Probability Foundations for Electrical Engineers
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Lecture - 05
Part B

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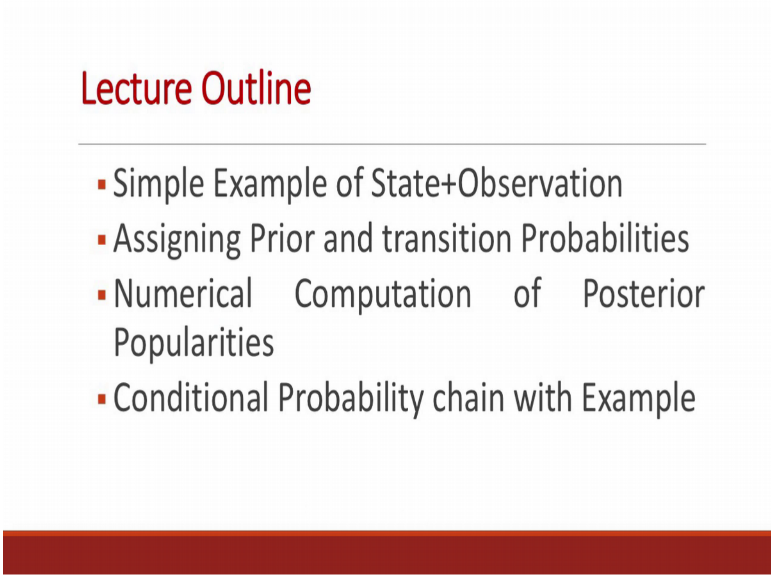

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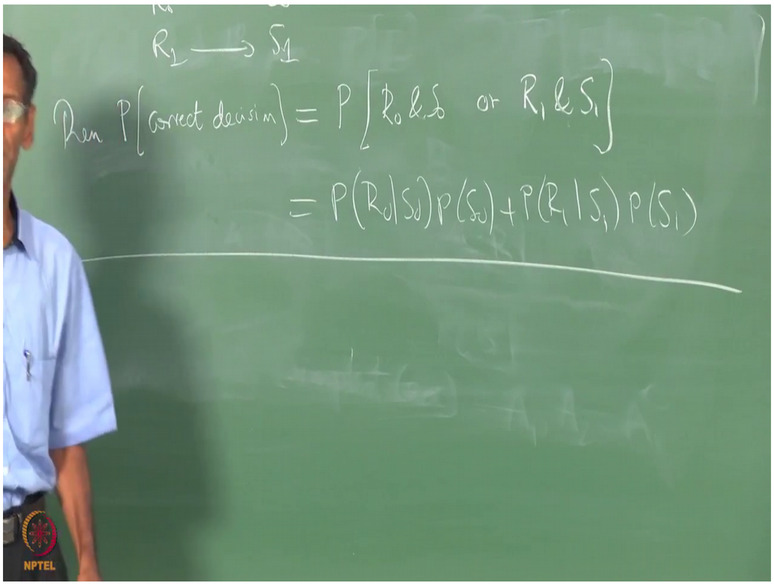

So, we look at a different kind of example where; which does not involve communications right.

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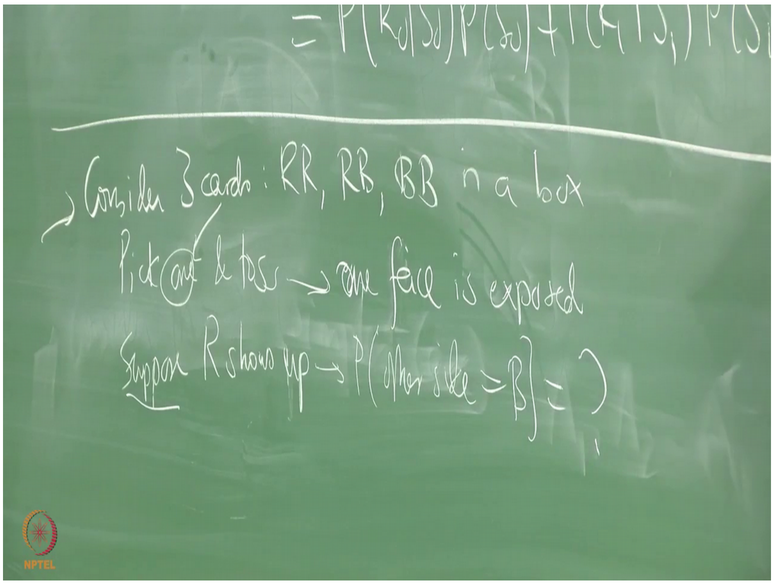

