Introduction to Nonlinear Dynamics
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Module -04
Lecture-09
1-Dimensional Flows, Bifurcations, Lecture 1

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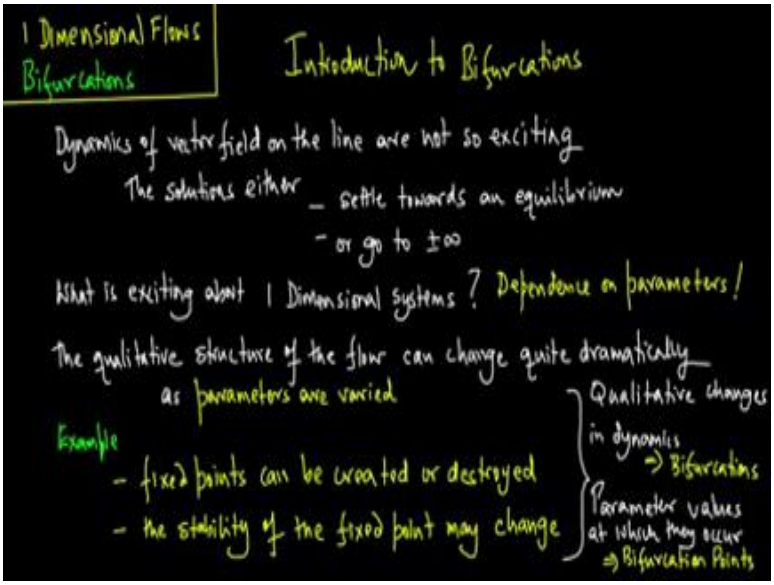

Fine so we are still dealing with one dimensional flows, but now we focus on an area called bifurcations. This lecture is going to be a brief introduction to the area of bifurcation theory. Now dynamics of vector field on the line are actually not so exciting. The solutions either go ahead and settle down towards equilibrium or they actually go off to plus or minus infinity. So in that sense they are not extremely exciting dynamics. So, what really is exciting about one dimensional systems? The answer turns out to be its dependence on parameters.

The qualitative structure of the flow can actually change quite dramatically as system parameters are varied, for example fixed points can be created or destroyed and the stability of the fixed point itself may change has system parameters are varied. So, the qualitative change in dynamics is what we referred to as bifurcations and the parameter values at which such bifurcations occur are called bifurcation points.

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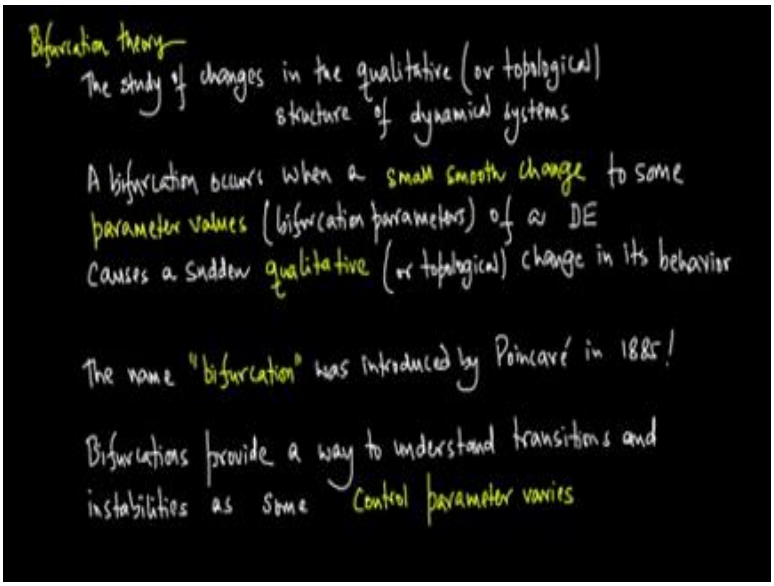

So, we are really delving into this area of applied mathematics called bifurcation theory. It is really the study of changes in the qualitative or topological structure of dynamical systems. So, a bifurcation occurs when a small smooth change to some parameter values referred to as bifurcation parameters of a differential equation causes a sudden qualitative or topological change in its behaviour.

