Analog Integrated Circuits
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Lecture - 07
Noise of some Simple Circuits

In today's class, we look at the Noise performance of some Simple Circuit. Now before we go there, we need to quickly recap way noises treated for a complex network.

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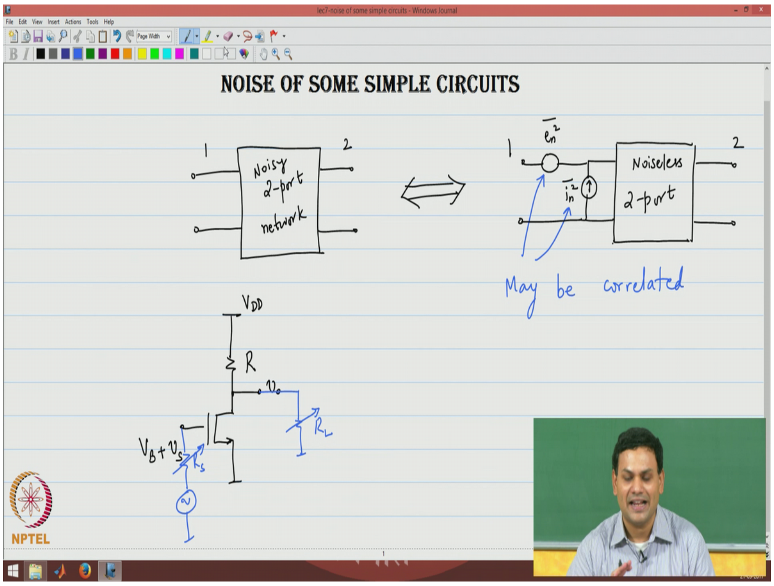

Let us say this is a noisy 2 port network and the input port is port 1 and the output port is port 2. Now it turns out that this is exactly identical to a noiseless 2 port with its noise referred as a noise voltage and a noise current at the input and I will call those input referred noise sources as e n squared and i n squared.

So, e n squared and i n squared represent the input referred noise sources of the network now this is unique to the network. So, different circuits will have different values of e n and i n the other thing to remember is that in general e n squared and i n squared may be correlated. This is because the same set of devices inside the 2 port contribute that are producing noise contribute to both e n squared and i n squared.

So, in general, you would expect e n squared and i n squared to be correlated this correlation maybe partial or total and that really depends on the exact circuit that is being considered the reason you need both e n squared and i n squared is because in general this 2 port may be excited with any type of source. In other words, it could be excited with an ideal voltage source an ideal current source or in general a the venin equivalent or a not an equivalent whose resistance might be any particular value.

So, to cover the whole range of the venin not and resistances you need both e n and i n to truly represent the input referred noise of the circuit now let us take some simple circuits and try to determine the input referred noise I will start off with a common source amplifier with a resistive load. Let us say the value of the resistance was R and at the input I am applying the signal plus some bias and the output is taken at this point.

Now, please note that the input referred noise is a function or is a characteristic of the 2 port itself it should ideally be valid for any value of load resistance and any value of source resistance. So, please note the input referred noise is a function of the 2 port itself. So, in other words the input referred noise should not be a function of R s or R L having said that.

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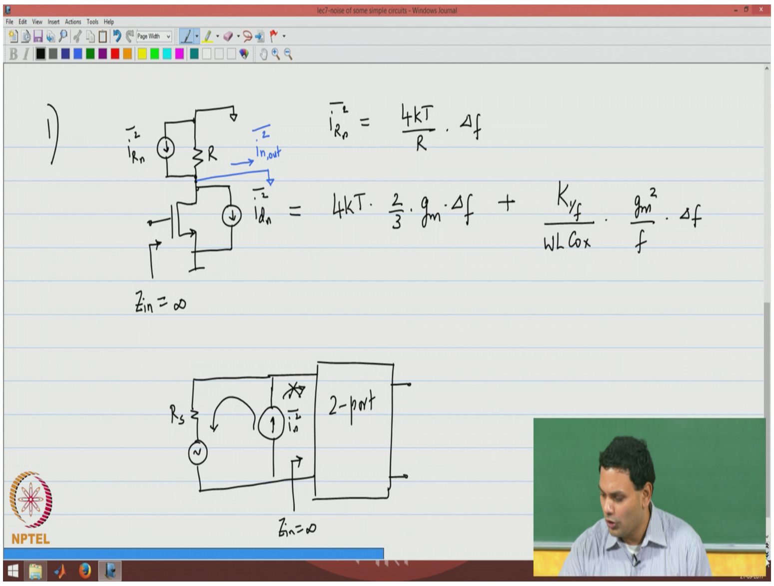

We can now say that the input referred noise can be calculated by considering all the noise sources in the circuit when you are doing noise analysis the input source does not matter. So, we will ground. So, the input not a signal of interest we are only interested in the noise signals inside the circuit.