So, this is the next example I am going to look at right is using well as you might expect right some sooner or later will have to draw cards out of a hat and flip them, right, we have coins we have hats, we have dies, these things we cannot do without right. So, we have to because all the time you know; we might be all electrical engineers here, but we cannot avoid going out of the examples that only interests us right we have. So, in their lie; in the spirit of that statement let me look at this example consider 3 cards right ill call them RR, RB, BB in a box; R; RR stands for painted blue red on both sides or something or marked R on both sides right RB is one side, R 1 side B, BB is BB both sides B and B.

Supposing you take one now you close your eyes take one box sorry one card out of the box at random then throw it in the air right just everything with our eyes closed right or somebody does it and you do not get to see what they are doing right or I mean I assume that the experiment is. In fact, done randomly then like a coin the card also is going to settle with anyone side showing exposed and you do not know what is the other side right you only open your eyes after its come and fallen on the ground let us say. So, I say pick one and toss one side is exposed or let me say one face of the card is exposed. So, it is obviously the expose face is going to be either R or B right there is no other possibly.

So, supposing R is exposed what is the probability the other side is B or R pick one card and toss right; one face is exposed suppose R shows up P of other side. So, this is given when some when someone says suppose that is the conditioning event. So, this is what we want to analyze no; no; this situation we want to analyze this little more formally rather than just simply informally trying some arguments.

So, we going to make you know before we even assign probabilities we have to write this the sample space of the experiment carefully that is the fundamental starting point. So, here this again we have you know there in all of these cases we will have 2 stage randomness right hidden randomness that selects the state which you write again cannot do anything about; obviously, the state here is which is this 3 right card you picked right and then the observation is which ca which side showed after being tossed then you make.

You can make an assumption that given any particular card it you know is equally likely show one face or the other right that standard argument.

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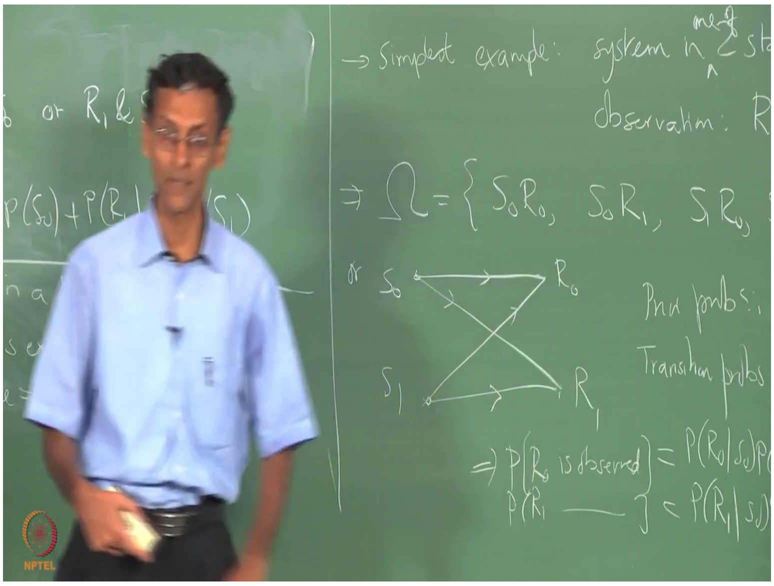

Therefore, in a drawing; this in a way, we are very similar to this right again, we are not instead of writing omega in a flattened manner like this it is; it makes more sense to think of omega with cause and effect type of diagram right rather than linear diagram write a linear list. So, keeping this available to us will draw a line here, the nice thing about using this borders you can selectively race and write. So, I am having instead of going back and forth between slides which is very painful for again for an observer right the board is contains a lot more space right and then a slide. So, I will exploit that and you know write draw a diagram here which is similar to this now except now that will have we need 3 spa 3 possibilities.

