Probability Foundations for Electrical Engineers
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Lecture - 17
Conditional Independence

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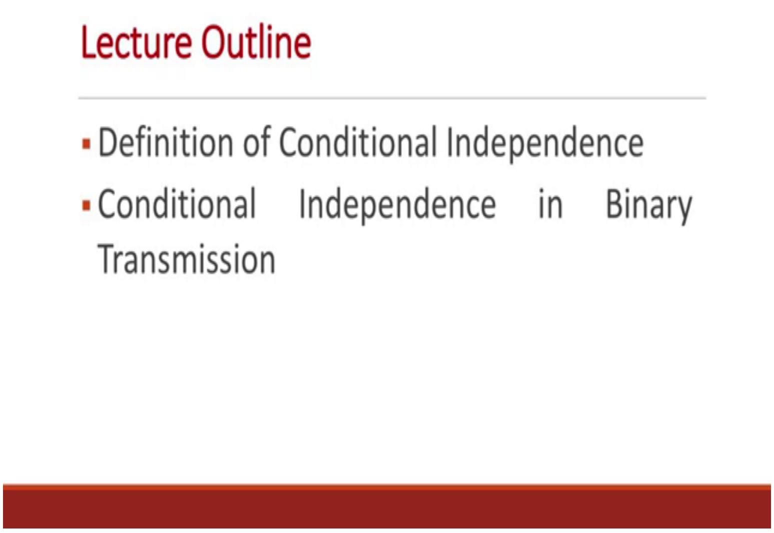

So, now, I will talk of 1 more a very important concept which is this thing about conditional independence.

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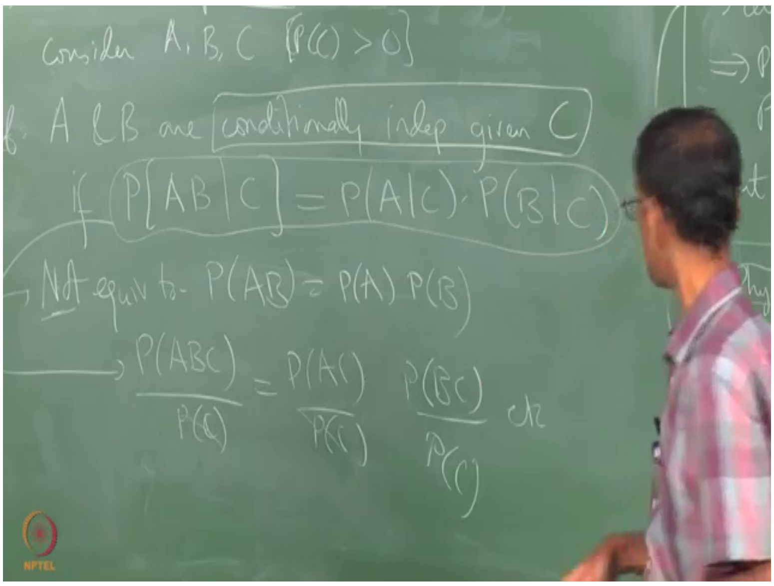

So, consider some three events in A,B, C with let us say PC alone we are interested in conditioning on C where if you the I am not going to keep on saying then saying maybe say maybe 1 last time right, anytime you write you conditional an event you assume implicitly its probability is nonzero right.

So, A and B definition right a and B are conditionally independent given C, if what P of A B given C they this intersection probability given C can be written as P of A given C multiplied by P of B given C. Note that this is not the same as the absolute or unconditional independence of A and B right. This is not equivalent to P of AB equal to P of A into P of B the 2 are not the same at all. You can have 1 without having the other right, but if you want to simplify this, you can there is no reason why you must write it only in this form you can write it in some alternate forms also right.