So, I am now going to draw the signal equivalent this is a resistance R and the resistance R has some a noise current across it; I am going to use this the not an or the current source equivalent noise equivalent circuit for the resistor for a very specific reason which we will see in a minute. So, i R n squared is equal to 4 k T by R times delta f and it turns out that the transistor also has a noise current which consists of 2 components one of them is the thermal noise which is 4 k T into 2 thirds into g m times delta f and the other component is the flicker noise which is K over W L c ox times g m squared by f times delta f.

Now, we are required to find out the total input referred noise. So, in other words if this way the input i want to refer all these noise sources back to its single to a set of single noise voltage and current. Now as it turns out the in general at 2 port network would have both an input referred noise current and an input referred noise voltage, but there are several types of circuits which have infinite input impedances. For example: this particular MOSFET has an input impedance that is infinity at very low frequencies which means that even if there were a noise current none of it would make its way into the circuit.

So, in other words if I were to Exide a 2 port with the noise current at its input and if z in were infinity all of this noise current would make its way through the source resistance R s and none of it would flow through the input of the 2 port. Therefore, for circuits that have infinite input impedances the noise current the input referred noise source is 0 we will not worry about it we are only interested in the input referred noise voltage source in other words e n squared or V n squared.

So, for this circuit for the common source amplifier we are going to calculate this input referred noise. So, this is the first circuit that we have looking at. So, since the noise equivalent is valid for all values of load resistances I will say that it is also valid for a load resistance of 0. In other words, the output node is shorter to signal ground what is this mean for us even though it is shorter to ground I can calculate the output noise current through this short circuit and if I divided by the overall trans-conductance of the 2 port network. In this case the overall trans-conductance of the amplifier I should be able to get the input referred noise voltage.

In other words, I am going to compare the total noise for this circuit with the following circuit where both the resistance and the MOSFET are noiseless.

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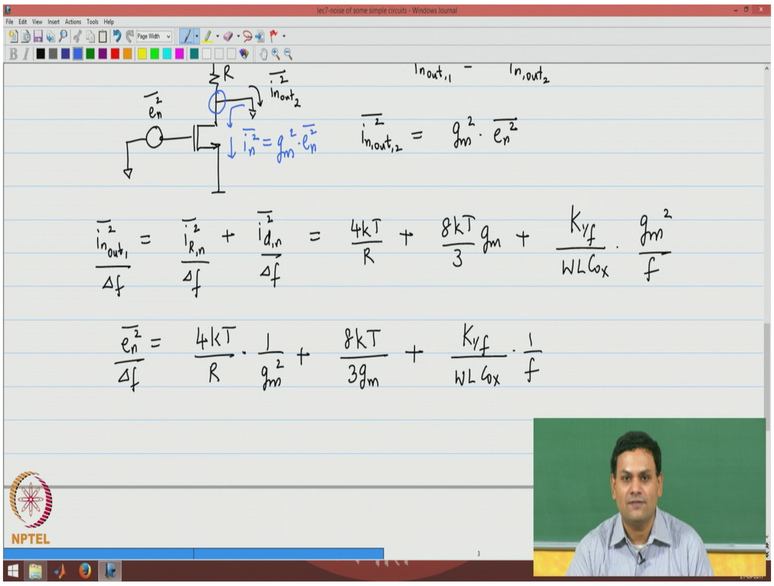

If the input referred noise voltage e n squared completely represented the noise inside the 2 port then the output noise current in the 2 cases should be exactly equal. So, let us calculate i n out 1 and i n out 2. So, we are saying that the 2 networks are exactly identical let us now calculate these 2 values.

Clearly i n out 2 i n out squared one should be the sum of the noises of the MOSFET and the resistor and since the noise of the MOSFET and the resistor are completely uncorrelated I can add the mean squared values at the output. So, I am going to add the mean squared values of the noise currents of the 2 devices and the point we need to note is that, because the output is a short circuit all of this current will flow through this short circuit similarly all of this current will flow through the short circuit. So, the complete noise from the MOSFET and the resistor will flow through the short circuit because it offers a 0 impedance for the current to flow.