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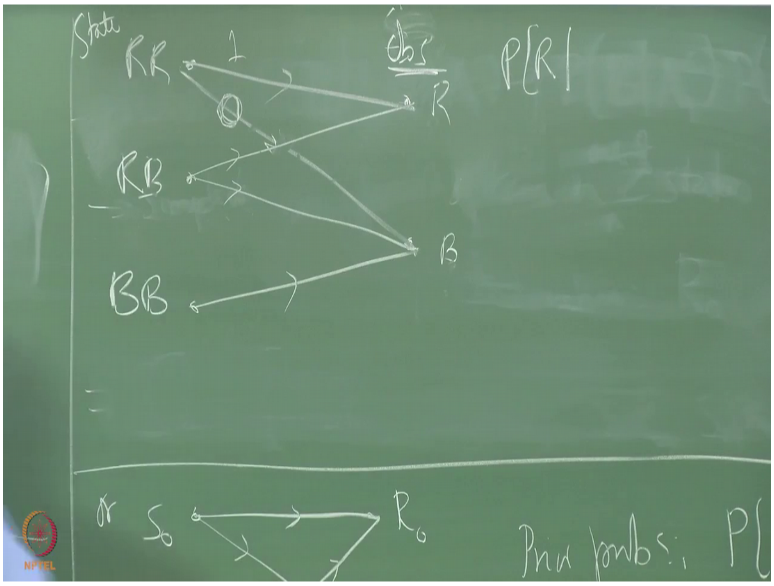

So, you have RR, RB, BB, remember again R is it is nominally red and B is could be any blue color; blue or black whatever you want which is not red or brown also right anything which is its not R right this R is same as this R and this B is the same as this B right. So, that you cannot tell by looking at what shows up exactly which card you are drawn. Obviously, then this is the hidden state right this is the state and this is the observation I was maybe; it is I guess; I will have to live with all of these six star fittings here right. So, again right here we can draw these connections, but some of these connections will have 0 probability. For example, regardless of the prior probability of selecting RR, RB or BB you cannot get to B from RR.

So, this is clearly going to be 0, similarly from BB you cannot get to R. So, that also will be 0 I can; I know, I do not even have to connect that right through the connections with this will all the probability; this will be concentrated in that branch given that you picked RR; you must you can only get R RB. However, tells you can go both to R as well as B and BB you can only go to B. So, let me write this 0 and then this is 0 here. So, what are now the conditional probabilities we assume that each of the cards is fair right and that when you toss any one of the cards just equally likely to show either faces; so therefore, P of R.

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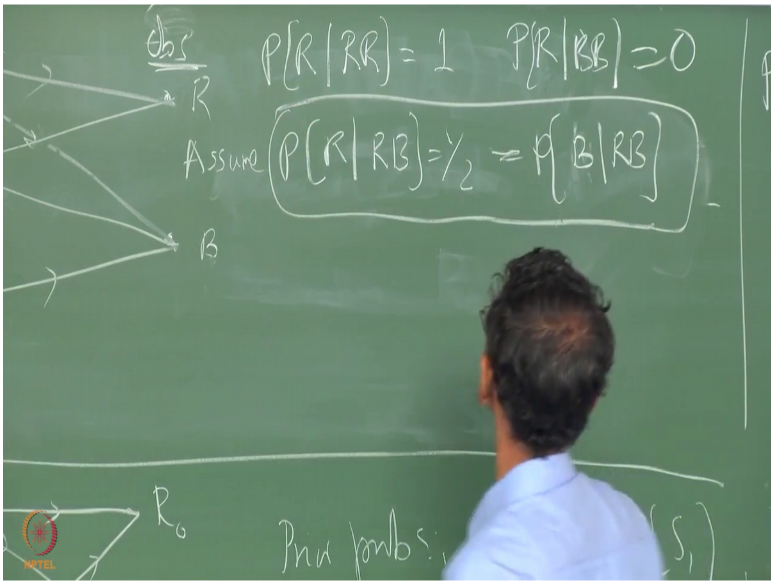

