Analog Integrated Circuits
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Lecture - 06

Noise

In today’s class, we are going to look at noise.Now, what do we mean by noise?Normally, noise refers to any type ofundesiredsignal.

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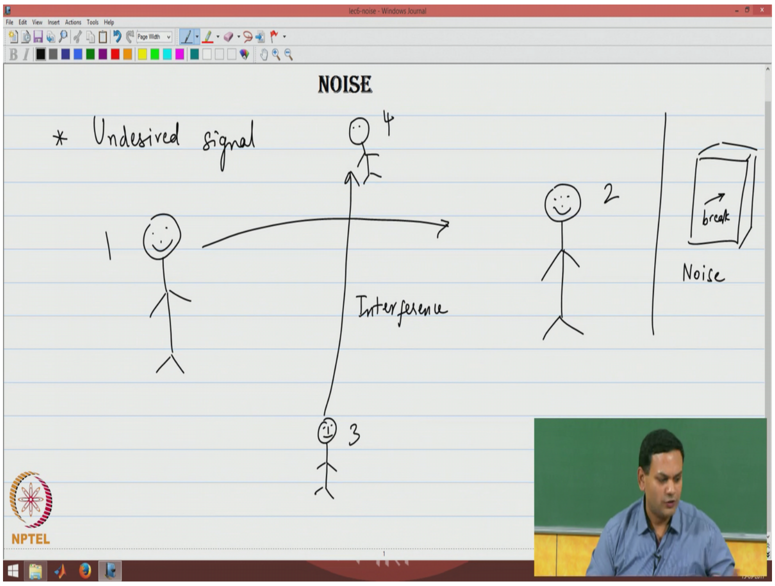

But, there are several different types of undesired signals. So, we will take the example of a person talking let us say you have a persontalking to somebody across the room. So, you could have many different things;for example, you could actually haveseveral people talking in the same room, maybe people are talking across the room in this direction alsoand somebody else is shoutingtofourth person.

So, let us say person 1 is trying to talk to person 2 and person 3 is trying to talk toperson 4 and let us say person 3 is shouting across the room to person 4, so that person 2 is not able to listento person 1. So, this type of signal would normally not be called noise, but it would be called an interferer, becauseit isyou know the source of theundesired signal and the undesired signal has very specific properties maybe it is another person trying to communicate withyou know somebody else.

So, this would normally be called interference, but let us sayjust outside the room somebody startsusing let us sayjackhammer or something like that to break some structure. So, outside this room somebody is trying to break or let us say breaking a wall.Somebody is actually breaking this wallwith a continuous din.In that case, there is no signal being communicated.This sound would be called noise because this is not somebody trying to communicate with anybody else it is an undesired effect of something some other activity happening.

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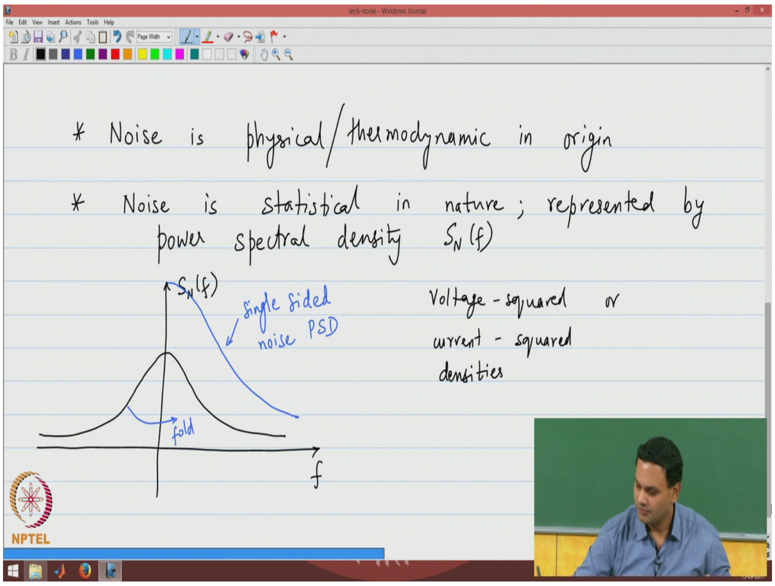

