Control Engineering
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Module - 12
Tutorial - 1
Lecture - 52
State Space Canonical Forms

Hello everyone. In the previous lectures, we were looking at State Space Models, we were introduced to them.

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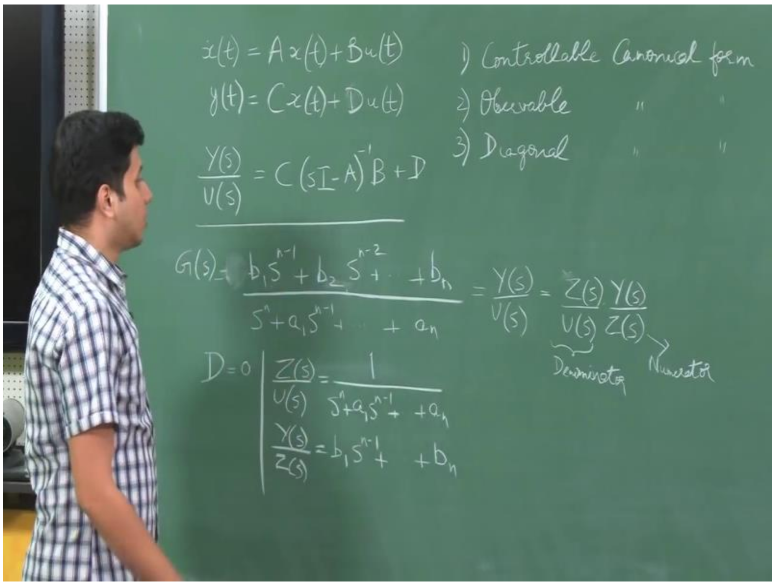

So, we looked at the standard form of the state space model as $\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{u}(\boldsymbol{t})$; and output $\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{C}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}\boldsymbol{u}(\boldsymbol{t})$. So, here $\boldsymbol{x}$ is the state vector, and $\boldsymbol{u}$ is the input vector, $\boldsymbol{y}$ is the output vector. And $\boldsymbol{A}$ is the dynamics matrix; $\boldsymbol{B}$ is the input matrix; $\boldsymbol{C}$ is the output matrix; and $\boldsymbol{D}$ is the feed forward matrix. So, these are the standard names for these matrices.

And so there are two things that we can do. So, given a state space model, we can go back to the transfer function model; and given a transfer function we can go to the state space model. So, earlier we looked at how can we go from the state space model to the transfer function model using this formula. So, $\frac{\boldsymbol{Y}(\boldsymbol{s})}{\boldsymbol{U}(\boldsymbol{s})} = \boldsymbol{C}(\boldsymbol{sI} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}$.

So, given a state space model with the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$, this is how you can transform it into a transfer function. And this transformation is unique. So, any state space model given you can only determine one transfer function, so that is the case of going from state space to transfer function

Now, what about the other way? So, when we go from transfer function to state space, the transformation is not unique. So, that means, when you are given one state space model you can find out multiple transfer function models. So, instead of finding some kind of random transfer, some random state space model, we will try to find out some standardized state space models which we call canonical forms.

And so today we will discuss about three canonical forms. First one is controllable canonical form, the second one is the observable canonical form, and the third one is the diagonal canonical form. So, while we discuss about each of these forms, we will see why these names come.

So, first one will be the controllable canonical form. So, to derive this, we'll start with a strictly proper transfer function on this form, sorry. So, this is strictly proper transfer function, because the numerator has a degree of $\boldsymbol{n-1}$, and the denominator has a degree of $\boldsymbol{n}$, that is the number of poles are greater than the number of zeros. So, what happens in this transfer function is $\boldsymbol{t=0}$ ok.

So, when we are given this transfer function, how do you find out a controllable canonical form of the state space model? So, in case you have a transfer function which is not strictly proper something like you can have here say $\boldsymbol{b_0 S^n}$. So, in that case, what you can do is you can simply perform polynomial division, and take this $\boldsymbol{b_0}$ out. So, that it will come something like this, and rest again remains in the same form, we can again apply the same transformation that we are doing now.

So, for now we will stick to a strictly proper transfer function and go ahead with it. So, given this strictly proper transfer function, how do we convert it into a controllable canonical form? So, what we'll do is, we'll deal with the numerator and the denominator separately. So, for that, I will take this as $\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}}$ is equals to I will introduce a new polynomial called $\boldsymbol{Z(s)}$. So, $\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}}$ can be written as $\frac{\boldsymbol{Z(s)}}{\boldsymbol{U(s)}}\frac{\boldsymbol{Y(s)}}{\boldsymbol{Z(s)}}$.