Now, I will represent the power spectral densities in this fashion. So, that is 4 k T by R plus 8 k T by 3 g m plus the flicker noise, this is the expression for the total output noise current in the first case what about the second case; I will write that up here. So, the noise current in the second case is nothing but g m squared times i n out squared because this voltage e n squared is going to cause a current through this MOSFET which is equal to the trans-conductance times the input gate source voltage. Since we are dealing with mean squared quantities this or this noise current squared mean squared value will be g m squared times e n squared and because of the short circuit all of this current will flow through the output short circuit.

So, therefore, this noise current in the second case is nothing but g m squared times e n squared. Therefore, e n squared is nothing but or rather e n squared over delta f is nothing but 4 k T by R times one over g m squared plus 8 k T by 3 g m plus K over K one over f over W L c ox times one over f. This is the noise total noise power spectral density at the output for this particular circuit and we divided that by the square of the trans-conductance to get the input referred noise voltage source e n squared.

Now, I hope you can see that you can follow this method for any given circuit we will do a couple of more examples in today’s class. Now the point to note is there are 2 significant points to note first of all this for this particular amplifier the input impedance was infinite. And therefore, we stopped with calculating only the input referred noise voltage, but there are MOSFET circuits where the input impedance is not infinite. And therefore, we would have to collect the; we would have to calculate the input referred noise current also.

We would follow the exact same procedure where we would apply an input referred noise current here and we would calculate the overall output noise current when we apply this input referred source. And then we would calculate the overall short circuit current without the source when the resistor and the devise are noisy we would compare the 2 cases. And we would determine the value of the input referred noise more importantly I have considered a short circuit at the output, because the MOSFET being a voltage controlled current source element it is often very easy to think in terms of the device currents. And we know that currents tend to take the path of least resistance or least impedance.

Therefore, if we short circuit the output we know that all of the current will flow through that output node. And therefore, the analysis simply becomes easier it is perfectly fine to consider the output node as an open circuit and find out the output referred noise voltage and divided by the overall voltage gain of the circuit. In other words you find out the output referred noise voltage in both cases with and without e n and you come equate the

2 and determine the value of e n squared rest you may rest assured that you will get the exact same value in both cases.

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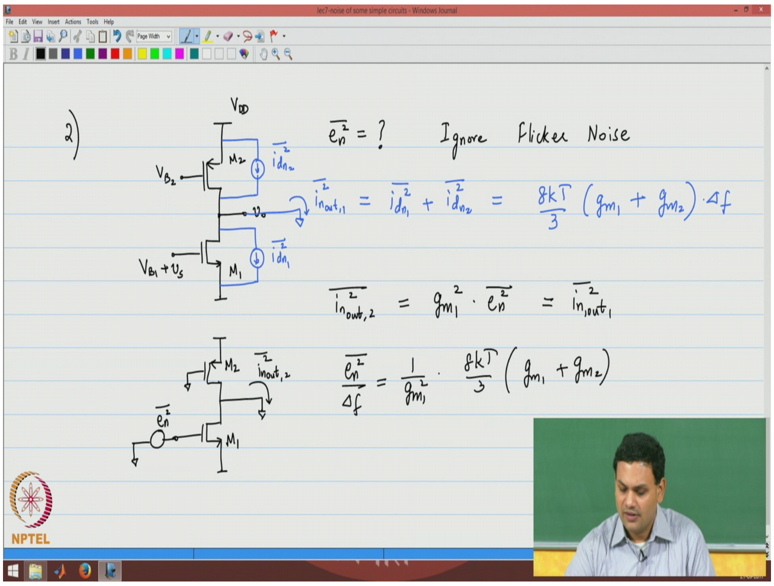

Let us try one more example I will take the same common source amplifier, but now I am going to use an active load in other words an active PMOS load instead often instead of a resistor .

So, let us say this transistor was M 1 and the PMOS transistor was M 2, I want to find out the input referred noise voltage for the circuit and clearly because the input impedance is very high the input referred noise current is irrelevant for this circuit; I will follow the exact same procedure to do this example. Also I will first note that both these transistors have drain thermal noises please note that when you are considering noise especially when the noise sources are uncorrelated the direction of the noise current does not matter because you are always adding them in mean squared terms and I have 2 drain thermal noises.