* Noise is physical / thermodynamic in origin

* Noise is statistical in nature ; represented by power spectral density $S_N(f)$

Voltage - squared or current - squared densities

So, for the purposes of this case, we will point out thatwhen we talk about noise;noise isphysical orthermodynamic in originand what that means, is; that means, thatnoise isnor or noise could be dependent on physical properties such as temperature and we will see thisin a few minutes.But,more importantly noiseis then statistical in natureand therefore, representedby a power spectral density.

Let us call this sumSNof f. So, if I were todrawapower spectral densityfor noise.It may have some representation I will show some behaviorlike this.Now, as it turns out when we work with circuits we normally look at the one sided representation in the frequency domain and therefore, what we will do is, we will take the signal content in the negative frequency and fold it over2 positive frequencies which means my noise will now get doubled. So, this would be the single sided representationof the noise power spectral density.Now, in the case of circuitswhen we say power we normally represented as voltage squaredorcurrent squareddensities.

Now, let us start looking at taking some simple circuits and we will start looking at the noise associated with those circuits.

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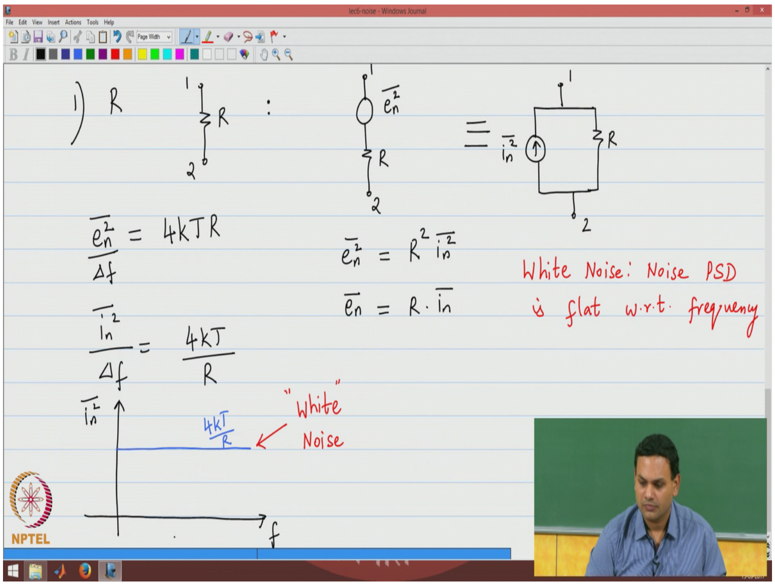

$$\frac{\overline{e_n^2}}{\Delta f} = 4kTR$$

$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R}$$

$$\overline{e_n^2} = R^2\,\overline{i_n^2}$$

$$\overline{e_n} = R \cdot \overline{i_n}$$

White Noise: Noise PSD is flat w.r.t. frequency

The simplestelement that we will consider is a resistor.Let ussay it has a value R, the noise associated with the resistor R can be represented in 2 ways; one is in terms of the voltage, the other is in terms of the current.If I represent it as voltage,let the 2 nodesbe1and 2, I will represent it as a voltage in series with the resistance Rand I am going to call this e n squaredwhich is the mean squared voltage in series with the resistance andan equivalent representationisa currentwhich is represented as a mean squared currentinsquared between the same 2 terminals,1 and 2.

And because you are talking about mean squaredyou can go between the Thevenin representation and the Norton representation and in terms of Theveninrepresentation you can say thatthedependence between the Theveninand the Norton equivalents is in the following way;e n squared is R squared timesin squared or if you are talking about the root mean squaredRMSquantities which I will represent as e n andin respectively.This is the relationship between the 2 quantities.