See right showing up which I am just going to write as R R showing up given RR is 1 P of R given BB is 0 this is where the half is going to come.

So, you are going to assume P of R given R B is half this is the assumption the fairness are equally likely option. So, you have half here and you have half here then what we are going to assume about the input state. Obviously, we will assume fairness again right and we will assume that they are equally probable, but note that that need not be the case right how can you easily have unequal probabilities all you have to do is have different numbers of are they are RR, RB and BB. Again the very you know intuitive or wait on its very clear that if we have many more of RBs, then you have of RRs or BBs in; obviously, you are more you are more likely to pick assuming that you pick you know the granularity is at a different level right you some pick card you picking at random.

So, if you know if you have 15 guys well pre probability of picking any one of those 15 cards is one by 15, but supposing they are divided unequally right.

So, there is a standard way of simulating unequal or actually getting unequal probabilities right very simple way of a you know for example, having coins with the unequal probabilities of head tail and so on is you map it to colors right so, but if you assume does not matter we can we will try to be in the case they are equally probable right. So, we will assume that they are equally probable because that is the most important I mean this case the way in which its stated in all the literature right the most arc right a common answer for this I mean what would you say is the answer here you all I am sure mean if you already worked it out, but if some if you said half its not the right answer. What is the right answer?

Student: One.

1 by 3; that might look counterintuitive right, but it is; that means, that if you pick if you see are you are more likely to pick RR right, but anyway let us do the whole thing out right. Now I think I can erase this because I have drawn this. So, I do not need the same diagram same kind of diagram.

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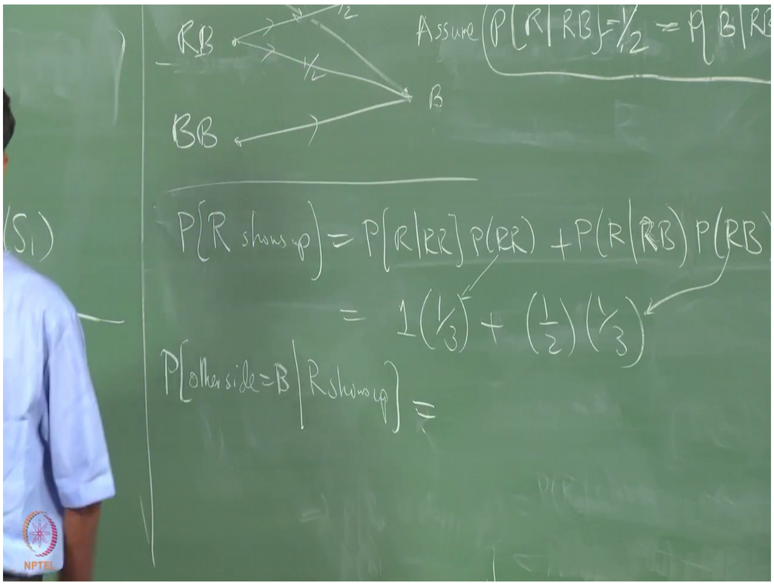

So, let me quickly get rid of this so P of R seem or R shows up, for example, is P of now I do not have to put for BB, because I know it is not right that term will not exist in this right in this summation because that 0. So, this is basically 1 into 1 by 1 into 1 by 3 plus this is RB here right this is half to 1 by 3.