So, I am just multiplying and dividing by a new polynomial $\boldsymbol{Z(s)}$. And this will be taken as the denominator part, and this part will be the numerator. So, I will just write it down. So, $\frac{\boldsymbol{Z(s)}}{\boldsymbol{U(s)}} = \frac{\boldsymbol{1}}{\boldsymbol{S^n + a_1 S^{n-1} + \dots + a_n}}$, so that will be $\frac{\boldsymbol{Z(s)}}{\boldsymbol{U(s)}}$. And then $\frac{\boldsymbol{Y(s)}}{\boldsymbol{Z(s)}}$ will be just the numerator. So, these are not any transfer function. We are just introducing them and divide what you say dividing them into two parts just for the sake of calculation convenience.

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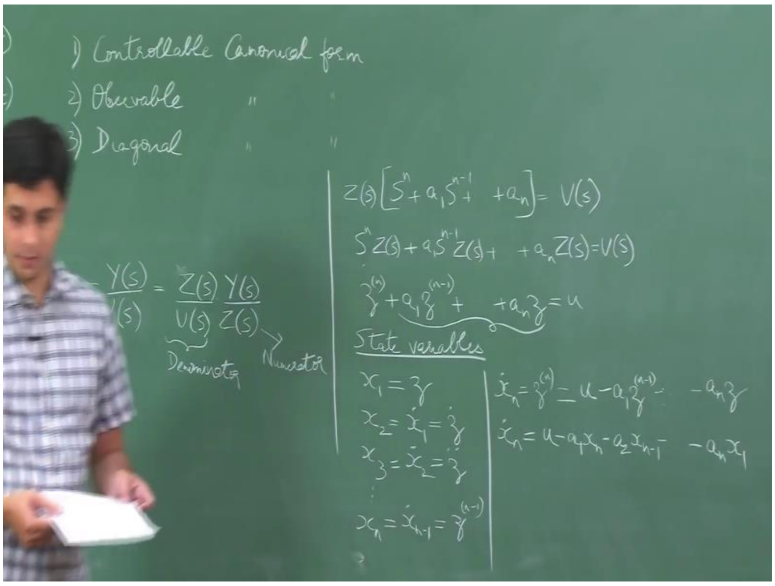

So, now, first we will take the first part, and we will do some algebraic manipulations as follows. So, I can take $\boldsymbol{Z(s)}$ times I can do cross multiplication on this, and then I will get $\boldsymbol{Z(s)[S^n + a_1 S^{n-1} + \dots + a_n] = U(s)}$.

So, now what I will do is, I will take inverse Laplace transform. So, when I do inverse Laplace transforms, $\boldsymbol{Z(s) = z}$. And so and it will be ok, I will just first multiply it $\boldsymbol{S^n Z(s) + a_1 S^{n-1} Z(s) + \dots + a_n Z(s) = U(s)}$.

Now, when we apply Laplace inverse, $\boldsymbol{S^n} = \boldsymbol{z^{(n)}}$(nth derivative of z). So, I can write it as $\boldsymbol{z^{(n)}}$ in the brackets. So, this is a notation to denote that we are taking the derivative of $\boldsymbol{z^{(n)}}$times, and so we will just follow the same thing $\boldsymbol{a_1 z^{(n-1)}}$ derivative of $\boldsymbol{z}$ and so on up to $\boldsymbol{a_n z}$ is equals to $\boldsymbol{u}$ (i.e., $\boldsymbol{z^{(n)} + a_1 z^{(n-1)} + \dots + a_n z = u}$). So, when we take the Laplace inverse, we are representing everything in small letters. So, $\boldsymbol{Z(s)}$ becomes small $\boldsymbol{z}$, and $\boldsymbol{U(s)}$ becomes small $\boldsymbol{u}$.

Now, on this differential equation, we will be defining our state variables. So, here is how we define? So, I will define my x_1 I need to define n state variables. So, x_1 I will take it as z, and x_2 I will take it as $\dot{x}_1$ will which will be equal to $\dot{z}$, so that's the first derivative of z. And then x_3 will be $\dot{x}_2$ which will be equal to $\ddot{z}$, and I will go on up till x_n is equals to $\dot{x}_{n-1}$, which will be equals to $(n-1)^{\text{th}}$ derivative of z. And x_n, I will write here $\dot{x}_n$ is equals to the $z^{(n)}$.

So, this $z^{(n)}$, I will take it from here. I will take all these terms and send them to the other side, and write it here. So, that will be $z^{(n)} = u - a_1 z^{(n-1)} \ldots - a_n z$. Now, you can see that we have $z^{(n-1)}, z^{(n-2)}, \ldots z$, and all these variables are already defined in terms of the state variables. So, we will just substitute them here, and my $\dot{x}_n$ can be written as $u - a_1 x_n - a_2 x_{n-1} \ldots - a_n x_1$.