What about the value?Now, the value of en andinare themselves physicalin origin and the expressionis givenin the following way; e n squared which is the mean squared voltage densityis4kTRand therefore, the mean squaredcurrent densityis 4kT by R.What does this look like in the frequency domain if I were to plot these 2 quantities?Let me plot e n squared and again I am going to this is the one sided value. So, if I were to plotin squaredas a function of frequency clearly because it has a constant power spectral density, thevalue is4kT byRand it does not change with frequency.

Sucha source of noise or such a type of noiseis called white noisewhite noise.White noise is something is a noise that has a constant power spectral density with respect to frequency.

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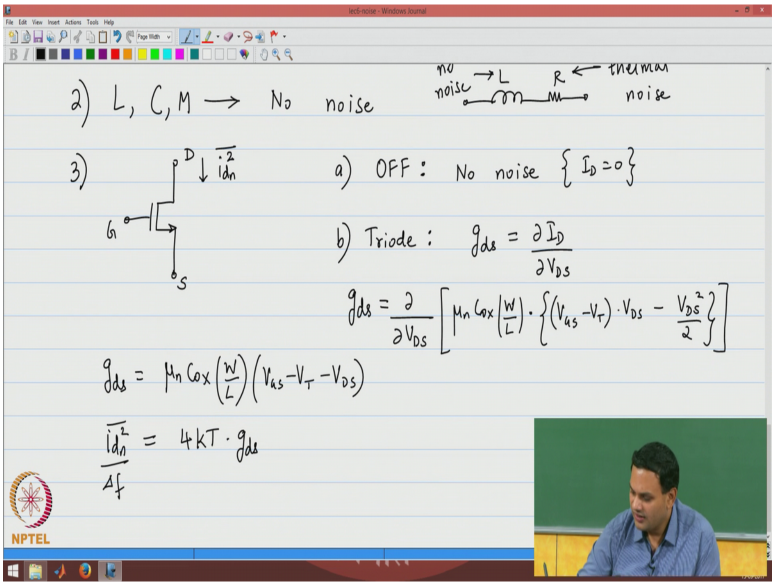

2) L, C, M → No noise

no noise → L R ← thermal noise

3)

a) OFF : No noise $\{ I_D = 0 \}$

b) Triode : $g_{ds} = \frac{\partial I_D}{\partial V_{DS}}$

$$g_{ds} = \frac{\partial}{\partial V_{DS}} \left[\mu_n C_{ox} \left(\frac{W}{L}\right) \cdot \left\{ (V_{GS} - V_T) \cdot V_{DS} - \frac{V_{DS}^2}{2} \right\} \right]$$

$$g_{ds} = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T - V_{DS})$$

$$\frac{\overline{i_{dn}^2}}{\Delta f} = 4kT \cdot g_{ds}$$

The second type of element that we will talk about are inductors and capacitors andit turns out thatinductors and capacitorsdo not produce noise they do not have any noise associated with themand that of course, includes transformers also. I will represent that as M. So, inductors capacitors transformers do not have any noise associated with them.

However, if you have non ideal inductors;for example, if you represent the series resistance of the coil used to create the inductance by a resistance R, this will have thermal noise.

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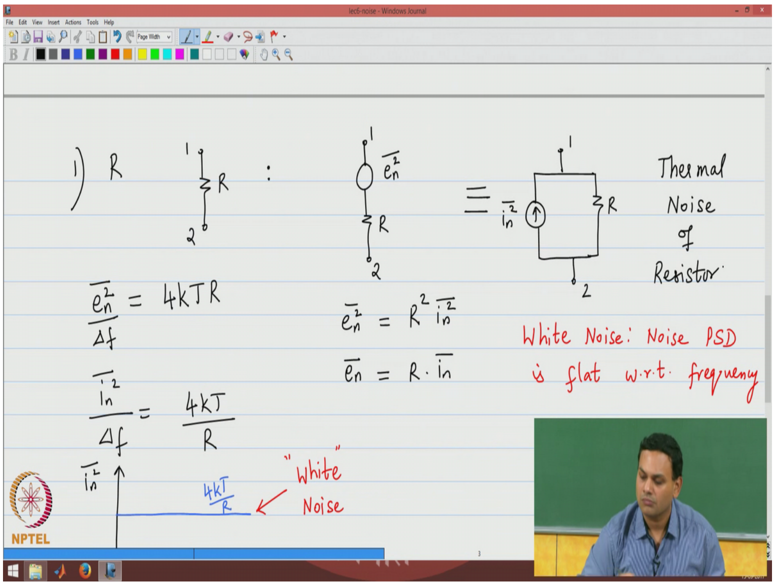

