Introduction to Nonlinear Dynamics
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Module -06
Lecture-24
2-Dimensional Flows, Limit Cycles, Lecture 1

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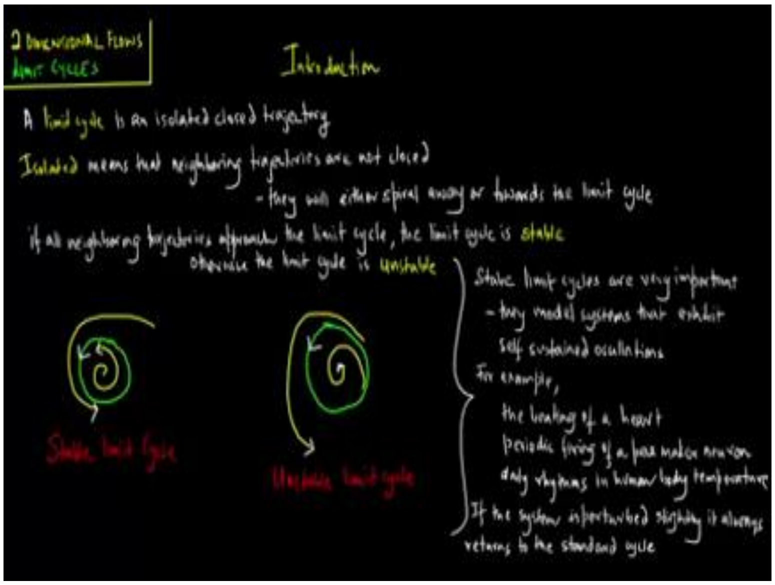

We are still dealing with two dimensional flows but now we focus on the important topic of limit cycles. So, this lecture is going to be an introduction to limit cycles. A limit cycle is an isolated closed trajectory. Isolated means that neighbouring trajectories are not closed, ie they will either spiral away or move towards the limit cycles. If all neighbouring trajectories approach the limit cycles and then the limit cycles is deemed stable, otherwise the limit cycles is deemed unstable.

So now let us try and visualise stable and an unstable limit cycles, here nearby trajectories are approaching the limit cycles and so we have a stable limit cycle. In this case neighbouring trajectories are moving away from the limit cycles. And so we have an unstable limit cycle. Stable limit cycles are very important scientifically. They model systems that exhibit self sustained oscillation.

For examples the beating of a heart, the periodic firing of a pace maker neuron or the daily rhythms in human body temperature in a sense, if the system is perturbed slightly it always returns to the standard cycle.

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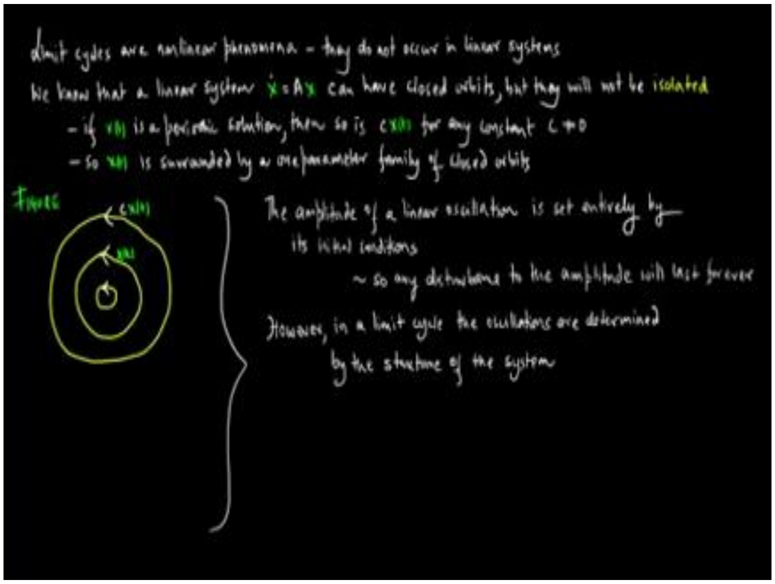

Limit cycles are inherently nonlinear phenomena they do not occur in linear systems. Now we know that a linear system x dot = ax can actually have closed orbits. But they will not be isolated. So basically if x of t is a periodic solution, then so is c times x of t for any constant c not equal to zero. So x of t is surrounded by a one parameter family of closed orbits. So now let us visualise this through a figure.

