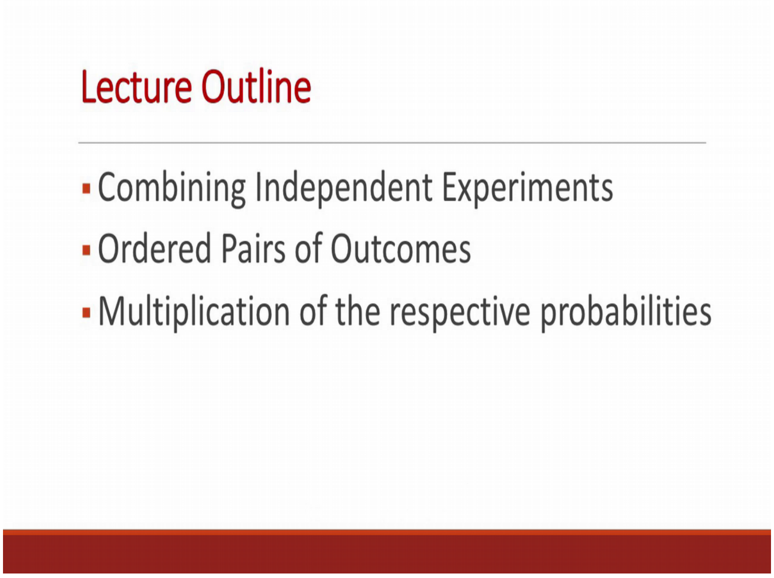
Probability Foundations for Electrical Engineers
Prof. Aravind R
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 11

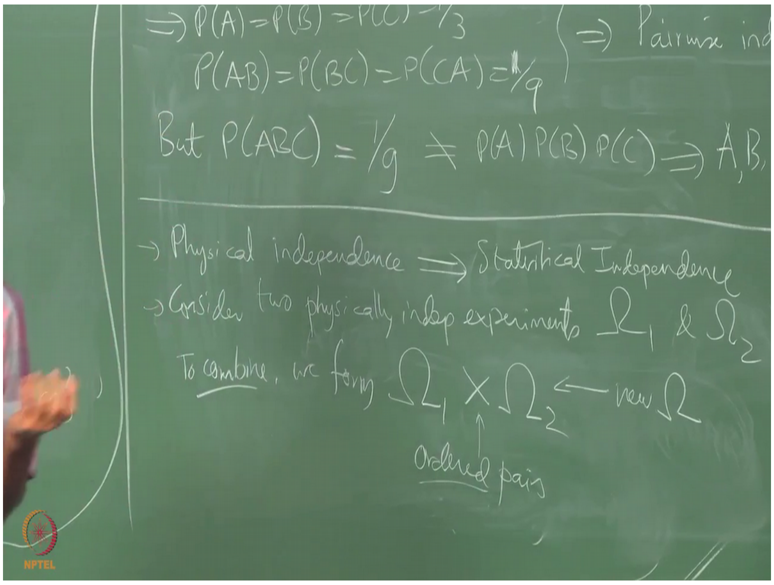
(Refer Slide Time: 00:13)



So, the next thing we want to do is look at I know look at this business of combining experiments which are known to be physically independent. So, all of you will have this intuitive idea that is what I checked with you in the beginning of this lecture or everyone understands physically independent means independent somehow.

If you toss two coins there is no reason that one coin should affect the other at all. So, how do you formally. Therefore, combine independent experiments? So, physical independent, physically independent experiments you always give you a statistical independence.

(Refer Slide Time: 00:59)

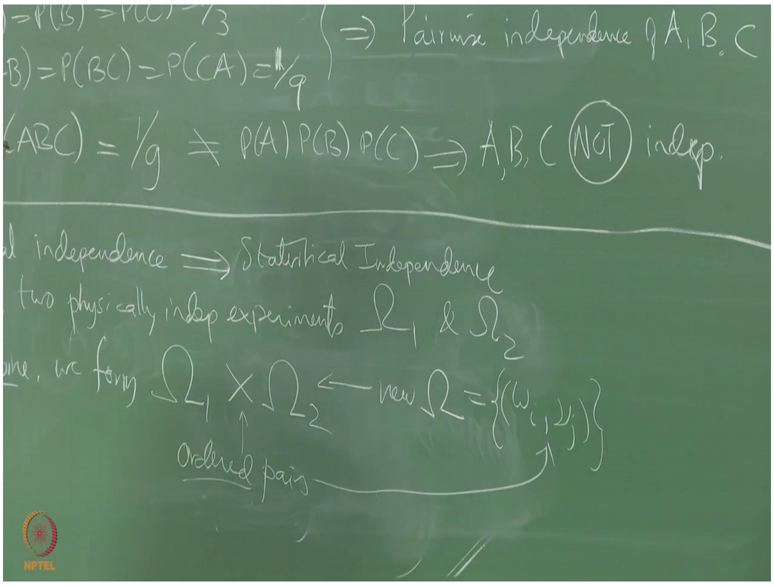


Let me just say physical independence always imply statistical independence. Statistical is going to be our objective to indicate independence the probabilistic setting the specific the word and the meaning connotation that we associate specific to all of this.