So, for example, this will become P of A B C divided by P C equal to P of A C by PC by times P of BC by AC and you can knock out 1 of these PCs and write this in some other way right. You can manipulate this to write it in some other which I am only do you do right it is not very important I am just I just from a manipulation point of view right nothing stops you from simplifying right and you can take 1 of these down here and look at right if what is P of ABC divided by P by B by PPC assume this is positive right if this is positive you can divided by this and put it down here P of ABC given sorry divided by P BC is nothing, but P of A given BC.

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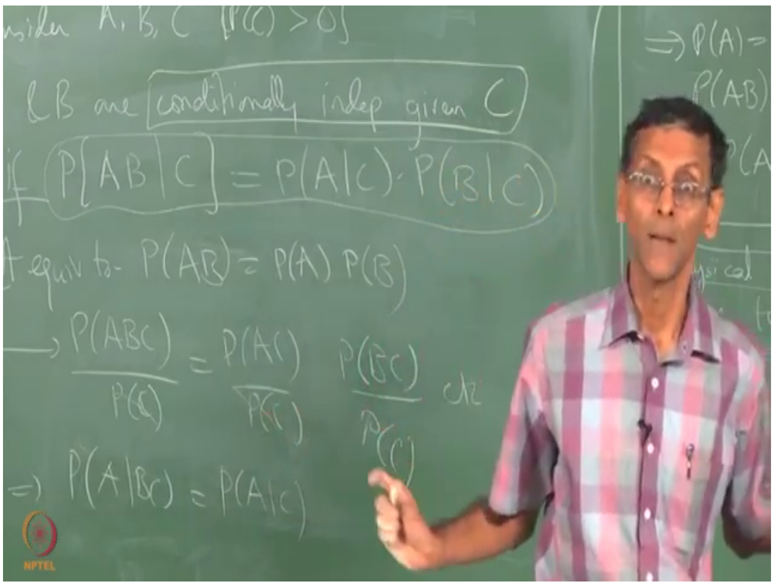

So, ie this means that P of what A given BC is going to be equal to what you will be left with is AC by PC which is a given C maybe I did not need to write the whole thing out, but. So, this will imply this and this also will apply this in the case that both BC and C are have positive probability right of course, if BC has positive probabilities C will definitely have positive probability right. So, basically it is right. So, this is another way of writing the same thing, but typically you know they do not do this manipulation, they stick to this more I write a simple definition like this which you can easily remember right anyway. But remember these 2 are definitely not the same at all right now when you encounter conditional independence in practice right, we go back to our communication channel that we did we talked about yesterday.

So, examples of conditional independence, supposing you have multiple uses of this channel.

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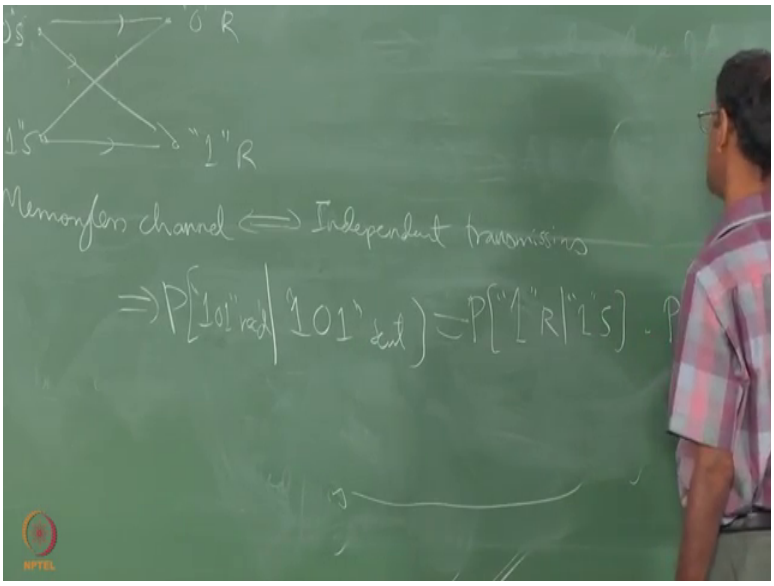

Let us say this is 0 sent and 1 sent right and 0 received and 1 received this is 0 S 1, S 0, R 1, R right. So, it turns out that in many cases right the memoryless channel means that successive transmissions are independent right. So, this memory less channel means what independent transmissions.