I will denote them by i d n 1 squared and i d n 2 squared for this particular example I will ignore flicker noise, but please note that it is very easy the behavior of the circuit for flicker noises almost exactly the same as that for thermal noise the only difference is that flicker noise frequency a flicker noise is frequency dependent, whereas the thermal noise is not it is white noise I am going to short circuit the output and I am going to calculate i n out 1 square.

Clearly this is equal to i d n 1 squared plus i d n 2 squared because both of those noise currents will prefer to flow into the absolute short circuit at the output node because the 2 noise sources are uncorrelated I can add them as mean squared quantities. And therefore, the total output noise current in this case is 8 k T by 3 into g m 1 plus g m 2 times delta f. Now I consider the other case when both these transistors are noiseless, and I only have the input referred noise source e n squared.

At the input and please note that noise is a small signal. So, all small signal quantities all small signal analysis is valid for noise and in this case i n squared 2 is nothing but g m 1 squared times e n squared. Now I know that this should be equal to i n out 1 squared. And therefore, e n squared by delta f is simply one over g m 1 squared times 8 k T by 3 into g m 1 plus g m 2. So, this is the input referred noise voltage squared for this particular circuit. Now, let us look at this in a little bit more detail because this is quite interesting.

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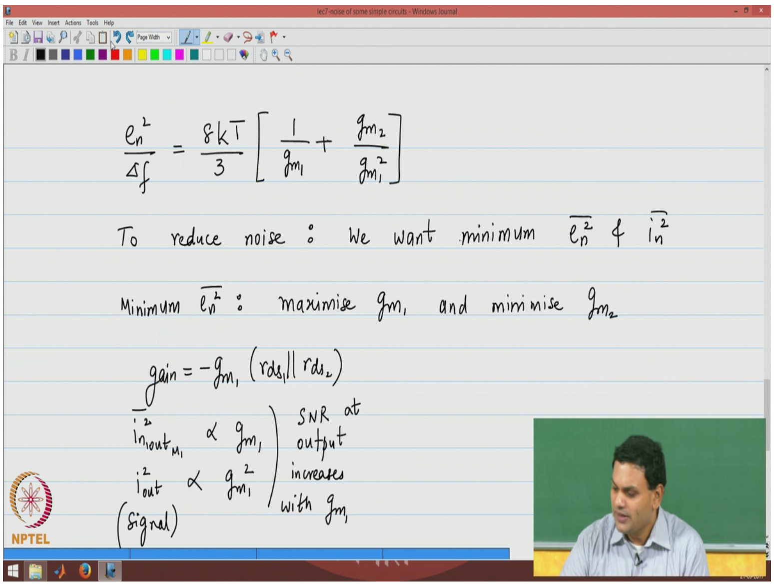

So, I will write it again on this page for your reference and now I am going to take the g m squared inside the phrases.

So, I obtain 8 k T by 3 into 1 over g m 1 plus g m 2 over g m 1 squared now clearly for an ideal noiseless 2 port e n squared would be 0 i n squared would be 0. So, if I want to reduce noise we want minimum e n squared and i n squared. In other words if you are trying to create a circuit that is not very noisy you want to minimize the magnitude of e n squared and i n squared. So, let us apply that philosophy to this particular circuit which is the common source amplifier with an active PMOS load.

Let us say we want to minimize e n squared what design implication does it have for us. So, this means clearly Boltzmann's constant and the temperature of operation are not under our control in a majority of cases, but there are some cases where the temperature might be under our control because if you want a really low noise circuit you might need to cool down the ambient temperature of the circuit and there are some cases where this is done.

But in a majority of cases you do not have control over the ambient temperature. And therefore, the only design parameters as you can see are g m 1 and g m 2. In our case this clearly points out that we need to maximize g m 1 and minimize g m 2. Now please note that the gain of the circuit is the function of g m 1. And therefore, for a high gain amplifier you do have some control over g m 1, because now the gain of this particular circuit for example, is minus g m 1 into r ds 1 parallel r ds 2 the gain of this actively loaded common source amplifier is minus g m 1 into r ds 1 parallel r ds 2 and there is some by controlling by choosing a particular value of r ds 1 and r ds 2 there is some design freedom for choosing the value of g m 1 for a given gain.

