think I guess A B is a subset of C that is it true that A B is a subset of C, A intersection B is a subset of C is that right; that is what I wanted to say right. A B is a subset of C not A is not.

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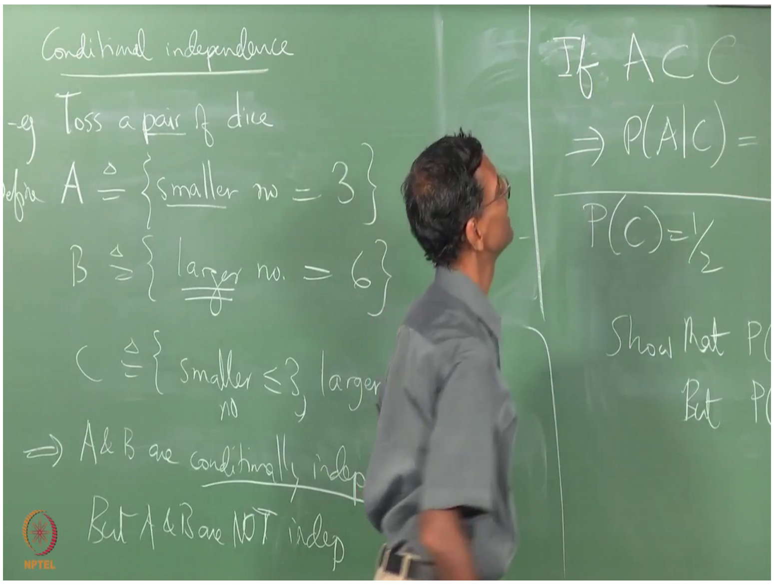

So, if I wanted to well I just replace is note with an if right this is perfectly. So, here A B is a subset of C, this is I am doing all this in spending half the lecture on this just to write strengthen your intuition in this important concept, you can have conditional independence which is different from absolute independence and we have seen 2 examples of that.

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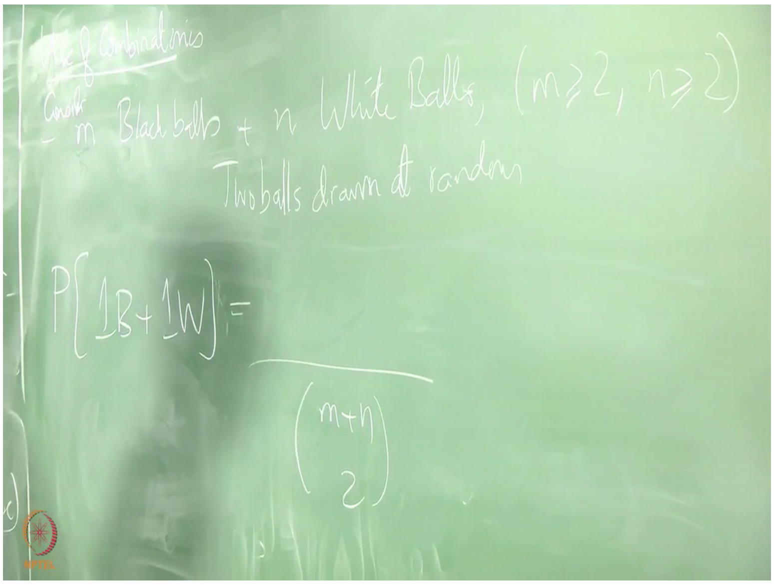

Say you have m black balls plus n white balls in the in sum this is unfortunately, here I have to do this for drawing you need to consider this kind of situation you do not have a choice. So, if you have a collection of mm m black balls what I say m black balls and n white balls and you are drawing let us say 2 balls, this is standard example where you are closing your eyes and pulling out 2 balls, they all feel a like you can pick out any 2 of them.

So, in this kind of a situation what is it that you can, what kind of outputs can you get either get 2 white 2 black, assuming that m and n are both greater than 2. Assuming both are greater than 2 right, you can get 2 white 2 back or 1; 1 by white or 1 black, you can also look at right drawing sequentially turns out that if the overall the aggregate probability, for example of drawing 1 white and 1 black it is the same right no matter how you calculate it; whether you calculate it as first ball is white second ball is black or the first ball is black second ball is white ,you can do it like that or you can say I am going to pull out both balls and use combinatory to write out that probability. So, it turns out for example, here the probability of 1 black plus 1 white here, let us there is no concept of which is first and which is second you are just pulling them out together.