Now interestingly the name bifurcation apparently was first introduced by Henry Poincare in year 1885. So, bifurcations essentially provide us with a way to understand transitions and instabilities as some system control parameter actually varies.

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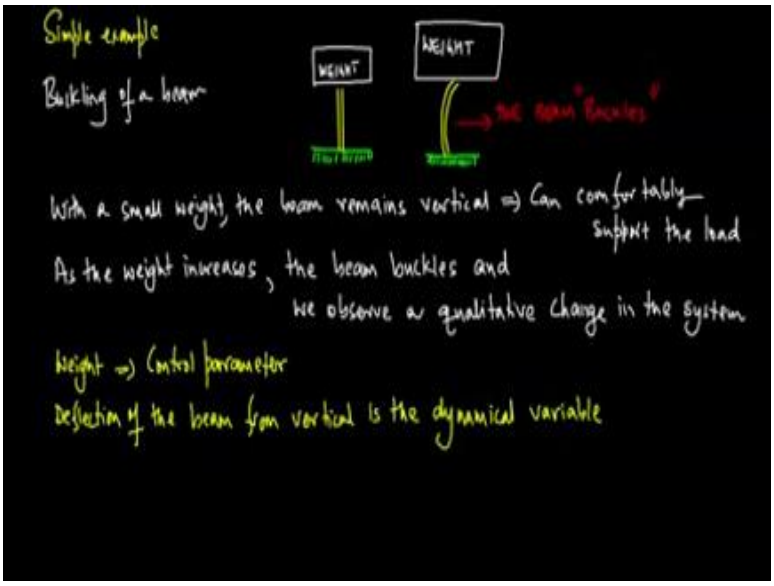

Consider a simple example bulking of a beam. So, you have a beam, the beam has a certain weight which is placed on top of it and if you go ahead and increase the weight beyond certain critical value the beam actually buckles. So, what we find is that beam as buckled under the weight, when the weight crosses certain value. So, with a small weight the beam remains vertical and can comfortably support the load.

As the weight increases the beam then buckles under the weight and we observe a qualitative change in the system. So, in this example the weight act as the control parameter and the deflection of the beam from the vertical is the dynamical variable. So, this is our first example may be observed bifurcation phenomena happening (()) (04:20), where there is a qualitative change in the system when a certain parameter varies.

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The dynamics of the vector field on real line are not very exciting. One of two things can actually happen, number one is that the solution can actually blow of to infinity or solutions which actually converge to some fixed point, so only these two things that really happening. You say what so exciting about dynamics or of vector filed on real line? And the answer turns out to be its dependence on parameters.

When you construct a model of the real world, almost every model that you find will have some parameter or the other in fact usually they too many parameters. So the question we are interested in, is that how does changes in the parameter of the underline model actually induce a qualitative change in the dynamics and that can happen in one or two ways. Number one, is that either fixed points can be created or destroyed as parameter varies or the stability of the fixed points themselves can change.

These are two important qualitative changes that can actually happen in the system as a parameter actually varies. And we give a very simple example, you can just get a motivation and get an intuition in order and that is that imagine you have beam on top the beam you have a certain weight and the beam can actually hold the weight without actually buckling. But now what we do is we slowly increase the weight, when the weight increases a certain threshold.

You will actually find that the beam buckles. So, this transitions from the beam being straight when the weight is small to the beam buckling when the weight is big. Actually, is a qualitative change in the system dynamics. So, the weight actually acts as a control parameter and as that parameter varies, there is a qualitative change in the system dynamics.

So, this is just you know brief motivation to bifurcation theory and why we would be interested in bifurcation in the system and in particular the role that parameters will play inducing bifurcation phenomena of certain type.