So, now you can see that we have represented all the state variables in terms of x now. So, now, we have a set of equations. So, what are those equations? The first one is $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$ so on $\dot{x}_{n-1} = x_n$, and $\dot{x}_n$ is this expression. So, using all these equations, we can write the state space model.

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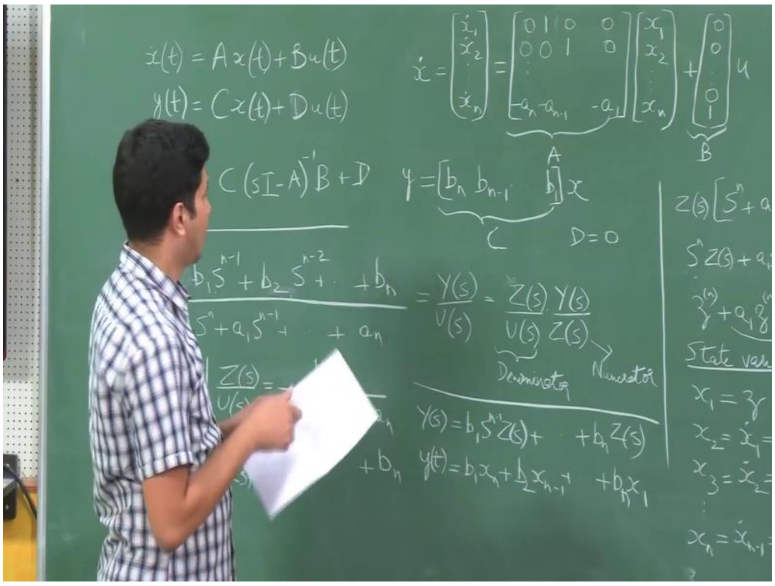

So, the state space model will be $\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \vdots \\ \dot{\boldsymbol{x}}_n \end{bmatrix}$. So, what is $\dot{\boldsymbol{x}}_1$? It is simply $\boldsymbol{x}_2$. So, I can write it 0 1 ok, I will just have to get a matrix here times $\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_n \end{bmatrix}$.

Now, you can see that this will be $\mathbf{0}$, this will be $\mathbf{1}$ and rest all again will be $\mathbf{0}$'s, because $\dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2$. Now, $\dot{\boldsymbol{x}}_2 = \boldsymbol{x}_3$. Now, you will get a $\mathbf{1}$ here and rest all will be $\mathbf{0}$'s. And similarly, finally, $\dot{\boldsymbol{x}}_n$ will be a_n minus $a_n x_1$. So, here it will be $-\boldsymbol{a}_n - \boldsymbol{a}_{n-1} \ldots. - \boldsymbol{a}_1$, so none of these first $(n-1)$ state variables have input. So, this will be all $\mathbf{0}$'s except at the end times $\boldsymbol{u}$.

Now, if you simply multiply these matrices and observe, you will get those set of equations exactly same ok. So, this is what we call the controllable canonical form. So, and this will be $\boldsymbol{A}$; this will be $\boldsymbol{B}$. And you can see output $\boldsymbol{y}$, so to get the output $\boldsymbol{y}$ we have to use the other set of equations.

So, I can take this and say $\boldsymbol{Y}(\boldsymbol{s}) = \boldsymbol{b}_1 \boldsymbol{s}^{n-1} \boldsymbol{Z}(\boldsymbol{s}) + \cdots \ldots + \boldsymbol{b}_n \boldsymbol{Z}(\boldsymbol{s})$. So, here again I can take the Laplace inverse, and just use those equations. Then my $\boldsymbol{Y}(\boldsymbol{s})$ will simply be, when we take the Laplace inverse, it will be just $\boldsymbol{y}$ or $\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{b}_1 \boldsymbol{x}_n + \boldsymbol{b}_2 \boldsymbol{x}_{n-1} + \cdots .. + \boldsymbol{b}_n \boldsymbol{x}_1$. So, this will be the equation for $\boldsymbol{y}(\boldsymbol{t})$.

And in the matrix form I can just write it as $\boldsymbol{b}_n$, sorry, this is $\boldsymbol{n}$, $\boldsymbol{b}_n$. So, in the matrix form, I can write it as $\boldsymbol{y} = [\boldsymbol{b}_n \boldsymbol{b}_{n-1} \ldots .. \boldsymbol{b}_1]\boldsymbol{x}$. So, $\boldsymbol{x}$ is the state vector. So, this matrix will become my output matrix $\boldsymbol{C}$. So, now you have $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ is equals to $\mathbf{0}$. So, when we use these matrices, the state space model that we get is the controllable canonical form.

