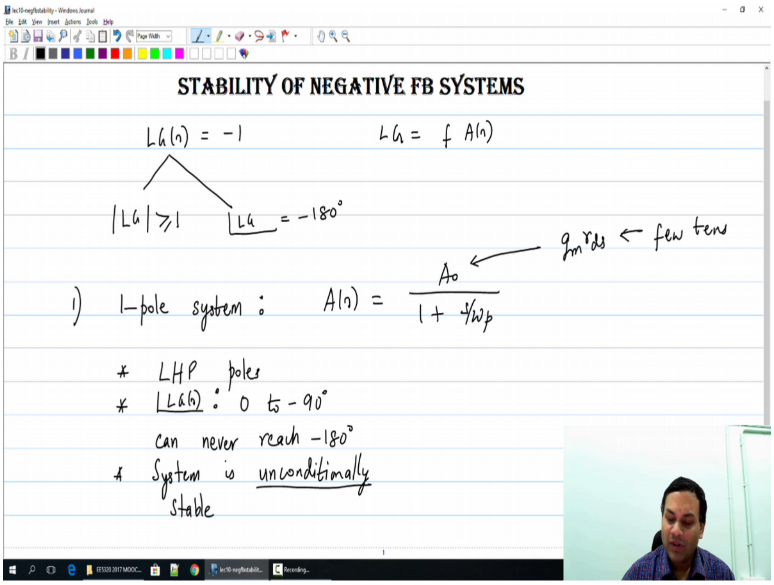
Analog Integrated Circuits
Prof. S. Aniruddhan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 10
Stability of Negative Feedback Systems

In this lecture, we will start by looking at the Stability of Negative Feedback Systems as a function of frequency.

(Refer Slide Time: 00:25)



So as we saw previously, we are going to compare the loop gain in the Laplace domain with minus 1 and as we said this can be split into two sets of conditions. The first condition is that the magnitude of the loop gain be greater than or equal to 1 and the second condition is that the angle of the loop gain be minus 180 degrees.

Now, these are called the Barkhausen criteria, but more importantly for us what this tells us; this gives is a bound on the stability of feedback systems on the; actually the sets the condition on the loop gain of the system. So, what we will do? We will start looking at some simple systems 1 pole, 2 pole, 3 pole systems and see what implication there is for stability. So, just a reminder that the loop gain is the feedback factor f times A of s.

So, we will take some sample systems, so let us start off with a 1 pole system. So, in other words A of s is; A naught by 1 plus S by omega p; this is a frequency response of a

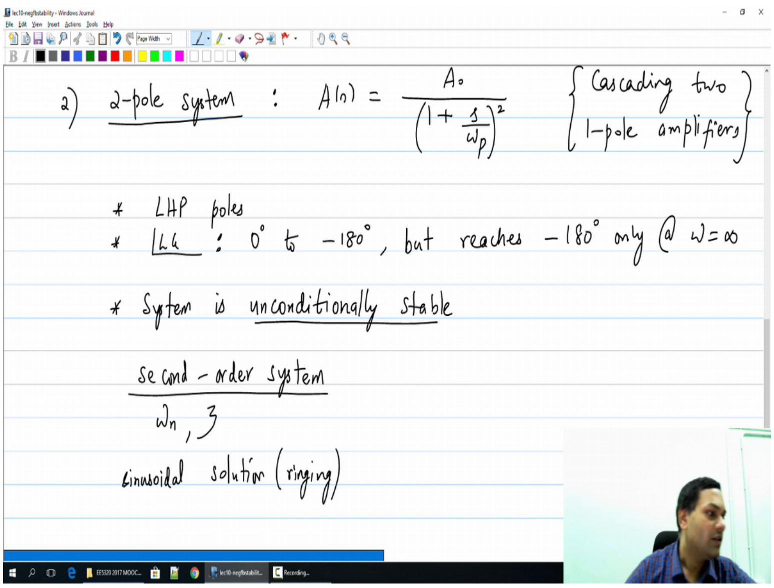
1 pole amplifier. And of course now we need to look at f times A of s, so what we can say about this system is that of course, the poles are all in the left half plane; which is good for us. Because normally when you have a resistor and a capacitor, you will get left half plane poles.