$$\frac{\overline{e_n^2}}{\Delta f} = 4kTR$$

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$$\overline{e_n^2} = R^2\,\overline{i_n^2}$$

$$\overline{e_n} = R\cdot\overline{i_n}$$

Thermal Noise of Resistor

White Noise: Noise PSD is flat w.r.t. frequency

This will have resistor thermal noise, but this inductance will have no noise.

Next, we will look at aMOSFET. The MOSFET of course, has 3 terminals;drain,gate and source.Since the gate current is 0,there is no noise associatedwith the gate current, but there is noise associated with the drain currentand I will represent that asid subscript n,the whole squared.

Now, clearly the MOSFET has 3 regions of operation and the noise of the MOSFET is different in each region.When the MOSFET is offfor example, when the gate source voltage issmaller than the threshold voltage, that device will have no noise because it does not conduct.In thetriode region,that device hasconductance g dsthat is given by dou I DoverdouV DS. So, gds in the triode regionis given by dou by douV DSof the current expression.The current expression is munCOX,W over LintoV GS minus V Ttimes V DSminus V DSsquaredover 2.

So, we have to take the first derivative with respect to the drain source voltage andthis g dsis given bymu nCOX,W over LintoV GSminus V TminusV DSin the triode region and therefore, it turns out that thethermal noise associated with the device in the triode regionis given bythe squared current density is given by 4kTtimes gds.

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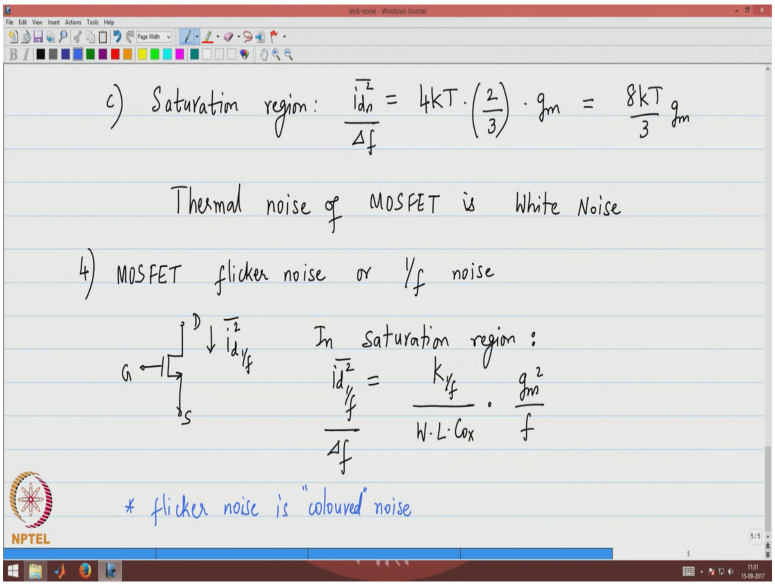

c) Saturation region: $\frac{\overline{i_{dn}^2}}{\Delta f} = 4kT \cdot \left(\frac{2}{3}\right) \cdot g_m = \frac{8kT}{3} g_m$

Thermal noise of MOSFET is White Noise

4) MOSFET flicker noise or $1/f$ noise

$\downarrow \overline{i_{d_{1/f}}^2}$

In saturation region:

$$\frac{\overline{i_{d_{1/f}}^2}}{\Delta f} = \frac{K_{1/f}}{W \cdot L \cdot C_{ox}} \cdot \frac{g_m^2}{f}$$

* flicker noise is "coloured" noise

The third region of operation of the device isthe saturation region.In this region the device thermal noisehappens to be dependent on the trans-conductance rather than g ds and this is given by 4kTtimesfactor twothirdstimesgmor 8kTby 3 times g m.