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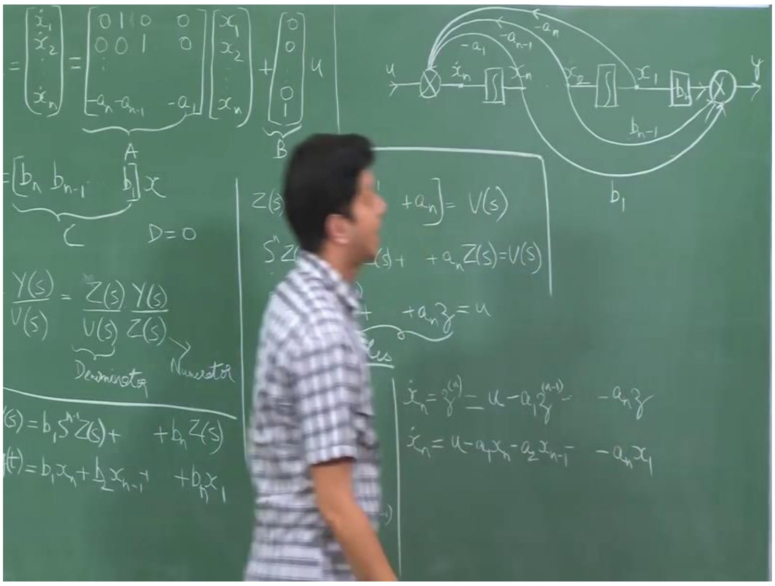

So, why is it called the controllable canonical form? I will draw a small diagram, and you will be able to see. So, we will draw a block, small block diagram to represent this. So, I will start with state $\boldsymbol{x_1}$ and so here I put an integrator block. So, this will be $\dot{\boldsymbol{x_1}}$, because when I integrate $\dot{\boldsymbol{x_1}}$, I will get $\boldsymbol{x_1}$. And this $\dot{\boldsymbol{x_1}}$ is nothing but $\boldsymbol{x_2}$.

So, similarly I can put a series of integrator blocks and get up to $\boldsymbol{x_n}$, and then I add another integrator block to get $\dot{\boldsymbol{x_n}}$. And to get the value of $\dot{\boldsymbol{x_n}}$, so as you can see $\dot{\boldsymbol{x_n}}$is summing up or over all the state variables with certain coefficients. So, we need to put a summer here and with the input, because it is $\boldsymbol{u}$ plus all this. So, $\boldsymbol{u - a_1 x_n - \cdots - a_n x_1}$.

So, my $\boldsymbol{x_n}$ is here I can just take a feedback from here and put $\boldsymbol{-a_1}$. And similarly it will be another feedback from $\boldsymbol{x_2}$ which will have a coefficient $\boldsymbol{-a_{n-1}}$. And similarly from $\boldsymbol{x_1}$, we will have another feedback with the coefficient $\boldsymbol{-a_n}$.

So, just forgetting the output for a while we can see that the input $\boldsymbol{u}$ is passing through all the state variables $\boldsymbol{x_n}$, $\boldsymbol{x_{n-1}}$, $\boldsymbol{x_{n-2}}$, ……, $\boldsymbol{x_2}$, $\boldsymbol{x_1}$. So, output $\boldsymbol{y}$ will actually come out here somewhere. So, it can be observed that input has a control over all the states in the system which is clearly observed to this block diagram, and that can be clearly seen in this matrix. So, this is the reason why we call the controllable canonical form, because the input has control over all possible, all existing states and that can be clearly seen.

So, if you want the output, you can further extend the block diagram by adding $\boldsymbol{b_n}$ here and so output $\boldsymbol{y}$ sums over $\boldsymbol{b_n x_1}$ and so it will be $\boldsymbol{b_{n-1} x_2}$ and so on up to $\boldsymbol{b_1 x_n}$, so that is the complete block diagram of the state space model. And you can clearly see why it is a controllable canonical form.

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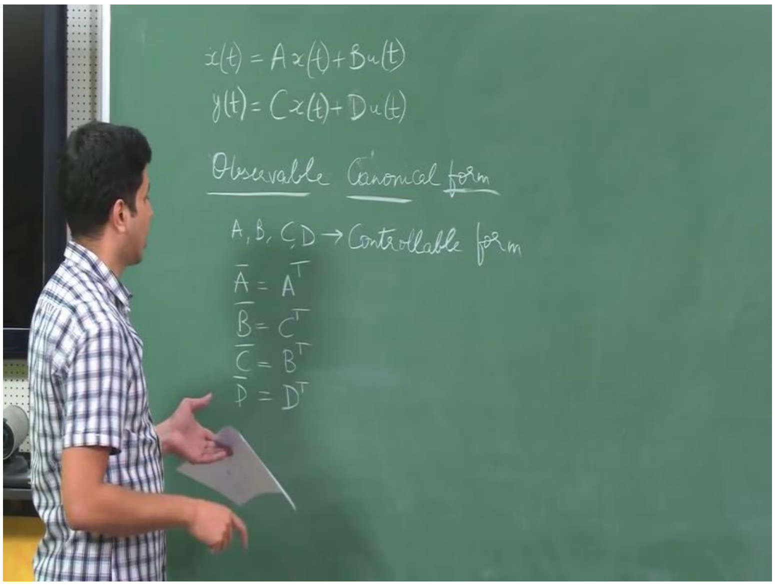