So, this is going to be the denominator of this is always m plus n choose 2. Why do I get this m plus n choose 2? that this a notation we are going to use for common combinations now right, not the capital C thing right this is the standard notation which is used for what choosing n objects out of. So, r objects out of n regardless of ordering.

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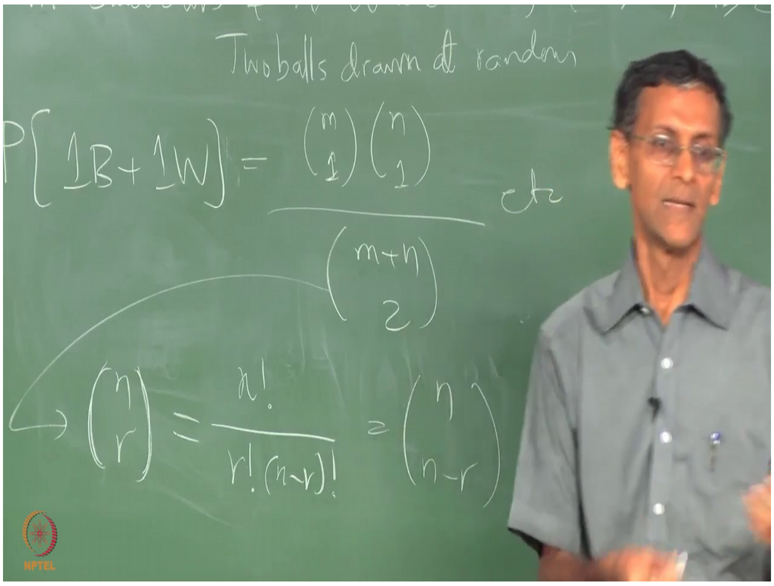

So, what is this call the car this binomial this coefficient equal to this combinatorial coefficient this is n factorial divided by r factorial divided by n minus r factorial. So, how do you write this is the internationally accepted standard right, no C stuff you write you put 2 brackets like that somewhat elongated right, you put the n on top you put the are at the bottom.

Please stick to this right and transform all your earlier I am sure all of you have seen combination combinatory at some point in the past right. But this have you seen this notation specifically yes you have in those of you have not seen this particular notation please note it and factorial also will just use the exclamation mark right. So, here this m plus n choose 2 is what the number of combinations you can have. The total number of combinations of chooses 2 is just 2, the number of ways of choosing 2 balls out of m plus n right.

So, here what we going to put the numerator it is m choose 1 time N choose 1 and these numbers can be generalized, I can have right I need not have just 1 and 1 here. I can obviously have the even the 2 and 0 for example, is going to be m choose 2 divided by this no n choose 0 right and anything chose 0 always 1, you all of you and you all know that. So, that n choose r is always equal to what n choose n minus r is it all very, please go brush up your combinatory is to make sure that you are completely comfortable with these things. So, you can write out the probability of right either choosing 2 black balls or white 2 white balls.

So, this 1 black balls plus 1 white ball using this now the interesting point I want to make is right in this business of you know this is called drawing without replacement, you are not putting back anything it is just taking it out. And so they aggregate drawing a 1 bite bottles on black ball. The probability of that does not depend on the order right and it does not you know whether you know look at first ball and I just said some minutes back, first ball white second ball black and then you also look at first ball white sorry first ball black second ball white you add them up you will get exactly this number right. And it turns out in this case the 1 more interesting point which we can make is this, what is the probability of the first ball itself being black it is m by m plus n, this I do not think anybody will complain.

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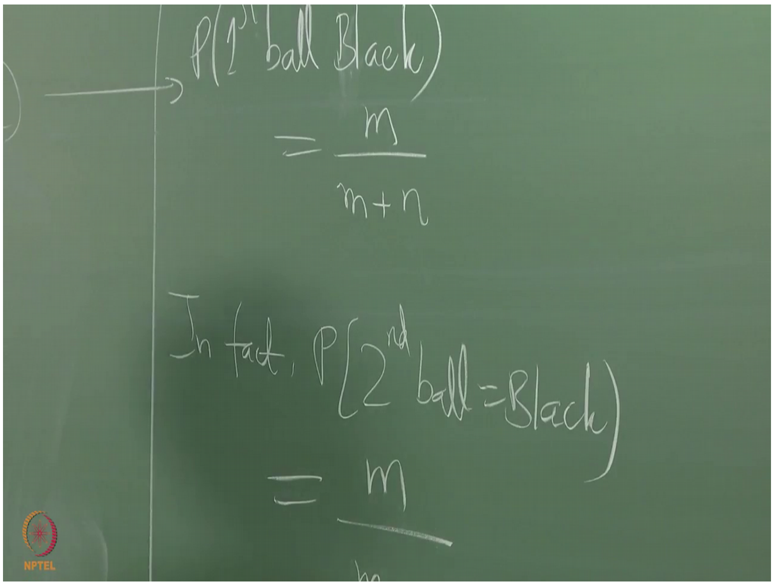

This is in fact a simple way of simulating what a biased coin if you want right, if you do not if you think that you have you got you can manufacture a coin that will land with heads is probably 1 by 10 or something all you have to do is pick the same that many white balls and black balls and simulate this.