What is this? Well, does not matter you can just leave it like this you do not have to simplify. So, the most important thing of course, is this thing P of either side equal to B given that R shows up this is what we want. So, what is how do you how do you how do you simplify this conditional probability this is the probability that your picked RB and I mean the numerator right that is that the let me first write the first point in English.

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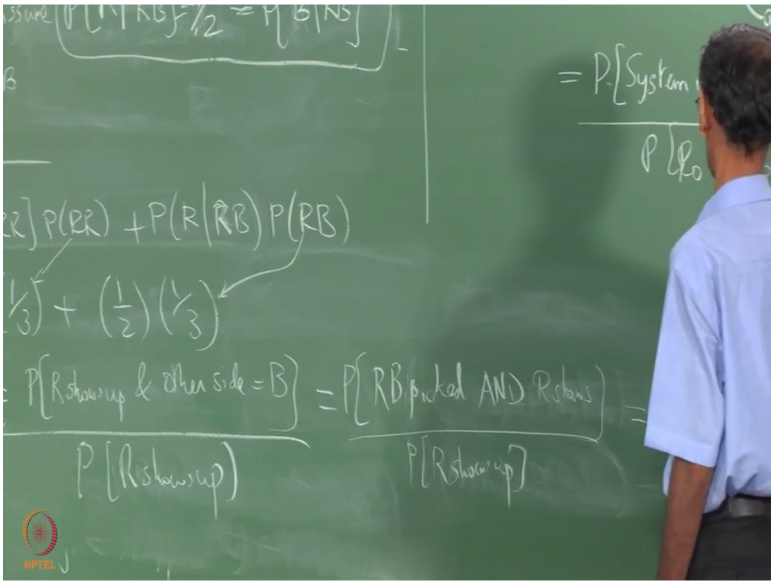

And then we will; so, this is R shows up and the other side is B divided by probability that R shows up. So, what is this numerator that is you picked RB and what R and R has shown up both things both they were both things have to happen. So, that is always a subset of the denominator this conditional probability can never be more than one remember RB picked and R shows divided by probability that R shows. So, what is the numerator?

Now, the numerator again you can write I; you know as in all our other examples you can condition on RB being picked again this base roll thing rights being. So, intuitive it right its. So, easy to jump into it and say this must be this given this multiply the probability of this right what is it probability of R showing up given that you picked RB is obviously this term.

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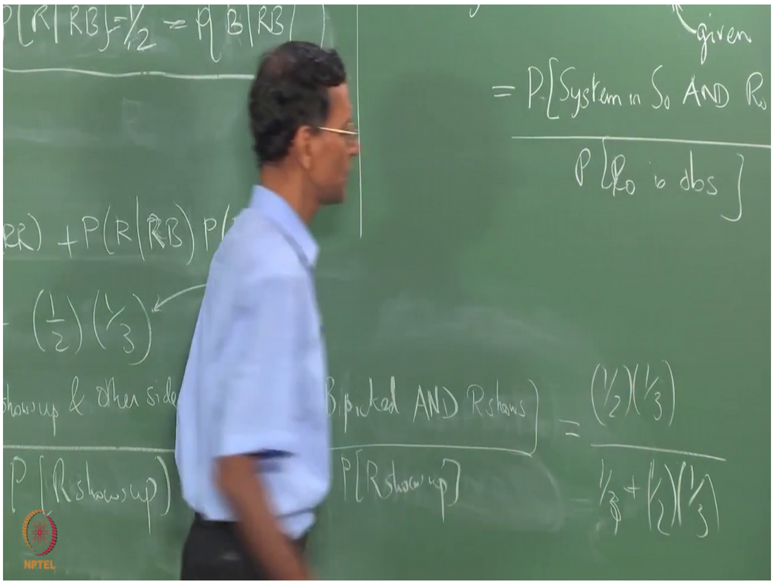

So, half into 1 by 3 divided by 1 by 3 plus half into 1 by 3; so this is why you get what you get which is what 1 by; 1 by 3 right. So, if you simplify this you get 1 by 3 is it not; wait a minute; yes, exactly. So, I noticed some quizzical looks out there. So, I thought even at this point, there should be right there should be no major problem with this number, earlier you may have thought well if I saw B A; sorry; if I saw R, I can only get B your R and by the symmetry the problems it should be equally likely, but that is not true.