So, now we look at the observable canonical form ok. Now, coming to the observable canonical form. This actually becomes very simple once you know the controllable canonical form. So, once say $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are the matrices pertaining to the controllable canonical form. So, the matrices pertaining to the observable canonical form can be written as follows. I will call them as $\overline{\boldsymbol{A}}$, $\overline{\boldsymbol{B}}$, $\overline{\boldsymbol{C}}$, and $\overline{\boldsymbol{D}}$.

So, $\overline{\boldsymbol{A}}$ is nothing but $\boldsymbol{A^T}$; $\overline{\boldsymbol{B}}$ is nothing but $\boldsymbol{C^T}$; $\overline{\boldsymbol{C}}$ is nothing but $\boldsymbol{B^T}$; and $\overline{\boldsymbol{D}}$ is nothing but $\boldsymbol{D^T}$. So, this will be the observable canonical form you. So, you can just find out the controllable canonical form, and take the transforms of those matrices, and arrange them in this manner to get the observable canonical form ok.

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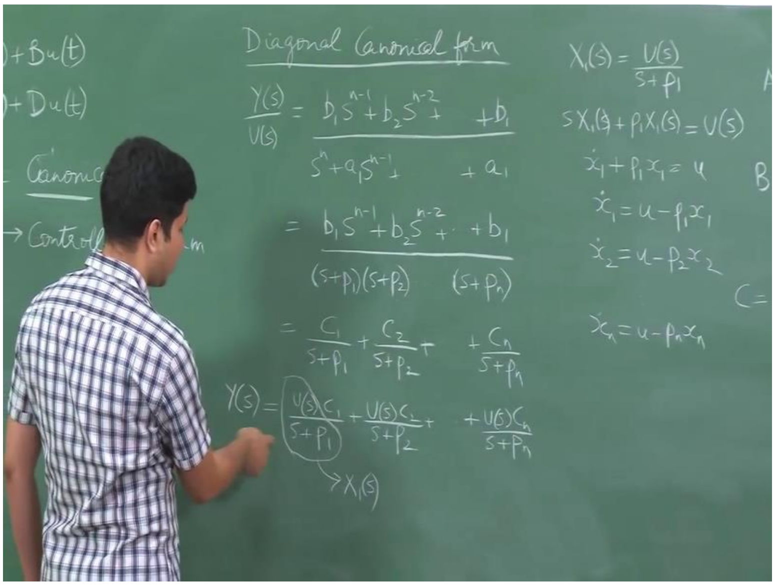

So, now we look at the diagonal canonical form. So, again we will start with the strictly perfect transfer function. So, what we will do is, we will take this denominator and factorize them, factorize the $\boldsymbol{n}$ roots by writing it in this form. So, $\boldsymbol{P_1}, \boldsymbol{P_2}, \boldsymbol{P_3}, \dots, \boldsymbol{P_n}$ are the roots of the denominator polynomial. And so we can write it where we can write the denominator in this form $(\boldsymbol{s}+\boldsymbol{P_1})(\boldsymbol{s}+\boldsymbol{P_2}) \dots\dots (\boldsymbol{s}+\boldsymbol{P_n})$.

Now, what we'll do is, we'll apply partial fractions. And I can write it in this form $\frac{\boldsymbol{C_1}}{\boldsymbol{s}+\boldsymbol{P_1}} + \frac{\boldsymbol{C_2}}{\boldsymbol{s}+\boldsymbol{P_2}} + \cdots \dots + \frac{\boldsymbol{C_n}}{\boldsymbol{s}+\boldsymbol{P_n}}$. Now, what we will do is we will send $\boldsymbol{U(s)}$ to the other side, and say $\frac{\boldsymbol{U(s)C_1}}{\boldsymbol{s}+\boldsymbol{P_1}} + \frac{\boldsymbol{U(s)C_2}}{\boldsymbol{s}+\boldsymbol{P_2}} + \cdots \dots + \frac{\boldsymbol{U(s)C_n}}{\boldsymbol{s}+\boldsymbol{P_n}}$ ok. So, now I'll define each of these parts, leaving the $\boldsymbol{C}$'s, I'll define that as $\boldsymbol{X_1(s)}$.