This is the thermal noise of the device in the saturation region.

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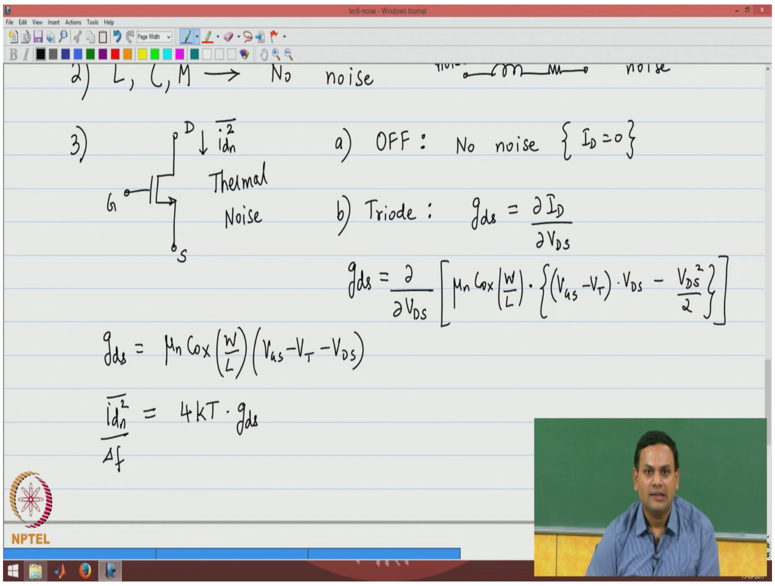

2) L, C, M → No noise

3) $\downarrow \overline{i_{dn}^2}$ Thermal Noise

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$$g_{ds} = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T - V_{DS})$$

$$\frac{\overline{i_{dn}^2}}{\Delta f} = 4kT \cdot g_{ds}$$

Sofar, we have been looking at the thermal noiseof the MOSFET.Itsohappens that the MOSFET also has a different type of noise. So, please note, that the thermal noise is flatin nature with respect to frequency.They all have constant power spectral densities and therefore, they are white noise sources.

The next type of MOSFET noise that we are going to look at is calledflicker noise. So, when the MOSFET is in the saturation regionthe MOSFET has one more source of noise and I am going to refer to that asid 1 over f squared.This is also called1overf noise because;the power spectral density is inversely related to frequency.In the saturation regionthe devicehas a 1over f noise that is relatedin the following way,the 1over f noise current densityis given by some constant k overWtimesLtimesCOXtimes g m squared over f.

So, there are several things to note for the flicker noise;number one,flicker noiseis colored noise,that is, the noise spectral density the current squared mean squared spectral density doesvary with frequency.

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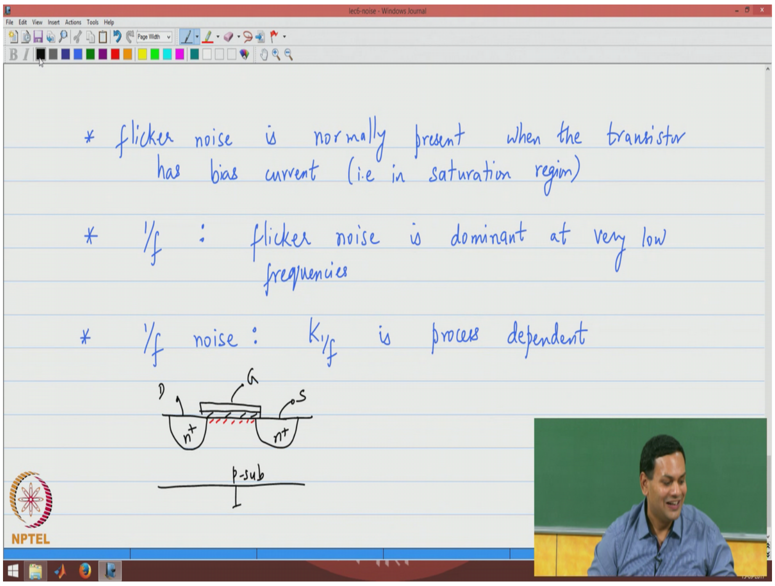