Physical independence; obviously, as any two things which may or may not be probabilistic which are done independently or which happen independently of each other. So, supposing I have two physically independent experiments omega 1 and omega 2. Now, again we are assuming that they are random experiments with some sample spaces. So, you have omega 1 for the first one and capital omega 2 for the second one, so how do I combine them. Obviously, the sample that the combining sample space, now each trial of the combined experiment means what you have to do both the constituent experiments omega otherwise you are not you have not completed the full trial mere. So, when you talk of combined you cannot just say I will do one and not do the other.

So, to combine them we form omega 1 this Cartesian product omega 1 comma cross omega 2. What is the meaning of this Cartesian product? It consists of all ordered pairs. So, this will be our new omega where this capital omega has all the ordered pairs of omega 1 and omega 2. So, this one has say supposing both are discrete and also countable with sorry finite that is if this has N 1 points in this has N 2 points then the overall omega will have N 1 into N 2, N 1 N 2 points. So, this Cartesian product I think, this mean means ordered pairs the ordering is very important you cannot change it you have you the by convention you have to agree that.

(Refer Slide Time: 03:48)

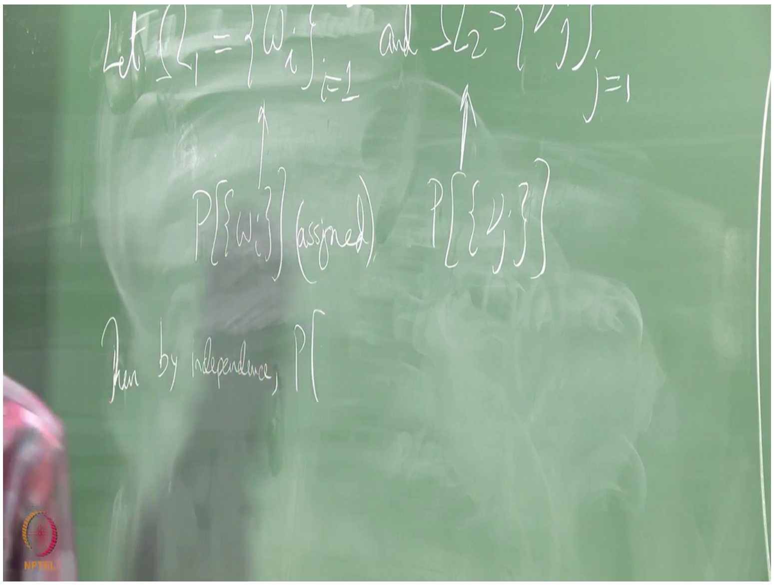


So, I will write here this will write the new omega will be something of this form ordered pairs omega small omega i nu j where omega i comes from experiment omega capital omega 1 and nu j comes from omega 2.

So, this is true in the case of let us say countable both omega, capital omega 1 omega 2 both being countable. And the same kind of thing also happens in the uncountable cases where if you have two random number generators or 2 pi spinning pointers you can there is no bar on finding or taking a Cartesian product of any two sets. So, at this point we just simply combining the two sets omega capital omega 1 where the two sets to form this new set omega.

So, obviously, for example, if both of them are identical that is omega 1 omega 2 are identical you can get things from 1 3 and 3 1 for example, if you do not know, if you throw dice right. But 1 3 is not the same as 3 1 that should be very clearly kept in mind 1 2 is getting one on the first die and 2 on the second die or 1 on the first throw or 1 throw and 2 on the second or whatever though you are very clear about what the first point refers to what and the second one. So, clearly omega that the cardinality the number of points and omega, the capital omega is a product if the both of these are finite and more importantly now we have to look at the probability that we want to put to associated with all these points.

(Refer Slide Time: 05:48)

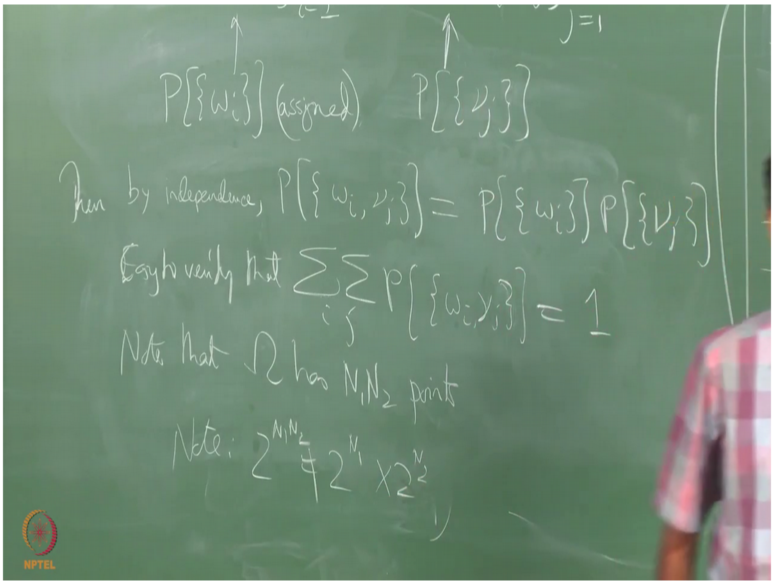


So, once again let me continue here. So, let me clearly write out. So, omega, so omega 1 is basically this nu small omega i let me say let omega i N 1 and omega 2 be nu j, j equal to 1 to N 2. So, this is very clear about what capital omega and capital omega 2 are.