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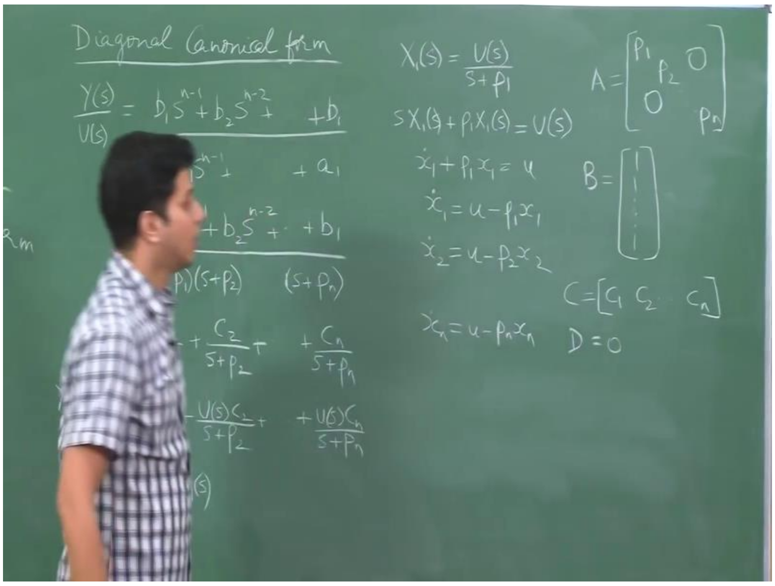

So, $\boldsymbol{X_1(s)} = \frac{\boldsymbol{U(s)}}{\boldsymbol{s+P_1}}$. So, I will cross multiply and now I take the inverse Laplace. So, when we take the inverse Laplace of $\boldsymbol{sX_1(s)}$, it will be just $\boldsymbol{x_1}$. So, earlier also and here also, when we do the inverse Laplace, we assume that all the initial conditions are $\mathbf{0}$. So, you should remember that when we apply this inverse Laplace we are assuming that all the initial conditions are $\mathbf{0}$. So, $\boldsymbol{sX_1(s)}$ when take in the inverse it becomes just $\boldsymbol{\dot{x_1} + P_1 x_1 = u}$. So, these are all again time based variables. So, I can just write $\boldsymbol{\dot{x_1} = u - P_1 x_1}$.

So, similarly I will take each of these as $\boldsymbol{X_2(s)}$, $\boldsymbol{X_3(s)}$,…. $\boldsymbol{X_n(s)}$. And just repeat the same process, and I will get $\boldsymbol{\dot{x_2} = u - P_2 x_2}$....... $\boldsymbol{\dot{x_n} = u - P_n x_n}$. So, now, I have all the state variables and their derivatives Now, I can write the canonical form I will just write the matrices directly. So, $\boldsymbol{A}$ will be $\boldsymbol{P_1, P_2, \dots, P_n}$ along the diagonal and all the terms will be $\mathbf{0}$'s. So, this is the diagonal matrix, so that is the reason why we have the name diagonal canonical form.

And then $\boldsymbol{B}$ will be all $\mathbf{1}$'s, because in every $\boldsymbol{\dot{x_1}}$, $\boldsymbol{\dot{x_2}}$, ….., $\boldsymbol{\dot{x_n}}$ we have a $\boldsymbol{u}$. So, it will be all just $\mathbf{1}$'s times ok, $\boldsymbol{u}$ won't come here, it's just $\boldsymbol{B}$. And $\boldsymbol{C}$ will be, so you can see why this $\boldsymbol{X_1(s)C_1 + X_2(s)C_2 \dots . + X_n(s)C_n}$. So, C n, $\boldsymbol{C}$ will be just $\boldsymbol{C = [C_1, C_2, \dots C_n]}$, and $\boldsymbol{D}$ will be anyway $\mathbf{0}$. So, these are the matrices pertaining to the diagonal canonical form. So, now we'll try to look at an example. So, we will just solve one example and try to derive all the three forms.

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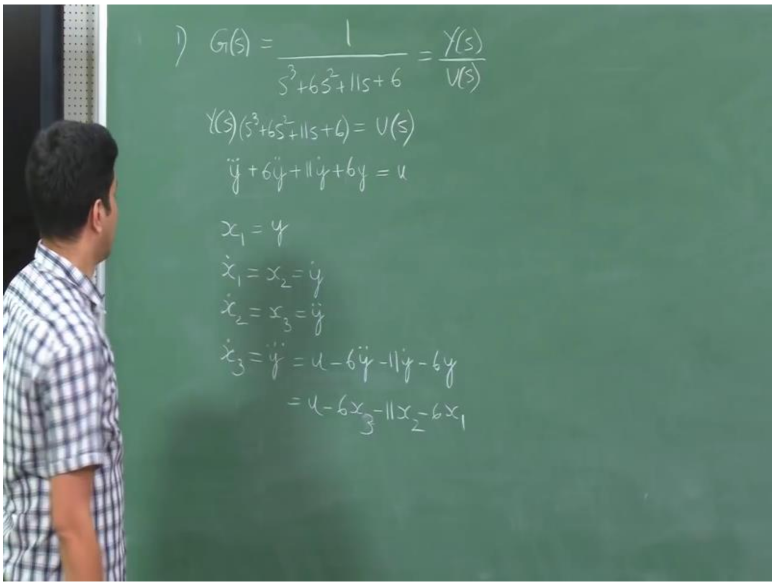