Now, what this tells us is you have to maximize the value of g m 1. Now let us go back here to the circuit and I wanted to point out that this may seem a little bit counterintuitive if you maximize g m 1 you are actually maximizing this noise current in other words.

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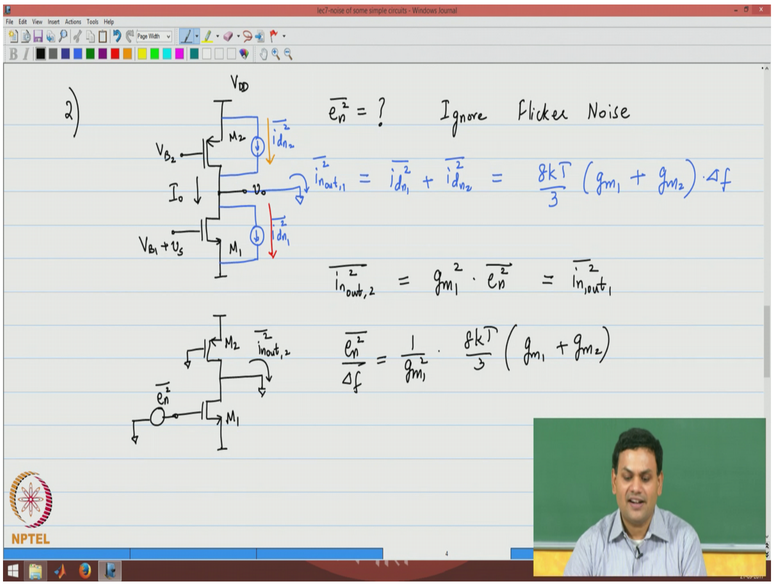

The noise current flowing through the output is increasing. So, it may seem at least at the beginning on the phase of it; it seems a little bit counterintuitive. But please note that the noise current squared increases linearly with g m, but the output signal increases as the square of g m.

In other words; i n out squared from M 1 is proportional to g m one, but this is the output signal is clearly the output current is clearly g m times V s which is the input signal and if you are looking at i out squared which is the output current power this is proportional to g m 1 squared. So, if you choose a larger g m 1 you will get a larger noise, but you will get a much larger signal. So, you will get larger gain r ds 1 and r ds 2 being constant if you choose a larger g m 1 the gain will be even larger and therefore, the signal to noise ratio at the output increases with g m please note this.

What about g m 2 in this case because g m 2 the PMOS transistor is being used as an amplifier is being used as a current source. And therefore, it contributes no signal gain, but it only contributes noise. Therefore, we want to if you make g m 2 larger and larger the second current source keeps getting larger and larger, but the signal power does not increase with g m 2. And therefore, you do loose on signal to noise ratio therefore, for a low noise circuit you want to minimize g m 2.

What is minimized g m 2 mean please note that the current through the circuit is some bias current i naught which is the same for both circuits for both transistors and has been chosen this bias current i naught has been chosen such that the transistors have a certain W over L and a certain g m.

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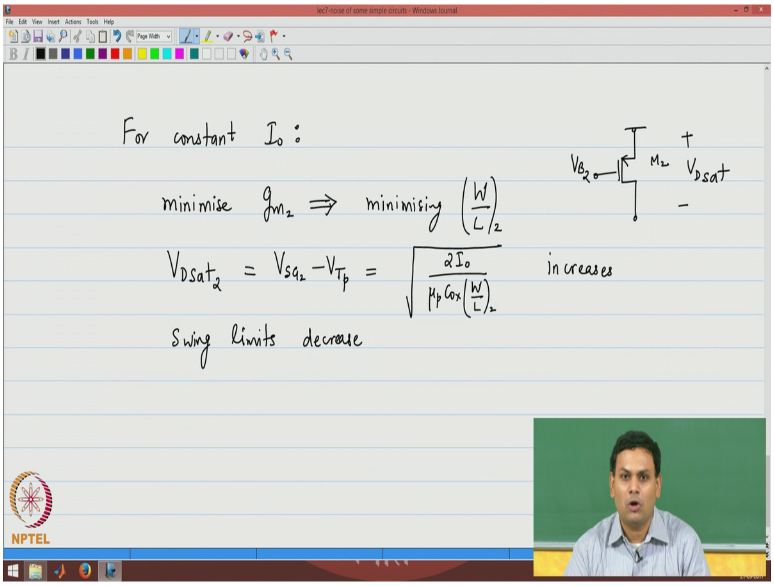

Now, let us assume that you are trying to keep i naught constant because you do not want to tamper with g m 1 which you have chosen in a particular way for noise for constant i naught; if you want to minimize g m 2 this can be done only by minimizing W over L of M 2 which is the PMOS transistor in this case.