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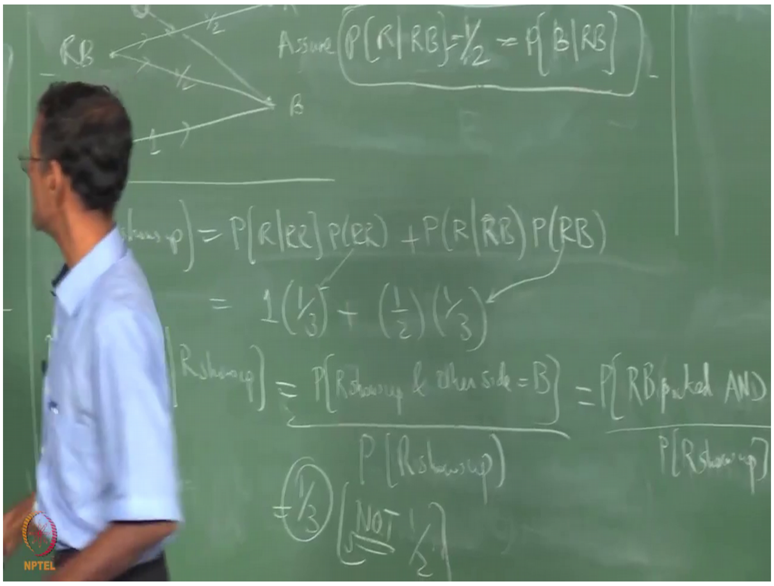

So, sometimes right not sometimes many times probability will right give you; what looks to be the case will not end up being the case right it will give you some counterintuitive results that you have to watch out for, but you cannot go wrong if you lay out the problem correctly you star you know you do not; you separate out in this case the 2 stage randomness the first right the selection of the state and then the observation right and then you write out the correct conditional probabilities and so on.

It will work no matter what right even if these conditions probabilities are exactly what they are if these change then these numbers can change obviously; clearly this, this 0 and this, this one I can add the one here. Now I think right; this 0 and this one are not going to change no matter what, but these numbers can be made to any set of 3 positive numbers between 0 and 1; just by as I said earlier by having more cards more identical cards which rather than just 3 after all if you can paint one card red and on one side black on blue on the other side you can paint more than one right and so on. So, you can you can create any scenario you know any setup you want by in a very simple way.

So, if you try to argue this from different and somehow you still got the correct answer, it may not help you right if you do not argue systematically you may not help you in the right in the general situation where the numbers are not exactly right as they are here any questions on this we can we free we certainly can have an exchange camera or not; does not matter, right, please do not hesitate, right, this is a normal lecture just like any other once again let me remind you right there need be nothing between us. If you have any day legitimate question you should ask. So, let me write you know I hope I have convinced you of the utility of this kind of approach right and as I said the same thing the same maths mathematics holds right in a bunch of situations of this kind right the important takeaway is that you should identify the states and observation correctly if you do that you can apply right ok.

So, let me not need; I do not need this anymore let me go onto something else.