So, the transfer function is $G(s) = \frac{1}{s^3+6s^2+11s+6}$ ok. So, this is the transfer function that we take. And so when we are deriving the canonical forms, there are two ways of going at it. One way is actually deriving all the state variables, and other way is remembering what we got the matrices A, B, C, D, and just trying to substitute the coefficients of these polynomials into them. So, you can do it either way.

But if you try to remember the formula, it might become bit complicated, because you need to know which coefficient pertains to which of those. Because even in the textbooks some people use it in a different way, some people use it from b_0 to b_n, and some people do it from b_n to b_0; and also in the denominator people also use it in the reverse.

So, when you remember the formulas, it might become a bit difficult. So, what I will do is I will try to derive the state variables using the way that we did earlier. So, I will just take $Y(s)$. So, this I will take it as $\frac{Y(s)}{U(s)}$. And so I can write $Y(s)(s^3 + 6s^2 + 11s + 6) = U(s)$. So, I just did cross multiplication.

Now, I will apply Laplace inverse assuming 0 initial condition. So, it will be $\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = u$. So, now, I got the differential equation. Now, I can define my state variables. I will define $x_1 = y$; $\dot{x_1} = x_2 = \dot{y}$; $\dot{x_2} = x_3 = \ddot{y}$; and $\dot{x_3} = \dddot{y}$. So, we just have three state variables x_1, x_2 and x_3.

Now, we can substitute them here? So, actually we already know $\dot{\boldsymbol{x}}_1$ and $\dot{\boldsymbol{x}}_2$. And $\dot{\boldsymbol{x}}_3 = \ddot{\boldsymbol{y}}$ which I will find out from this equation; get it as $\boldsymbol{u} - 6\ddot{\boldsymbol{y}} - 11\dot{\boldsymbol{y}} - 6\boldsymbol{y}$. And again $\ddot{\boldsymbol{y}}$, $\dot{\boldsymbol{y}}$, and $\boldsymbol{y}$, I can substitute it from these. So, I will get $\boldsymbol{u} - 6\boldsymbol{x}_3 - 11\boldsymbol{x}_2 - 6\boldsymbol{x}_1$. So, now I have the values of $\dot{\boldsymbol{x}}_1$, $\dot{\boldsymbol{x}}_2$, and $\dot{\boldsymbol{x}}_3$.

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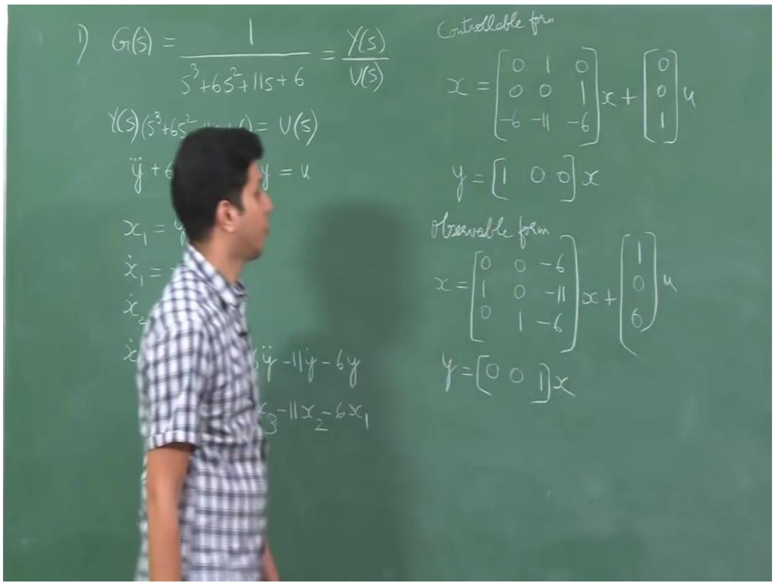

Now, you can simply write the model as $\boldsymbol{x} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{-6} & \mathbf{-11} & \mathbf{-6} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix} \boldsymbol{u}$. And $\boldsymbol{y}$ is just $\boldsymbol{x}_1$ in this case, so it will be just $\boldsymbol{y} = [\mathbf{1} \quad \mathbf{0} \quad \mathbf{0}]\boldsymbol{x}$. So, this is the controllable canonical form. As you can see it is very easy if you can derive it out directly instead of remembering the formulas. So, now we have the controllable canonical form. So, I will say I will call this ok, this is the controllable form.