Now, if you keep the bias current constant and try decreasing W over L what happens is the VD sat or the VSD sat of the PMOS transistor in this particular case, it is the source gate voltage minus V T p please note that this is inversely related to the W over L of the device. In other words if you have a constant VB 2 at this point you are by for a constant current by minimizing g m 2. And therefore, minimizing the width over length you are compromising the minimum voltage required across the transistor M 2.

The minimum voltage required to keep the transistor in saturation and that is extremely important to get large gain from the circuit. So, if you try to decrease VD sat of the PMOS transistor increases and swing limits of this particular amplifier decrease. So, clearly there is a tradeoff between noise performance of the circuit and the swing limits of the circuit and this is a classic trade off an analogue circuits, because as processes scale as i c processes are moving from longer technology longer channel length technologies to shorter channel length technologies you find that the power supply scales and for the same noise your overall dynamic range is being constraint.

Because on the upper side has the power supply voltage decreases your constraining the voltage and therefore, there is a certain limit s n R s n R limit that is being imposed upon by this process scaling.

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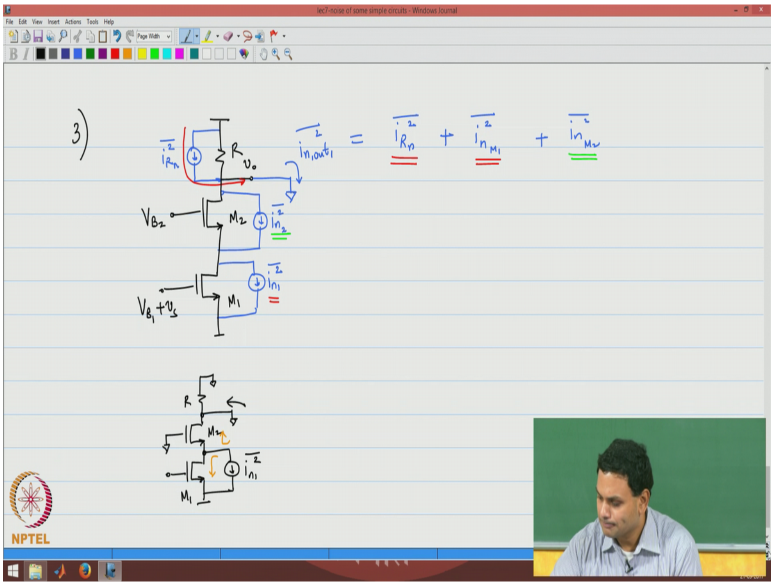

Now, let us look at a slightly different variant of the circuit I am going to look at the cascode amplifier let us take the resistively loaded because that will give us one more noise source. So, let us say I have M 1, I have M 2 and I have a resistance R and I have a bias voltage VB 1 plus the signal applied here at the gate of M 1 and I have some bias voltage VB 2 at the gate of M 2 and now I want to find out the input referred noise voltage of this particular circuit and as before I am going to short circuit the output.

Look at the output short circuit noise current squared there are 3 noise sources for the circuit i n 1 squared i n 2 squared and i R n squared now these 3 noise sources are completely uncorrelated and I will need to find out the output component of the noise from all 3 of them. So, in fact I will write down the 3 components here now we need a small change in notation.

So, let us say that the output noise due to M 1 is i n M 1 squared and the output noise due to M 2 is i n M 2 squared. So, this are the 3 components flowing through the output please note that the noise from the resistor flows directly into the short circuit. So, that is why I have written it straight away at the output now we need to calculate the component of noise current i n 1 that is flowing through the output and I am going to call that i n M 1 and I need to find out the component of i n 2 flowing through the output and that I have called i n M 2 squared.

Let us calculate these 2 quantities; so to do that I will consider each one intern. So, I am interested in the noise current flowing through the output short circuit due to i n 1 squared. So, now, at the drain of M 1 clearly the noise current has 2 paths to flow one is back into the transistor the other is into M 2.