Normally, such a system will not behave in an unexpected manner. So, then we will look at the angle of the loop gain. Now, it turns out that for this particular system; at very low frequencies the angle of the loop gain is 0. And at very high frequencies, the angle of the loop gain goes to minus 90 degrees. So, the loop gain angle stands 0 to 90 degrees; it can never reach 180 degrees.

So, what this means for us is that the system is unconditionally stable and this is great news for us. What is the problem with 1 pole system? As we have seen before the 1 pole system can only have limited gain. So, the maximum gain that you can get for example, from a common source amplifier; which was an example of a 1 pole system that we saw; is only the intrinsic gain of a transistor which is approximately g m r ds.

Now, this g m r ds number is probably a few tense to maximum maybe hundred or so in modern processes. So, therefore you just cannot get enough gain; let us say g m r ds was 100, that is just not good enough to build a good ohm amp.

(Refer Slide Time: 04:08)



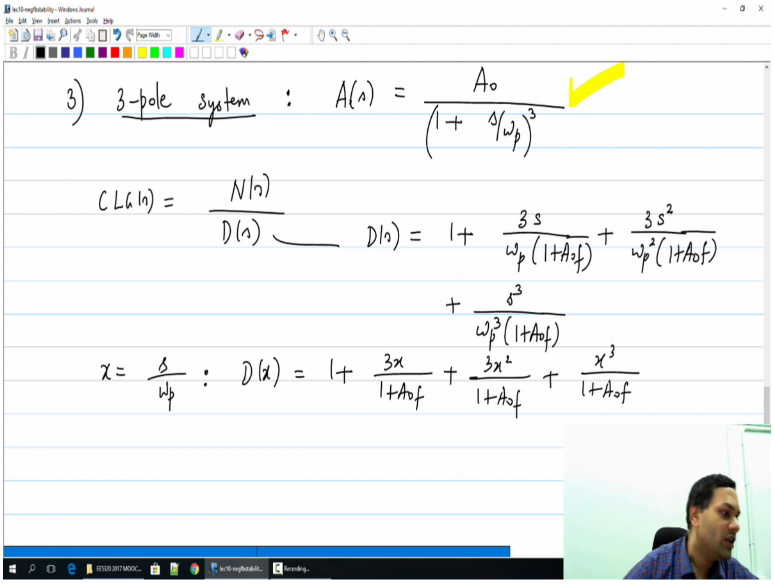
Then next we try to build a 2 pole system, if you look at a 2 pole system; we say that A of s is some A naught by 1 plus s by omega p, the whole square.

So, in other words what I mean is; I am cascading two 1 pole amplifiers. So, for example, if the gain of 1 pole amplifier was around 100; if you cascade them, you can easily get a gain of 10000; so that is good news for us. Now, we need to look at the impact of doing this on stability. So, now what do we know about this system; so the nice thing is again it has left half plane poles, the phase spans 0 to minus 180 degrees but reaches minus 180 degrees; only at omega equals infinity.

So, technically this system is unconditionally stable; so this is also great news for us. Unfortunately, if you look at the time domain response of the system; as we know at 2 pole system or rather a second order system is defined by two parameters, which is the natural frequency and the damping factor.

So, what this means is that will be a particular value of A naught on omega p; where the system will start to ring. What do we mean by saying the system will start to ring? So, normally you expect a well behave system to only have an exponential solution. It turns out a second order system is also capable of having a sinusoidal solution, which means the step response of the system could have ringing. And we can easily calculate the value of gain at which this happens and it turns out that this happens at a fairly low value of A naught f, so this is a problem. So, even though the system is technically stable; the transient response in closed loop could take a long long time to settle, so that is a bad thing for us.