So, what about the observable form? Observable form will be simply $\boldsymbol{x}$ is equals to so as $\boldsymbol{A} = \boldsymbol{A}^T$, I just had to take the transpose of this, so it will be $\boldsymbol{x} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-6} \\ \mathbf{1} & \mathbf{0} & \mathbf{-11} \\ \mathbf{0} & \mathbf{1} & \mathbf{-6} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \boldsymbol{u}$ plus $\boldsymbol{B} = \boldsymbol{C}^T = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \boldsymbol{u}$. And $\boldsymbol{C} = \boldsymbol{B}^T$, so it will be $y = [\mathbf{0} \quad \mathbf{0} \quad \mathbf{1}]\boldsymbol{x}$. So, this is the observable form.

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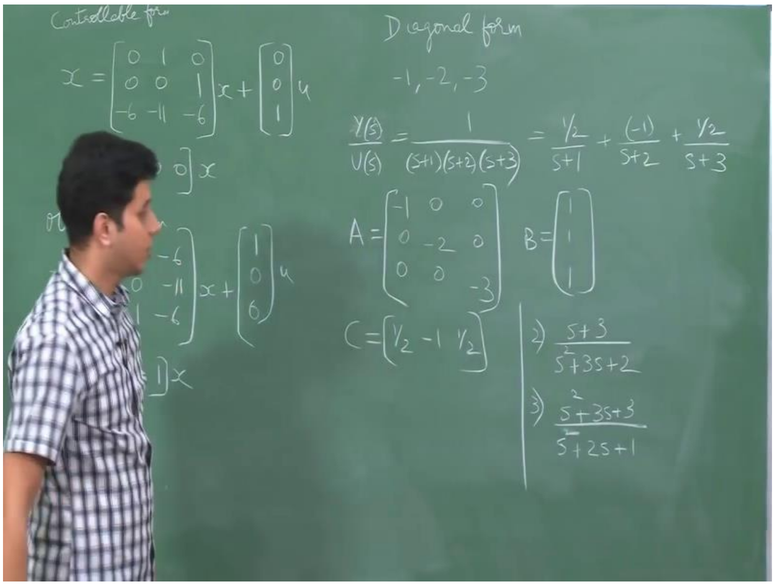

So, finally, the diagonal form. So, to get the diagonal form, we need to find the roots of the denominator polynomial. So, I am just I already found them to be $\mathbf{-1, -2, -3}$, you can verify. So, I can write my $\frac{\boldsymbol{Y(s)}}{\boldsymbol{U(s)}} = \frac{\mathbf{1}}{(\boldsymbol{s+1})(\boldsymbol{s+2})(\boldsymbol{s+3})}$. And this I will when I apply partial fractions, I will get $\frac{\mathbf{1/2}}{\boldsymbol{s+1}} + \frac{(\mathbf{-1})}{\boldsymbol{s+2}} + \frac{\mathbf{1/2}}{\boldsymbol{s+3}}$. So, this also you can verify yourselves.

And so, now, we have this partial fractions my $\boldsymbol{A}$ is nothing but these three coefficients sorry these three roots coming in the across the denominator, so it will be $\boldsymbol{A} = \begin{bmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-3} \end{bmatrix}$. And then $\boldsymbol{B}$ is all just 1's (i.e., $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$). And $\boldsymbol{C}$ is the numerators of the partial fraction, so it is $\boldsymbol{C} = [\mathbf{1/2} \quad \mathbf{-1} \quad \mathbf{1/2}]$. So, that is this is the diagonal canonical form.

So, we just took one example and we try to derive all the three canonical forms. So, there might be some special cases where you might not be able to derive the diagonal form, because there can be repeated roots and not unique roots as in this case. So, that is when you go to something called a Jordan canonical form which we are not doing as of now.

So, these are the canonical forms which we discuss. So, I will try to put up some more problems which you can try out as an exercise (ii) $\frac{\boldsymbol{s+3}}{\boldsymbol{s^2+3s+2}}$, and (iii) $\frac{\boldsymbol{s^2+3s+3}}{\boldsymbol{s^2+2s+1}}$. So, you can just try out these two examples and you can see that this last example actually has a nonzero $\boldsymbol{D}$. So, you need to perform polynomial division first, and then apply whatever we did until now. So, you can just try out those, and maybe we will try to give them as assignment problems.

Thank you.