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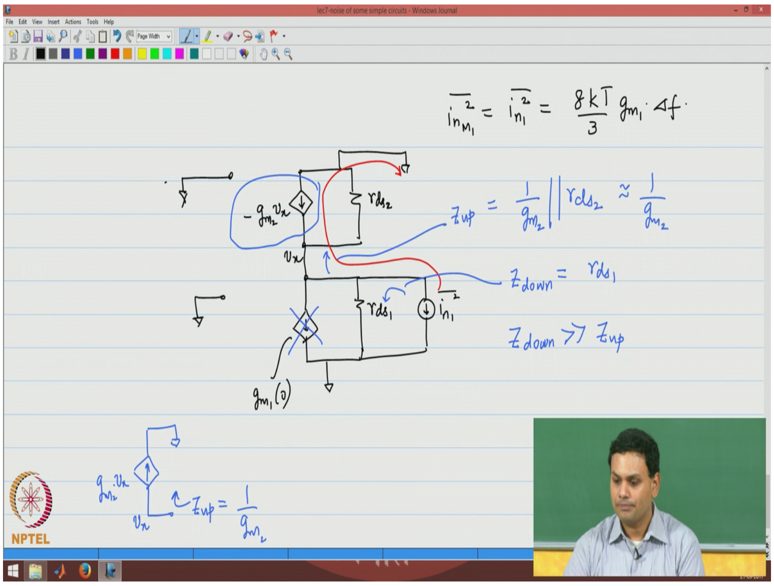

So, to understand this a little bit further we will need to draw the small signal equivalent circuit I will call this intermediate node voltage as V x and therefore, this current source for M 2 is minus g m 2 times V x because the gate is a small signal ground.

Please note that for noise purposes the gate of M 1 is also grounded and therefore, this is g m 1 time 0 and at this point I am applying the drain thermal noise of M 1 which is i d n 1 squared. Now this current has to flow in only 1 of 2 paths, it either flows through r ds 1 or it flows up here now it turns out that the impedance looking up is simply 1 over g m 2, because this current source comes completely in parallel with r ds 2; this current source is please note that the direction of the current source can be changed and if you are trying to find out z up; this is purely 1 over g m 2 and that is of course, in parallel with r ds 2.

The impedance looking downwards is r ds 1 and as you can see clearly for typical values of g m and r ds z down would normally be much larger than z up because z up is approximately one over g m 2, whereas z down is an R d is r ds 1. And therefore, z down is much larger than z up and we can say that almost all of this current will make its way to the output. So, in other words most of this current almost all of this current will flow up into M 2 and eventually into the output.

This means that i n M 1 squared that we have been trying to calculate is simply i n 1 squared or i d n 1 squared which is 8 k T by 3 g m 1 times delta f what about i d 2.

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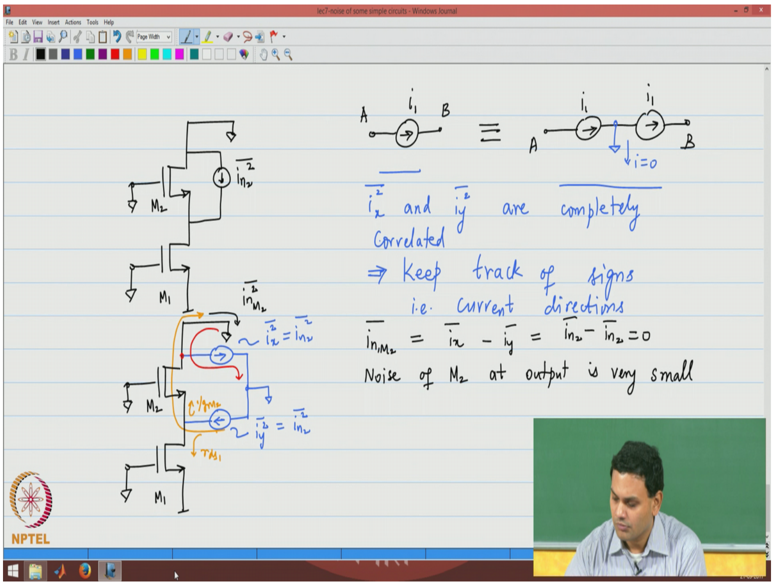

Now, let us take that circuit; now for this particular case both the gates of M 1 and M 2 are grounded and I am now looking at a current source between 2 nodes. Now to analyze this circuit we will use a small network analysis technique. So, if I have a current source i 1 as far as the rest of the networks across A and B are concerned this is exactly the same as putting 2 current sources in series.