And then independently separately we have let us say we already give I know you start out with some probability assignments to the two. So, here we have let us say the probability assignments capital P of omega i these singleton events here have all you know we already assign probabilities to them and here you have. So, both of these are assigned this is anyway our starting point for the discrete case we have already said that we are going to start by assigning points to all the sample points of the discrete space.

So, now here we have the new point omega i or the overall the ordered pair omega i comma nu j and the experiments are known to be physically independent. Then how do we differ, then by independence that is physically independent the physical independence implying statistical independence.

(Refer Slide Time: 07:34)



Independence is our is our justification for what for writing this point this ordered pair to assigning what probability are we going to assign to this product, at the product of the two.

So, this is a formal way of saying well, if an experiment consists of tossing two die twice or two die to dice once whatever then all 36 points have the same probability how did you give a 1 by 36 and how do you get that, just by invoking this as many times as you want. Similarly it do not stop if you know it does not stop it does not have to stop it does not have to only apply to two dies you can also have a coin being toss out here and it died being tossed out there is no bar on what omega 1 and omega 2 can be as long as it makes some sense to combine them you can always combine them. After all we only interested in what we can do mathematically and not whether everything that we do mathematically has some physical meaning or not.

So, and you can see the basically this, the only thing you care about is that these numbers should not become negative which obviously, they will not because the product of two non negative numbers and then do they have up to unity and answer is yes. So, it is easy to see that it is easy to verify, what is easy to verify that the double summation over i and j of this elementary event will be 1. It is like I know you get a rectangular table of numbers and if you add all the horizontal numbers you get either nu i or omega j depend or omega nu j depending on how you wrote the entries. And you add them all vertically, you get all those marginal probabilities as we will call them later on and the whole thing has to add up to 1.

So, I am not going to know am going to like you people go and look at this in more detail if you want. So, now, what happens is we have a sample space with N 1 N 2 points note that omega has N 1 in two points. And therefore, if you look at events now there will be sub sets of this omega and you can have that many more events because this two power N 1 N 2 which is a power set of omega is not equal to 2 power N 1 into 2 power N 2. So, know that is bigger. So, in some sense especially in the discrete case you can clearly see you have more ways of combining the events.

You can also go back to the event that you own, that you see omega without any concern for the other one that would be the combination of all of these points with omega where omega is common and nu j hap is you sum over all point all of us all the points nu j we did that you get back probability of omega i. That is if I what if I have, so let us say coin and a die just to make them different.

What is the probability just getting a head; that means, you are closing your eye to what is happening with the die if you are only interested in the probability of head. So, you will add up head 1, head 2, head 3, head 4, head 5, head 6 you write out all those 6 events there see elementary events for this set space and add them up. If you did that you would get back the probability of head with, but that does not mean that you are not I mean that you are that the other experiment is not performed, it is if you look into the sample the combined experiment then their understanding is every trial of this combined experiment means both are performed.

But know all these, in these probabilities that also there it is not that they have gone away. This is clear. You can always, you are permitted always to close your eye to what is happening on the other experiment that is best perfectly allowed. So, this is how basically this is going to be a model for come for considering combining experiments and I am again going I am going to go fairly fast over this because I think it is fairly say you know elementary and not particularly.

Just the one point to note is that 2 power N 1 the power set 2 power N 1 into 2 is not equal to 2 power N 1 multiplied by 2 power N 2. So, you cannot take the power set, you cannot form this power set by form forming the Cartesian product or the product of a f 1 cross f 2 that does not work. This is a point that was actually made by some students' long back so I thought I will include it and also state and said out here.

Why does that happen? Because if I take omega 1 comma nu 2 that event cannot be or may not omega 1 comma nu 2 union omega 3 comma nu 4 or something that cannot be formed as form this way. So, you can go and think about it, that there will be events in omega which are not expressible in this form ok. So, this is the basis for constructing more complicated experiments assigning probabilities to the points in it and all of whatever we have said everything will apply.

So, they said we have to move keep moving. So, what is the next point that we want to look at. Important thing is of course this physical independence is often happens and it always will mean statistical independence. Whenever we start with the assumption that we have two different things that are happening they can always they will be considered to be physically independent.

So, this will come up throughout the course, we will always see you can always assume that we have any number of physically independent events there are you know experiments that are going on. There is no shortage of random number generators or spinning pointers and so on after always just hypothetically constructing them. So, we can construct any one of them. So, you can clearly combine more than two also, there is no reason why you should stop at two.