But what is interesting is that this same number is also the probability that any you how many balls can you draw you cannot m plus n balls, you can keep on drawing balls what at any point the probability of drawing a black ball will be that n by n plus n as long as; I mean a fact that is true as long as there are balls in the box that you can draw right, the second ball being black that is here regardless of the first ball you do not care. If you draw 2 balls you do not care about the first 1 look at the probability second ball is black that also will be m plus n.

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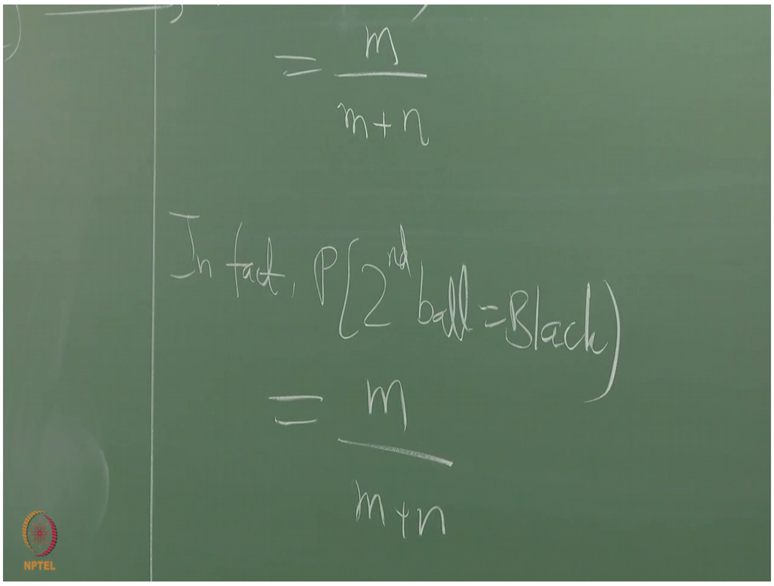

So, here we do not care about the first ball, you can easily show this in the way that we did earlier calculation right. You can condition on the first ball being either white or black right.

So, how do you get assuming that m is m and n are both greater than 2 rights? So, you can have first white second black or first black second black right and if you add it you will get this.

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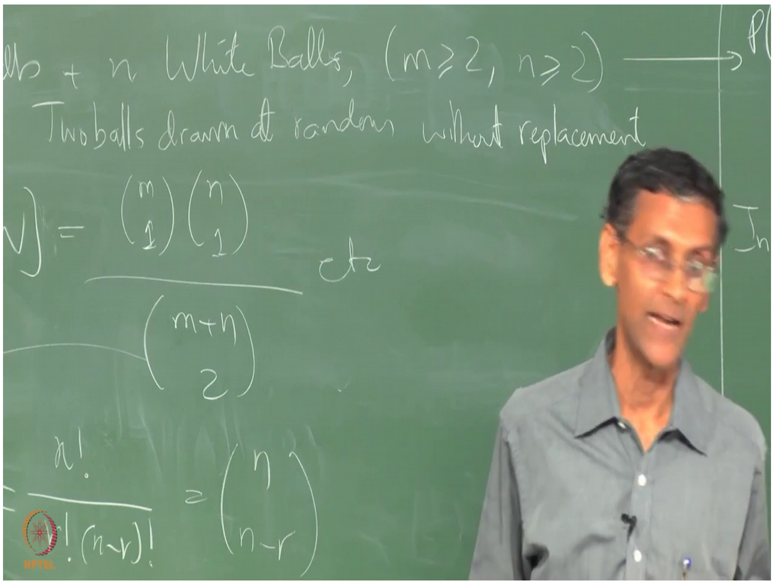

No they are not with drawing we are assumed it was drawing where it same with drawn at random without replacement. Actually it is true even with replacement that is the interesting thing. Even if you put it back you get the same m by m plus n, but even without replacement you still get m by m plus n.