Whose value is exactly the same there will be no violation of either K V L or K C L if this is done number 2 even if I decide to ground or apply any specific voltage at the intermediate node. Please note that there is no current through this node you can clearly see that by applying K C L at this intermediate node. And therefore, the rest of the network is not disturbed across it this configuration looks exactly the same as this configuration across the nodes A and B. And we will use this particular technique to analyze the circuit. So, we will split i n 2 squared has 2 sets of sources to do this I will make a copy of this particular network I will show this in blue I am maintaining the same direction and I am going to short the intermediate point to incremental ground.

As you can see now it is the network looks a little bit more complex, but it is actually easier to analyze I am going to call these 2 sources as i x squared and of course, this are both and I am going to call them i y squared to distinguish the two, but in reality both of these are equal to i n 2 squared. Now it is very important to note that i x and i y are completely correlated.

So, in other words since we are splitting the same i n 2 squared into 2 current sources in series these 2 noise sources as i x and i y are completely correlated what this means is. Whenever you hear the word correlation with respect to noise you need to keep track of science of the noise voltages and currents in this case we need to keep track of current directions. So, we are to summarize we are trying to calculate i n M 2 squared. So, i n M 2 squared will have 2 components one from i x the other from i y.

So, the first come component from i x will clearly be equal to i x itself in other words the current source i x has the option of flowing back into M 2 or flowing out of a flowing into the short circuit since current always takes the least resistance path all of i x will flow into the output short circuit. What about i y if you look at i y it has the option of flowing into the lower path which has a resistance r ds 1 or the upper path which has a resistance one over g m 2.

And therefore, most of this current i y would normally flow through the transistor M 2 and eventually flow into the output node the only thing we have to be careful is to keep track of directions. Now let me I notice that I have used different directions let me please correct them. So, since I have taken a specific direction this current the current flow path for i x is downwards and the path for i y is upwards. I will show that in a different color like this.

So therefore, now we no longer deal with mean squared quantities because we have to keep track of correlations and therefore, we have to talk about the signal with the science involved. So, i n M 2 will have a component from i x which is i x itself minus. Please note that the 2 current directions are opposite minus i y, because almost all of i y flows through the output also in the opposite direction this means that most of the noise from M 2 does not make its way to the output.

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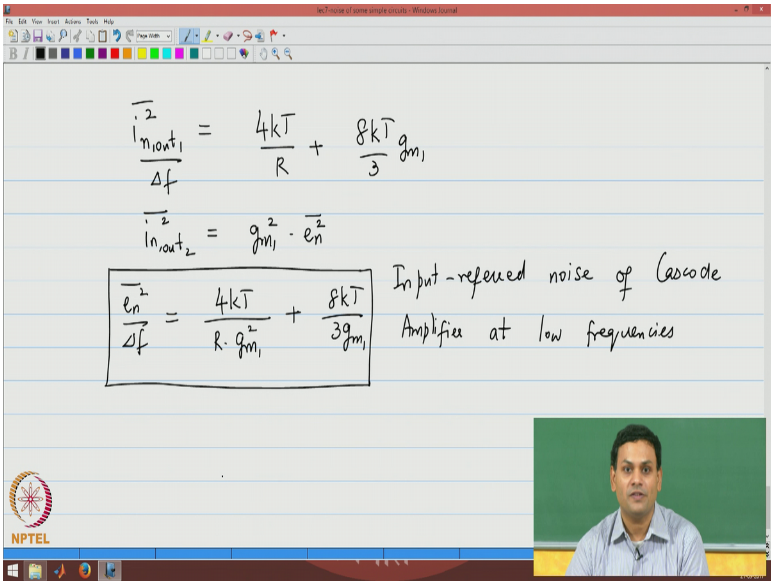

And therefore, now we can write the total noise current at the output which is 4 k T by R plus 8 k T by 3 g m 1.

In the case where I have a noiseless circuit with an input referred noise we know that the overall trans-conductance of the cascodes is the same as that of the device M 1 and therefore, i n out 2 squared is g m 1 squared times e n squared. And therefore, e n squared by delta f is nothing but 4 k T by R into 1 over g m squared plus as you can clearly see the noise from the cascode is at the appearing at the output in other words appearing in the input referred noise source is very small the cascode increases the output impedance, but it contributes very little noise to the output especially at low frequencies.