So, individual transmissions when transmissions are independent then what do you say. So, this implies that, the probability for example, of supposing you send given let us say 1 0 1 is sent right and let us say you are looking at the event that, the same on 0 1 also received right the whole thing is fixed, we have already said that right in our discussion now we will assume the whole right. We can only take measurements, we cannot change our decision rule or whatever right everything is fixed about this you know that you know you have sent you can send a 0 and receive a 0 or receive a 1 also right.

So, all of that is fixed right. So, this probability of 1 0 1 received, even that 1 0 1 is sent. So, here now you are freezing the input sequence.

So, how does conditional independence come into the picture now? Given this in this sequence of sent if the channel is in fact, memoryless right, then you would write this the probability and this conditional probability as what as this would be like A B given C right where A B is 1 receiving in this case it is abc; that means, also the abd whatever three events I am talking about right I am talking about three different transmissions.

So, this would be the probability of 1 R given 1 sent multiplied by the middle 1 which is 0 right given 0 sent.

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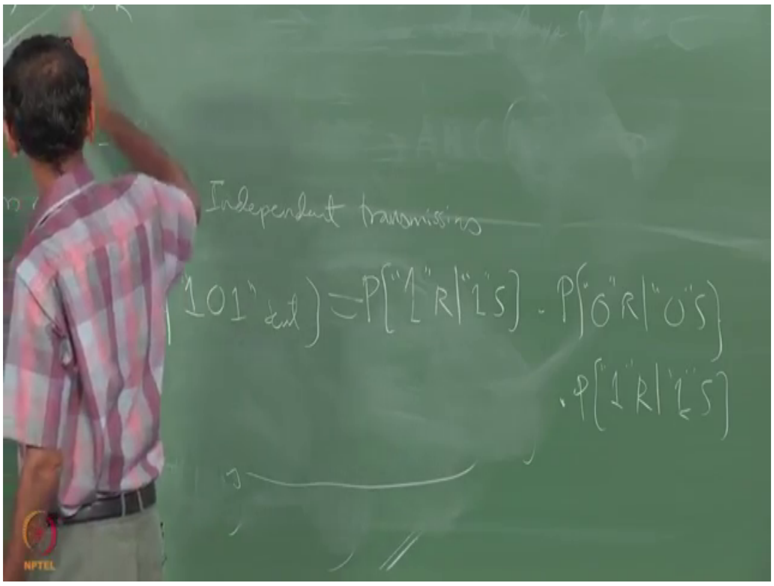

And then again another 1 R given 1 sent no of course, I could have a different sequence here, I could say well what is given the same input 1 0 1 sent I could have let us say just says look at 0 0 0 or 111 whatever all I would do in that case would be replace these three guys while keeping this 1 fixed. So, in a memoryless channel right acts independently on each 1 each (Refer Time: 07:11) it does not care about why the bits came from.

So, this is basically an example of conditional independence, but if you looked at right if you looked at the overall probability of getting 1 0 1 that will not be equal to in general right probability of receiving 1 multiplied by the probability of seeing 0 multiplied by the product receiving 1. Again because these three that would depend on the prior probability that you will attach to or you know the sequence 1 0 1. Supposing, for example, right the in the same input bit 1 was mapped to the sequence 1 0 1 some coding was done right that is an example of a simple example of coding right, where you are sending more bits than is actually needed. Then in such cases right clearly right the probability of receiving I mean probability of receiving 1 and probability of receiving 0 and probability 1 are not independent, I mean they are not independent events right you would this will not be right.

So, in general what I am saying is that, what am I saying? I am saying 1 0 1 received is not equal to the probability once 1 R probability of 1 0 sent sorry R into probability of 1 (Refer Time: 08:40), ok.