We have a closed orbit, we have another closed orbit x of t and we have another closed orbit c times x of t. So the amplitude of a linear oscillation is set entirely by its initial conditions. So any disturbance to the amplitude will last forever. However in a limit cycle the oscillations are determined by the structure of the underlined system.

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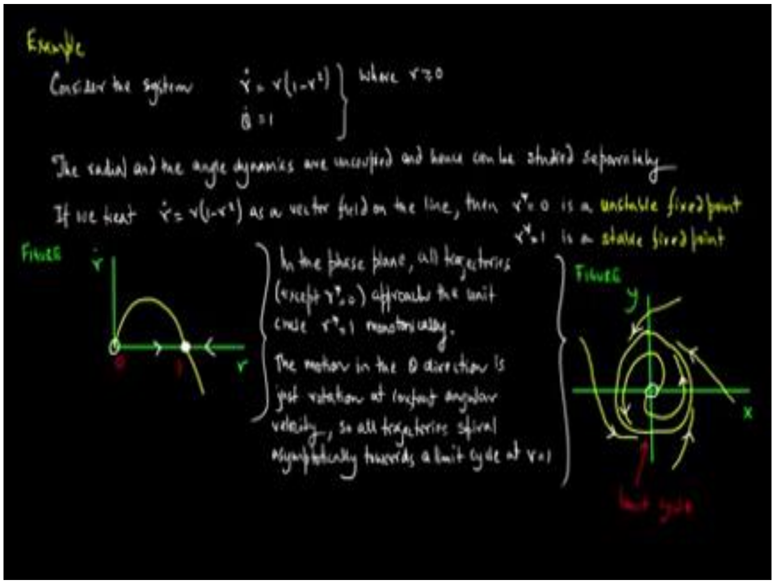

Now let us look at an example, consider the system r dot = r 1 - r squared and tita dot =1 where r is greater than or equal to zero. The radial and the angle dynamics are in fact uncoupled and hence can be studied separately. So if we treat r dot = r 1- r squared as a vector field on the line when r star = 0 is a unstable fixed point and r star =1 is a stable fixed point. So now visualise this through a figure.

So we plot r dot versus r, we highlight the unstable fixed point and the stable fixed point, so that is 0 and 1. In the plane all trajectories except r star = 0 approach the unit circle r star =1 monotonically. The motion in the tita direction is just a rotation at constant angular velocity. So all trajectories spiral asymptotically towards a limit cycle at r =1. So let us visualise this again through a figure.

We have the limit cycle, we have the unstable fixed point at zero and note that the limit cycle that nearby trajectories are actually approaching the limit cycles.

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In this lecture, we had a very brief introduction to limit cycles, a limit cycles is a closed trajectory that is isolated. Isolated essentially means that any nearby trajectories will not be closed they will either spiral in towards the limit cycle or spiral away from the limit cycles. Just has a fixed points, we can have stable fixed points and unstable fixed points. Similarly with limits cycles, we can have stable limit cycles and we can have unstable limit cycles. Now recall that in the linear system. So we have x dot = ax we could have closed orbits.

The key different is that those closed orbits are not isolated. So if x of t is a periodic solution coming from a linear system then c times x of t where c is a constant which is not equal to zero will also be a periodic solution. So the key difference between periodic orbits coming out of a linear systems and the limit cycle is that in the limit cycle nearby trajectories were converge to the limit cycle or will spiral away.

As suppose to closed orbits coming from linear system even you perturbed it a little bit or have another solution in a silent neighbourhood of that particular periodic orbit that itself could to be periodic solution that is what.