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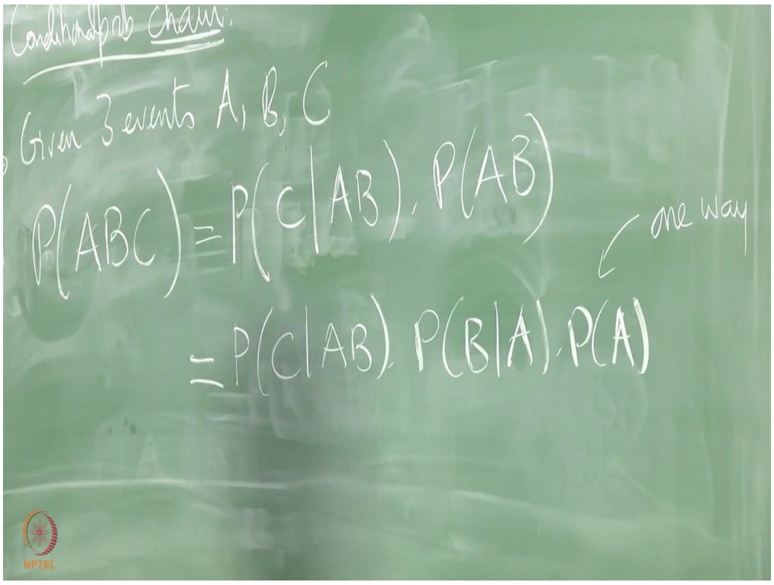

So, let me make sure the next thing I am going to before moving on to the different topic let me look at this conditional probability chain; a chain right. So, if I have given 3 events A B C, how do I can; I write this AP of the joint event A B C when I write this. This is a joint probability of observing A B and C in a particular trial all 3 of them right you are assuming that this joint probability is non zero right that. In fact, you can observe the 3 events jointly this is let say right one way of simplifying this is to condition on the job on the event A B. So, I can condition on a B and get this C given a B times P of a B this is exactly by definition assuming that a if abc is probability of a B C is non zero probability a B cannot be 0 either right.

So, we are assuming that all the conditioning events have I have some probability not 0 right in turn this P of AB can itself be simply for can be written as P of let us say B given into P of A its one way right of course, you can have ma many other possibilities here why should I only write it this way I can also write it equally well as a given B C or B given C A whatever, but starting out like this means I have P of A B coming out here. Therefore, this is P of C given A B times P of B given A times P of A, this is one way right of note this is not the only way this is one way of writing this conditional probability chain. So, I have right one the last term is going to be kind of a prior probability of observing one of those events the others are all conditional and you can extend it to more than 3 also.

So, when does this property become very useful let us the can be write this one this exact this property becomes very important in the analysis of sequential RA; you know here we had only in the in a discussion prior to this we had like 2 things 2; a 2 stage randomness. If you have more than 2 stages randomness and so on right like 3 3 successive things happening mo right you can use this to analyze that. So, for example, if you have again I pick this out of one of the out I guess. So, it seems to be the simplest example of this kind.

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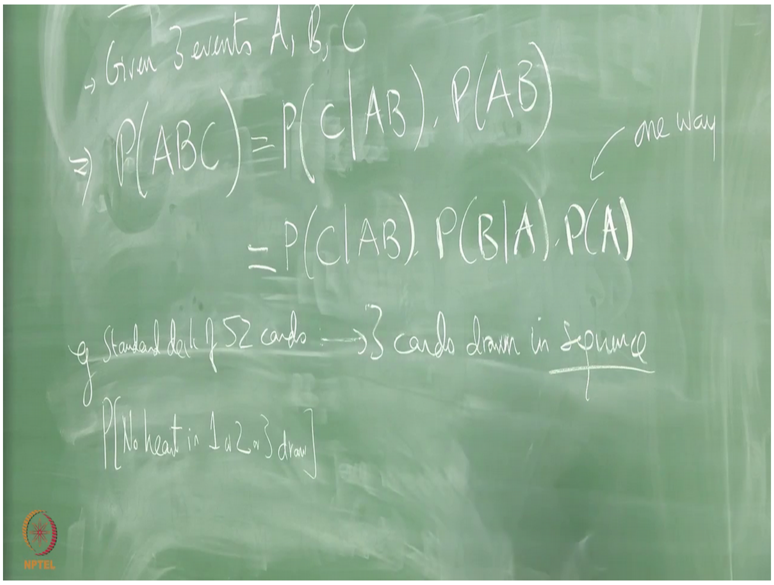

So, supposing I have the standard step deck of 52 cards and 3 cards drawn one after the other drawn in sequence right one after the other in right without replacement; obviously, when you say in sequence you do not put anything back right. So, now, if somebody as you what is probability that none of those 3 cards is a heart right in one first draw or second draw or third draw? So, this is clearly a joint right event exactly like this. So, how do you mathematically write simplify it using this notation clearly you can condition I right it makes sense you know you can start with the first that is the easiest one to think of you draw one card was it probably that is not a heart right then condition on that given that that first card is not a heart what is the probability that the second one is not going to be heart.