(Refer Slide Time: 07:35)



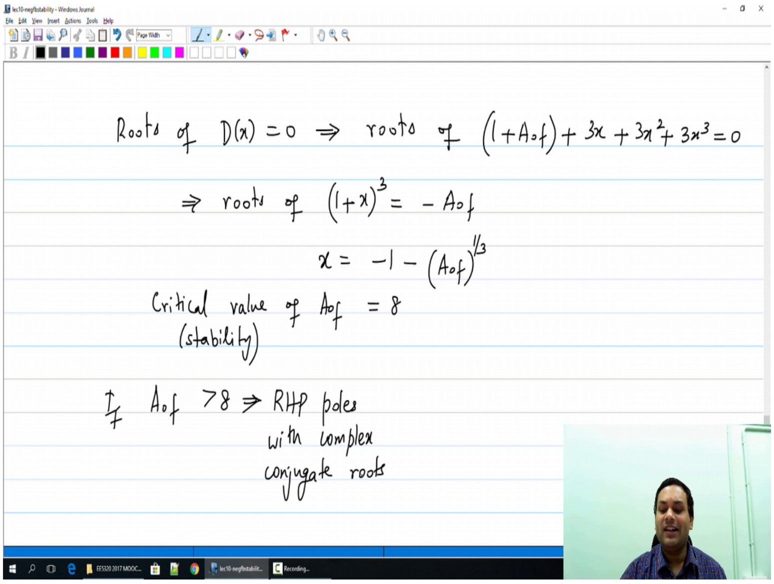
So, now we will take a 3 pole system and look at its response. So, what that means, is that f times rather A of s is A naught by 1 plus s by omega p whole cubed. In other words, this system is created by cascading three 1 pole systems; clearly the maximum possible value of A naught can be quite high.

Now let us look at 1 plus L g of s or rather let us look at the closed loop gain of the system. It turns out that the closed loop gain, you can write it as some numerator polynomial by denominator polynomial. I will give you the final expression; I will leave this is the homework for you. So, it turns out the denominator polynomial D of s can be written in this fashion, it can be written as a polynomial in this fashion.

So, D of s is 1 plus 2 s by or rather A naught f omega p plus oh sorry; so, it is 1 plus; 3 s by omega p into 1 plus A naught f; plus 3 s squared by omega p squared into 1 plus A naught f; plus omega p cubed by; I am sorry s cubed by omega p cubed into 1 plus A naught f. So, all I have done is I have taken this particular expression; I have plugged it into the closed loop gain expression, which is 1 over f times; f A of s by 1 plus f; A of s. And I have made it look like a polynomial n of s; over another polynomial D of s and I am looking at the denominator polynomial.

We know that the system becomes unstable when the denominator polynomial goes to 0. So, what does that mean for us? We need to find out the roots of the denominator polynomial D of s. So, now, I am going to make a simple substitution; I am going to say x is s by omega p; just to make my writing easier. So, what I am going to look at is; D of x is 1 plus 3 x by 1 plus A naught f; plus 3 x squared by 1 plus A naught f; plus x cube by 1 plus A naught f.

(Refer Slide Time: 10:57)



And which means I need to find out the roots of the polynomial. So, roots of D of x equal to 0 are the same as roots of 1 plus A naught f; plus 3 x; plus 3 x squared plus 3 x cubed equal to 0. Now the roots of this particular polynomial are the same as roots of 1 plus x the whole cubed equals minus A naught f or x is equal to minus 1; minus A naught f; whole power 1 by 3.

Now, it is clear that the cube-th root of A naught f will have three roots and each one of them will give you a particular solution. However, it turns out that for values of A naught f larger than a particular value, the system starts to have sinusoidal solutions and more importantly the complex conjugate roots corresponding to the sinusoidal solution, move into the right half plane which clearly points to an unstable system because it points to a sinusoidal solution whose amplitude is increasing.

So, it turns out that happens at a critical value of A naught f; which is 8 for stability. So, what I mean is; if A naught f is greater than 8. So, you have right half plane poles with complex conjugate roots. Now, as you can see this is clearly a problem, because we started of assuming that you can cascade; multiple single pole amplifiers to get more and more gain.

But this is telling you that if you try to do that for example, for a 3 pole system; even if the loop gain is greater than 8, the system will start to become unstable. Remember that we want a loop gain A naught f; which is much much larger than 1. So, you are talking about thousands of tens of thousands type of numbers. So this is clearly not the way to go.

In the next lecture we will see how we can achieve large A naught f while still maintaining stability.