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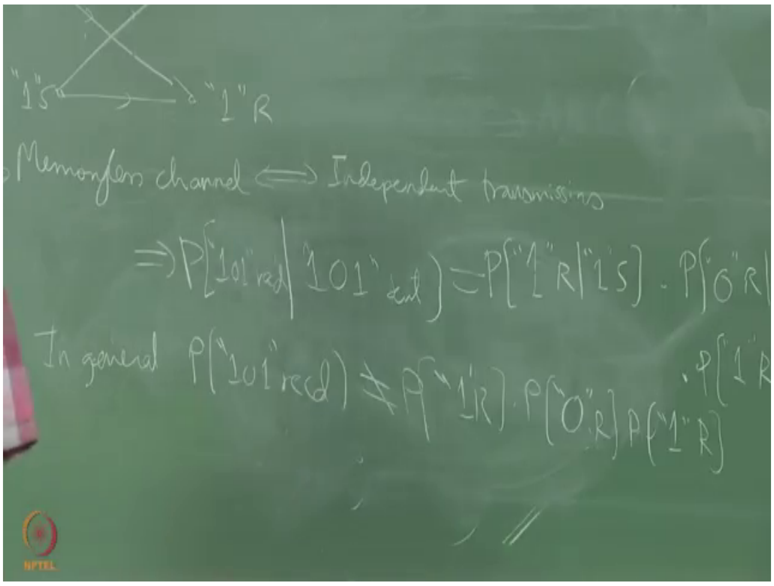

So, because I am not right this one requires the input right you require this overall probability requires you to know how the input sequence 1 0 1 was formed. This one does not this one looks only at transmission this one look is a combination of both the transmission and the input and prior probabilities, this one requires only transition probabilities right.

So, if you know for example, these condition 0 given 0 1 given 0 those things it is enough that is enough to write this guy out whereas, for this that is not enough that clear any questions on this. So, this is an example why you do have conditionally difference, but you do not have overall independence of the received bits right. So, we have come to the end of today's class we will continue or tomorrow or with a different topic I am going to speed up right. We are going to revisit this example that we did yesterday in a bit. So, you know we did it quite fast.

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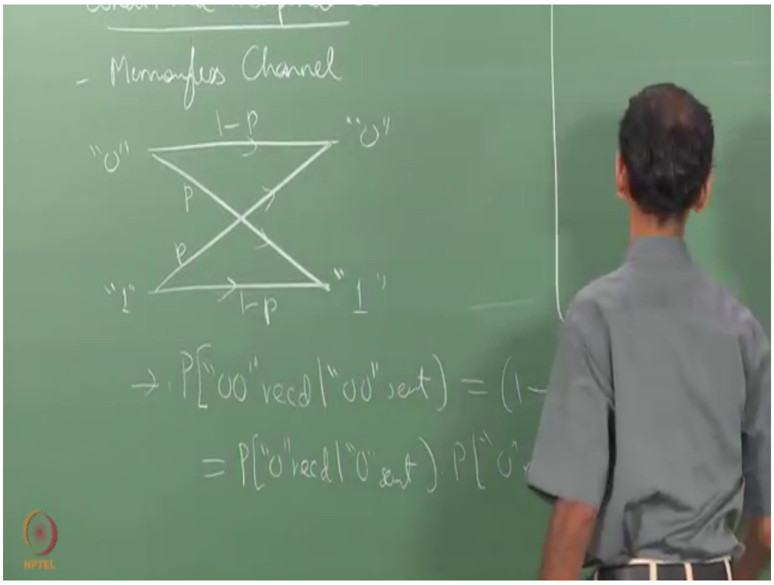

So, this memoryless channel basically it does not remember what it did.

