

Analog Integrated Circuits
Prof. S. Aniruddhan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 09
Negative Feedback

In this lecture we will have a quick introduction to negative feedback. We will take up the more advanced aspects of negative feedback in a different lecture. So, this is meant to be a refresher because in this advance course you are already assume to have encountered negative feedback in an earlier course in a prerequisite course, but we will quickly review it in a very short lecture So, what is a negative feedback system consist of? It consists of broadly operationally it consists of three major portions, you have a desired quantity, you have the actual quantity and you want to drive the desired quantity toward the you want to drive the actual quantity towards the desired quantity or maybe a replica of that.

(Refer Slide Time: 00:52)

NEGATIVE FEEDBACK

Amplifier
 $V_o = k V_i ; k > 1$

$V_e = V_i - f V_o$
 $V_o = A V_e$

$$\left. \begin{aligned} V_o &= A (V_i - f V_o) \\ V_o &= A V_i - A f V_o \\ V_o (1 + A f) &= A V_i \end{aligned} \right) \quad \frac{V_o}{V_i} = \frac{A}{1 + A f} = \frac{1}{f} \frac{A f}{1 + A f}$$

In the case of electrical engineering we are trying to build amplifiers. So, we will assume that you are trying to build a negative feedback amplifier where V_o is some k times V_i and of course, because it is an amplifier k is greater than 1.

How do we do this using negative feedback? As you all probably know you take a small portion of the output quantity in this case I am representing the output quantities by

voltage quantities. So, you take a small replica of V_o and we will call that sum f times V_o . We compare that with V_i by taking a by performing a subtraction operation. So, if you subtract f times V_o from V_i you will get what is called an error quantity we will represent that as V_e . Now the magnitude and sign of V_e we will tell you how far away V_o is from V_i .

And then you magnify this error quantity using a forward amplifier whose gain we will term as A . So, now, the quantity describing the system are V_e equals V_i minus f times V_o and V_o equals A times V_e . And now we can now manipulate these two to get the actual relationship between V_o and V_i and V_o is nothing but A times V_i minus f times V_o and V_o is $A V_i$ minus $A f V_o$ and therefore, this expression holds true and finally, we can write V_o by V_i has been equal to A by $1 + A f$.

Now, this is the common way of writing this, but now I will write it in a slightly different manner, I am going to write it as 1 over f times $A f$ by $1 + A f$. Now, the reason I am going to write in this form is twofold, number one, we know that the ideal value is in reality V_o should be some replica of V_i , now when does that happen? That happens when you have $A f$ being as large as possible. Let us just write that down separately.

(Refer Slide Time: 04:23)

$$\text{If } A_f \rightarrow \infty \Rightarrow \frac{V_o}{V_i} = \frac{1}{f} = k \quad f < 1$$

$A_f = \text{loop gain}$

Frequency Dependence

$A \rightarrow A(s)$

$$\frac{V_o}{V_i}(s) = \frac{1}{f} \frac{f A(s)}{1 + f A(s)}$$

If $A(s)$ is freq. dependent
 \Rightarrow Loop gain

If $f A(s) \gg 1 \Rightarrow \frac{V_o}{V_i}(s) = \frac{1}{f}$

So, if $A f$ tended to infinity then V_o by V_i will become equal to 1 over f which is the same as k . So, now you have achieved your amplifier that you have been wanting to design so clearly f should be less than 1 since k is greater than 1 .

Now, this quantity $A f$ is called the loop gain of the system and very often it is the most important quantity associated with the negative feedback system. Why do I say it is a most important quantity? So, in reality even though you might have studied you know earlier that you want A to be as large as possible which is true, it also you will find that the actual quantity that matters for most of your feedback systems is A times f because it is primarily only when you have A times f that is much larger than 1 that the quantity that I have circled here goes to 1.

Now this is definitely looks good, now what happens if you have systems that are frequency dependent? So, we will club all of this together and assume that it is only the forward block that is frequency dependent. Now, what we mean is the forward block is no longer A , but is it is sum A of s in the Laplace domain if that happens then V_o by V_i of s is what we are trying to derive and that is clearly 1 over f times f times A of s by 1 plus f times 1 plus f times A of s . And now you can see that if A of s starts reducing with frequency then you can see that is frequency dependent for example, its starts reducing with frequency you can see that loop gain will start reducing with frequency.

And as we know negative feedback is operating only when the loop gain is extremely large. So, at frequencies were f times A of s is much much larger than 1, V_o by V_i of s becomes very close to 1 over f . If A of s starts degrading with frequency that you will reach a point where the loop gain becomes comparable to 1 and negative feedback is said to be no longer operational.

(Refer Slide Time: 07:59)

The image shows a digital whiteboard with handwritten mathematical notes. At the top, it says $A \rightarrow A(s)$. To the right, it says \Rightarrow Loop gain. Below that, it says $\text{If } f A(s) \gg 1 \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{f}$. The main equation is $\frac{V_o(s)}{V_i(s)} = \frac{1}{f} \frac{f A(s)}{1 + f A(s)}$, where the denominator $1 + f A(s)$ is circled in yellow. Below this, it says $\frac{f A(s)}{1 + f A(s)} \leftarrow \text{If } 1 + f A(s) = 0$. A small video inset of a person is visible in the bottom right corner.

Now, we are specifically going to look at the denominator of this expression f times A of s by 1 plus f times A of s . So, if you look at this the denominator expression there will be frequencies where it is possible for 1 plus f times A of s to become equal to 0 because now A of s is a function of frequency it has both magnitude and phase therefore, it is possible for 1 plus f times A of s to become 0 .

What implication does this have for the system? So, if you go back and look at the expression for V_o by V_i of s you can clearly see that if the denominator goes to if the denominator goes to 0 then V_o by V_i goes to infinity or is indeterminate really, what it really means is V_o is no longer a function of V_i or to put it in different terms you can have an output V_o of s even if V_i of s were equal to 0 .

(Refer Slide Time: 09:25)

$fA(s)$
 $1 + fA(s) \leftarrow$ If $1 + fA(s) = 0 \Rightarrow$ It is possible to have finite $V_o(s)$ even if $V_i(s) = 0$
 \Rightarrow Instability
 $1 + fA(s) = 0 \Rightarrow LA(s) = -1$
 $\begin{cases} |LA(s)| = 1 \\ \angle LA(s) = -180^\circ \end{cases}$

So, what this means it is possible to have finite V_o of s even if V_i of s were equal to 0. In other words your system is unstable. So, normally a stable system is a bounded system you will get an output only if you apply an input. So, this means that there is a possibility of instability. Now, you would have seen this in many forms in your undergraduate courses or in your prerequisite courses in either analogue circuits or control engineering or linear IC design basic idea is that this can be put in many different forms, but in this particular course we will see instability in this very specific form because we are dealing with systems which have certain number of poles and zeros. So, we will express it in this manner.

So, we will say that we will write it in this form $1 + f$ times A of s is 0 we will rewrite it as f times A of s or I will just write it as loop gain of s is equal to minus 1 and this has two portions as you might imagine the magnitude of the loop gain should be exactly equal to 1 and the phase of the loop gain should be minus 180 degrees. If these two conditions are satisfied it is there is potential for instability in the system. So, these are the two criteria that we are going to apply.

We will go one step forward and say that it is possible for the system to be unstable even if the loop gain were greater than 1. You will learn the reason for this; you would have learned the reason for this in your control engineering course. So, we will not cover that in

this course, but I just want to point out that this is the condition that we will be applying for instability.