* flicker noise is normally present when the transistor has bias current (i.e in saturation region)

* $1/f$: flicker noise is dominant at very low frequencies

* $1/f$ noise : $K_{1/f}$ is process dependent

Number 2;flicker noiseis normally presentwhen the transistor has a bias current, in other words,in the saturation region.You do not expect to see flicker noise in the triode region where normally the transistor is used as a resistor.Number 3;because of 1over f dependenceyou expectflicker noise to bedominantat very low frequencies.We are in a minute we are going to draw the spectral content.Finally, this flicker noise or 1over f noise,it has a process dependent parameter which is calledK 1overfand therefore, different processes could have different amounts of flicker noise.

Now, the reason this happens is because flicker noise is related to unwanted traps in the silicon to silicon dioxide boundary inside the MOSFET at the gate oxide to the channel interface. So, when you have when you build the MOSFET, you may have unwanted traps in this region.In the interface between the substrate the channel region and the oxide and when you have them you could have charge carriers periodically being trapped in a random manner and released in a random manner again.And that causes flicker noise and therefore, if you are able to build a process which has lower density of traps you might very well have lower flickernoise for that process.

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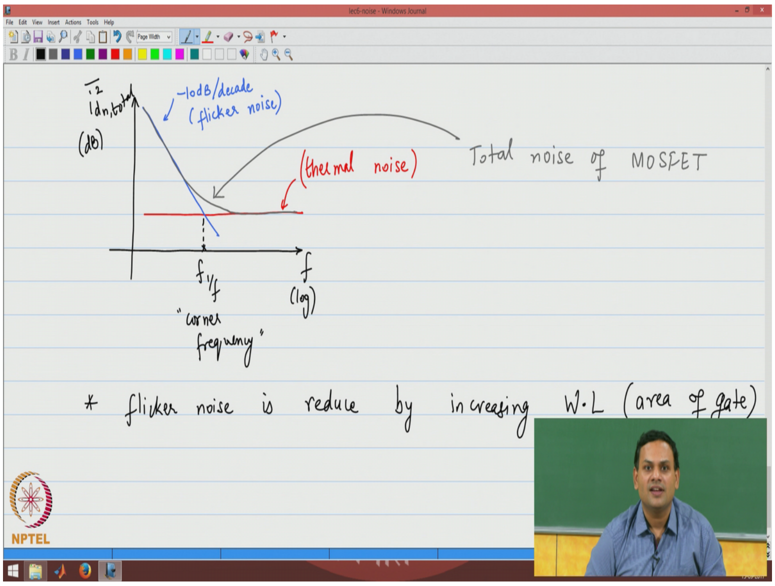

* flicker noise is reduce by increasing W·L (area of gate)

Having said this, let us now draw the noise of the MOSFET.Let us draw the total noise of the MOSFET. I will first show the flicker noise; let us assume that this is the, I am going to plot thetotal with the individual contributions also shown. So, I will first show the flicker noise in blue and I am going to show it in a log;log scale. In fact, even better is, if I show it in a decibel scale. So, if I do this,1 over f noise wewill have a power spectral density which looks like this.Since, I am drawing itin a decibel scale I will seethat the magnitude of this noise drops at a rate of minus10 dBper decade change in frequency.

The blue color shows the flicker noise.In red color I am going to showthermal noise of the device.Note that; thermal noise is white noiseand it has a flat power spectral density and therefore, if you look at the total noiseat very low frequenciesthe noise is dominated by the noise from flicker noise and at very high frequenciesthe noise is dominated by thermal noise.

So, normally you would expect that flicker noise is dominant at very low frequencies and thermal noise is dominated very high frequencies.Now, how do you decide at what frequencyeach one is dominant that completely depends on the values of gm that is the thermal noise level and k and g m and W L C OX, sothat, you would have a resultantnoise corner frequency which is called1over f,corner frequency.This corner frequency is completely dependenton the values of gm, k,W L C OXetcetera.