So, if I have 0 sent 0 or 1 sent and 0 or 1 received as per yesterday's notation what did we say? So, what is it? So, if I want to say a 0 0. So, let me put some now net numbers here or whatever symbols. So, this small pay will represent the cross, probability; probability of receiving 1 given 0 and receiving 0 given 1 right. So, that these are 1 minus P and 1 minus P I think yesterday I did I put any or use this notation or not no. So, let us use it today so that, if I have the problem if I want to look at this probability of 0 0 received given 0 0 sent right just for any 2 right 2 instances to transmission instances. So, this is going to be 1 minus P square right how do I get this 1 minus P square? Is 1 minus P times

1 minus P if I want to look at let us say 1 0 to receive given 0 0 sent it will be P times 1 minus P like that right.

So, that is a property of a memoryless channel right each transmission is independent of any other transmission the channel does not care. This is it a channel model and does not say anything about how you get these symbols in the source right is this clear. Now if you want to look at right the probability of receiving a z 1 0 and then another 0 after that those 2 events will not be independent right. So, you can see that this, this is the.

This let me write it even more explicitly, 0 0 receiving even 0 0 sent is a probability you have 0 received given 0 sent multiplied by the probability of 0 received another 0 received 1 and are the zeros and these 2 of course, are the same right. So, that this is how the channel works, but if. So, you can say that that the bits are independently transmitted by the channel right this, this is a version of conditional independence that we talked about yesterday.

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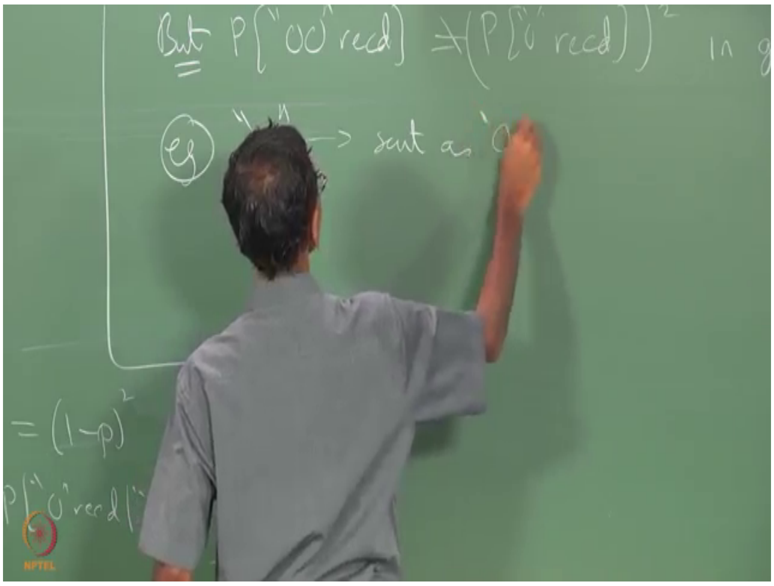

However if you look at the probability of receiving 2 zeroes in succession, it will not they those 2 events will not be independent in general do you see what I mean.

This will not be the probability of z 0 received squared. So, this is a case of conditional independence without in some sense absolute independence right now why is this happening? This is happening because you right you do not know how you are forming the input bits right. Supposing right I am looking at this 0 0 as I said yesterday I this was a coded version of an input 0 that is some supposing an input 0 or 1 was chosen independently with probability half right.

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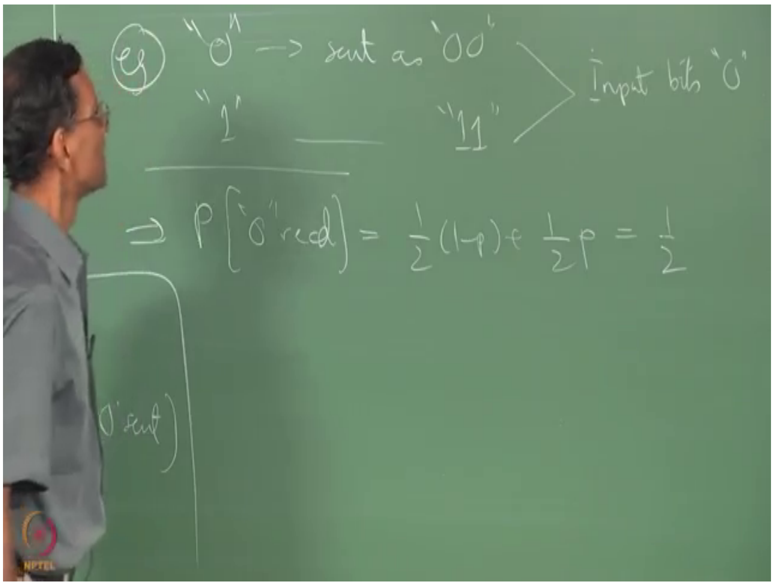