So, it turns out that you know I know I cannot write all; that is you know all the stuff on the board otherwise I would be reproducing textbook on that on the board is not possible.

So, please right associate this first draw with A the second draw with B and the third draw would C.

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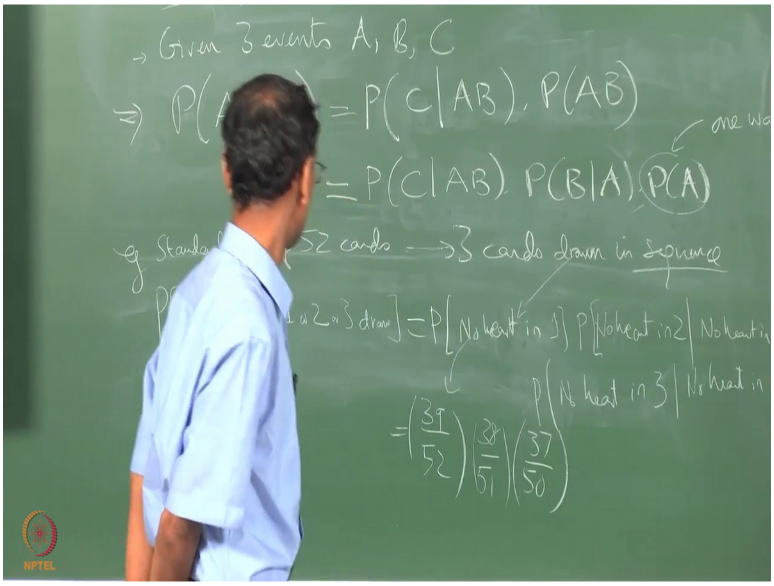

So, the therefore, this is P of. no heart, no heart in 1 or 2; sorry; no heart in one times that would be P of a no heart in one right that is the easiest one you know you would you would proceed only in this sequence. So, maybe you can think of this is starting of PFA, then doing this first stage second stage third stage right. So, no heart in 2 given no heart in one multiplied by P of no heart that would be P of right B given A.

So, what would be P of C given a B B no heart in 3 only given no heart in one or 2 one lab ill just write one comma 2 because that is a joint event right you have you saying no heart either in the first row or second. Now, it is easy enough right to write down each of these things. So, this is clearly what give me that year 52 cards and 13 hearts what is the probability that one draw one card is not going to you know assume you know pick any card at random right probably that you will not get a heart is what.

39 by 52 exactly right. So, you are picking right anything he said those 13 cards are 3 by four does not matter right this no heart in 2 given that you have pulled out something which is not a heart; that means, you have e one cards and you have only thirty eight cards in there right which are not hearts. So, this second term becomes thirty eight divided by if you want the third thing is what thirty seven divided by 50 given that you pull out 2 cars which are not both hearts; that means, they are all 13 hearts still there in

that given right and you have only thirty seven now cards which are not hearts. So, thirty seven divided by 50; this same right number can be obtained using combinatorial analysis also right.

Basically it is nothing, but thirty nine C choose 3 divided by 52 choose C, but that ill say for a different lecture because it I have to review little bit of combinatorics for the benefit of those of you that have forgotten them; forgotten that subject. We will need some combitorial; this course, we cannot again avoid that. So, we will I will do it at the right time right. So, I think with these examples I know I will not do any more examples just unconditional probability. I think I have done enough right the whole of in fact today's class we have pretty much devoted to examples.

So, what are we going to take up next is the study of independence, and look at how to combine; there are two kinds of things that we can talk about it will do it tomorrow.