Finally, one last thing to note;you can reduce flicker noiseby increasing the area of the gate.So, the gate area of the MOSFET is WtimesL and if you increase that the absolute value of flicker noise is reduced because flicker noisesquared noise density is inversely proportional to that WLCOX.

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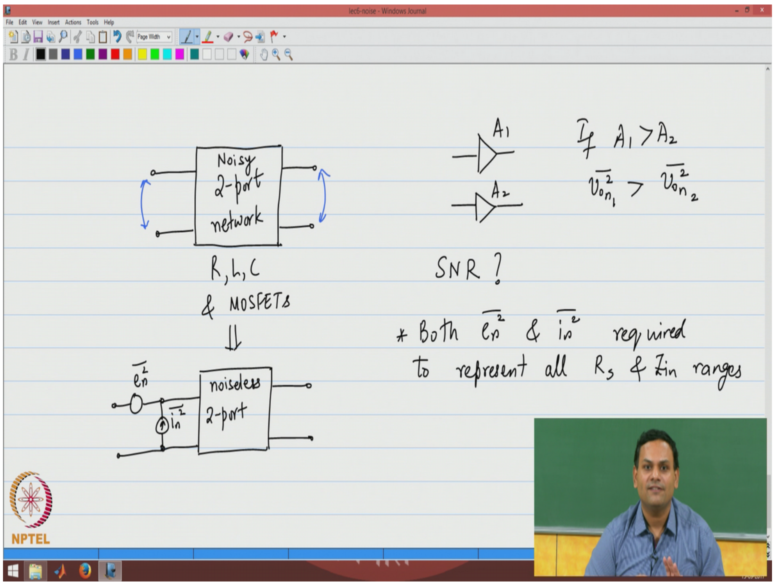

* Both $\overline{e_n^2}$ & $\overline{i_n^2}$ required to represent all R_s & Z_{in} ranges

In today’s lecture we will look at one final topic for just a couple of minutes. Let us say, now that I have learnt about sources of noise, let us say I have some2 port network.That consists ofR, L, C and MOSFETs.Clearly, this2 port network is going to generate noise. So, I will call it a noisy 2-port. I want to find some way to represent the noise of this2 port.

Now, clearly I can do it in2 ways. I can represent, I can take all the noise sources and if I want to represent it in asystem manner I need to represent it with respect toeither the input port or the output port.Just like at a very high leveljust as we would represent the system parameters by the z, y, g, h parameters.We can represent the noise the total noise produced by the 2 port either at the input or at the output.

Now,if you represent itat the output,it turns out this is not a fair way of comparing different 2 ports.The reason we want to represent the noise at the input and the output is to compare 2 different networks and see if one is noisier than the other.For example; I have 2 amplifiers,with gatesA 1and A2. I want to find out in a particular application whether I should use A 1 or A2 with respect to noise. I need to find some way to represent this.IfA 1 was much larger than A2,if I look simply at the output,it is very possible that VO n squaredis much larger thanVO n1squared is much larger than VO n2 squared.

So, in other words the output noise of A 1 may look much larger than that of A2,because if I keep increasing the gain of the circuit, the noise levels will look larger and larger.The important thing to note is we are really interested insignal to noise ratio.If you increase the gain A 1 the signal at the output of A 1 is also larger. So, we want to find out of fair way of representing it and the fair way of representing thenoise of a 2-portis at the input. So, we convert itto a noiseless 2-portwith a representation at the inputand in general,they are represented by 2 sources at the input;a voltage source and series with the input and a current source in shuntwith the input.

So, you need both e n squaredandin squared, in general, to represent2-ports with all range of input resistances and all range of source resistances. So, in general any 2-port will have ae n squaredvoltage noise source in series with the input and a current noise source in shunt.

In the next lecture, we will look at some example circuits,MOSFETs resistors where we will calculate the input referred noise current and voltage sources.