That part can be independent, but supposing a 0 is sent right 0 is sent as 0 0, and then 1 is sent as 1 1 right this is a simple example of a repetition code right, you are why you are introducing redundancy just to improve the performance I know the reliability of the communication channel it turns out it is not a very good thing to do.

But it is one of those classical examples that start first in the classroom realist rate how right you can use you know you can sort of lower the bit error rate of course, you would not want to repeat it in even number of times you repeat it an odd number of times. And you take a majority count or something and then in the receiver to figure out which was actually sent right so. But anyway, but we are not going that far in this course we are only looking at let us say just a simplest version which is just repeating what it just wants just to prove a point right.

Now, in the supposing you are 0 and 1 are equally likely right the input bits 0 1 let us say they are equally likely that is probability is half, then the probe what is the probability of receiving a 0? This that is what is it probability of receiving a 0 no man it just with just 1 use of the channel right. Then the prop which this will imply this the input which are equally likely the probability of receiving 0 is what you can sure that it is half into 1 minus P plus half into P how did I get half into 1 minus P plus half into P? This is just for using the formulas or whatever we did last week right or whenever already, but this week we conditioned on the input bit being a 0 or 1 and write this out. So, this is going to be what is this going to be this is half is not it, but what is the probability of getting a 0 double 0, this the same 0 send is being sent as a double 0. So, the probability of sending a double 0 is also half right because you are repeating that 0, right.

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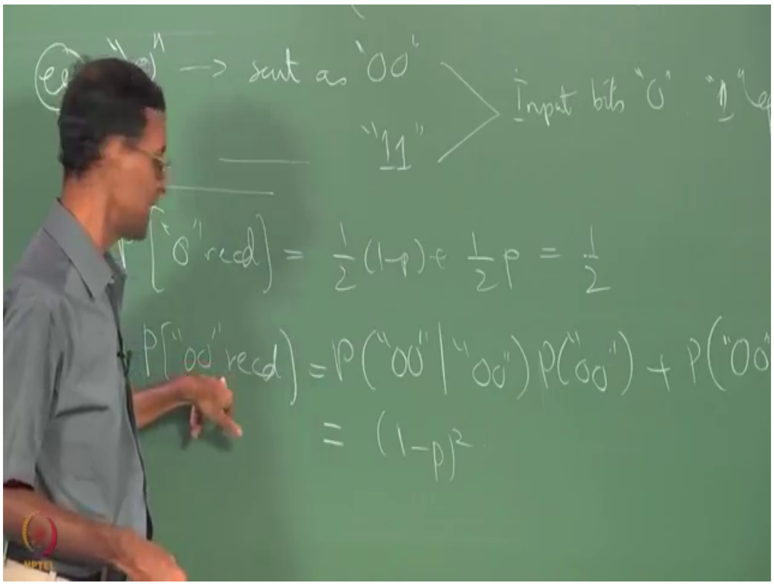

So, probability of receiving double 0 is basically, right. You have to write this as probability of 0 0, given 0 0 plus or times the probability this is the input right you can also get it with 1 if the bit where one, but you do not have this combination of 0 1 and 1 0 is at the input because you are sending each bit is being doubled before being sent, right.

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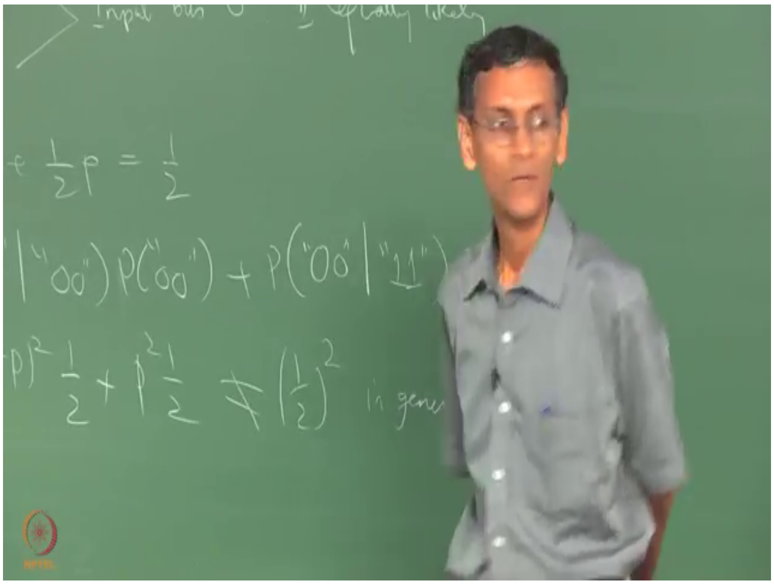

So, you do not have the combination 0 1 or 1 0 right. So, you only have this now you write this out, this is going to be this 1 one minus P squared and this is half because this is the same as this input bit 0 which are right. I am looking at right suppose you are looking at a single 1 was transmission of a single input, which is sent as it double zero. So, what is the probability that received sequences another double 0, right. The input bit could have been just as well been 1, in which case 1 one would have been sent, but you can still get 0 0.

So, this is the probability if they have overall probability of getting this doubles double zeros 1 minus P squared multiplied by half right the probability of sending this is half what about this P squared into half. So, this is not equal to half times half in right in general it is not equal to this. Especially if P is right very close to you would like this P to be very close to 0 actually right. So, if you say if you are allowing a small value of P for example, right you can easily see that this is not going to be half squared. In fact, if P is very small it will be close to half not half squared right why we are not taking 0 1 and 1 0.

Because those combinations will you know I am looking at after the coding 0 sent is double 0 right I am looking at the event, I mean looking at the transmission of this coded bit 0 which is coded as 0 0. So, in the reception of that that can either be received as 0 0 1 one, but the 0 0 can come also from 1 being sent which is which will be sent as 1 1, but the channel does not know about the coding right. The coding is done independently of the channel the same channel is used for all the coded bits independently right.

So, that that is a conditional independence which is imposed by the channel, it is a memoryless channel; those of you that want to study information theory you better take the example very seriously right I do not know how many of you out there right well I want to get into the wireless information theory business, but for all of you all those students right this kind of stuff is just it is a basic right starting point right the discrete memoryless channel as they call it out there right.

So, there you have right any a regardless of any coding done and the input bits right, that the probability of a string of bits being received given a certain string of bits being sent you can write that in for a memories channel in terms of the individual transmissions. What is the probability of 0 0 0 received given that 0 0 1 was sent it is, you can write it out right in terms of the individual transmission it means what 0 to 0 0 to 0 0 to 1 whatever you can look at the patterns of the input and output and figure out, how do and all you have to do is multiply out they right, the various conditions that are happening there and you get the right for this block of bits, this the output block given this particular input block right you can write it out there is no multiplication.

So, that is conditionally bends without the independent independence in the deeper sense of is this probability of getting a output 0 in the first as the first bit independent of the 0 in the second bit it is not, that is what we are showing here. So, this is a right a more real life again no more write electric communications, I should say example inspired example rather than taking coins and flipping them and so on. Although, the same mathematics can be made to apply to that is setting also, but I think this is a nicer way to understand right. So, I think this should be sufficient to explain it.