Introduction to Nonlinear Dynamics
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Module -06
Lecture-22
2-Dimensional Flows, Phase Plane, Lecture 2

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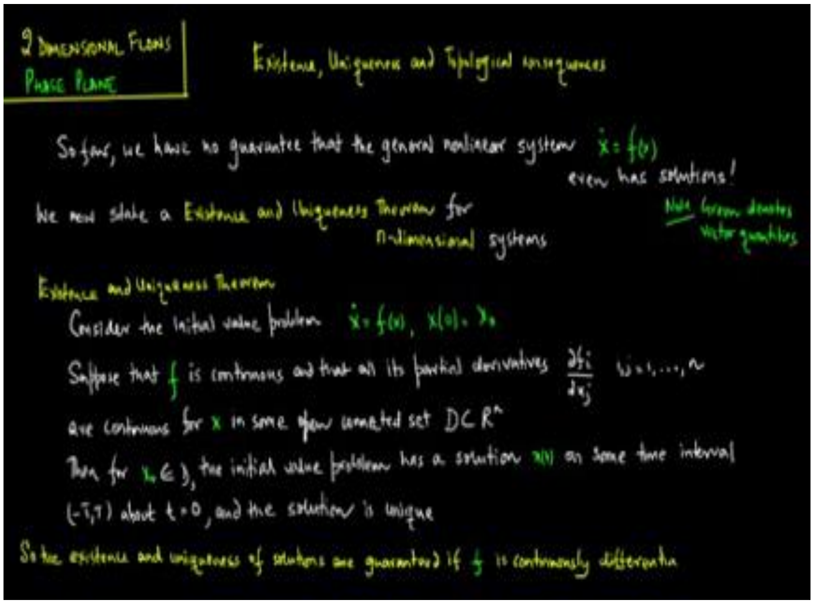

In this lecture we discuss the existence, uniqueness and some topological consequences. So far we have no guarantee that the general nonlinear system x dot = f of x, even has solutions. Note that green denotes vector quantities. We now state the existences and uniqueness theorem for n dimensional systems.

So the existences and uniqueness theorem states that, consider the initial value problem x dot = f of x where x0 = x not and suppose that f is continuous and that all its partial derivatives dfi dxj for i j, to 1 to n are continuous for x in some open connected set DLRN. Then for x dot an element of d the initial value problem has a solution x of t on, some time interval - to capital T about t = 0 and the solution is unique. So, the existence and uniqueness of solutions are guaranteed, if f is continuously differentiable.

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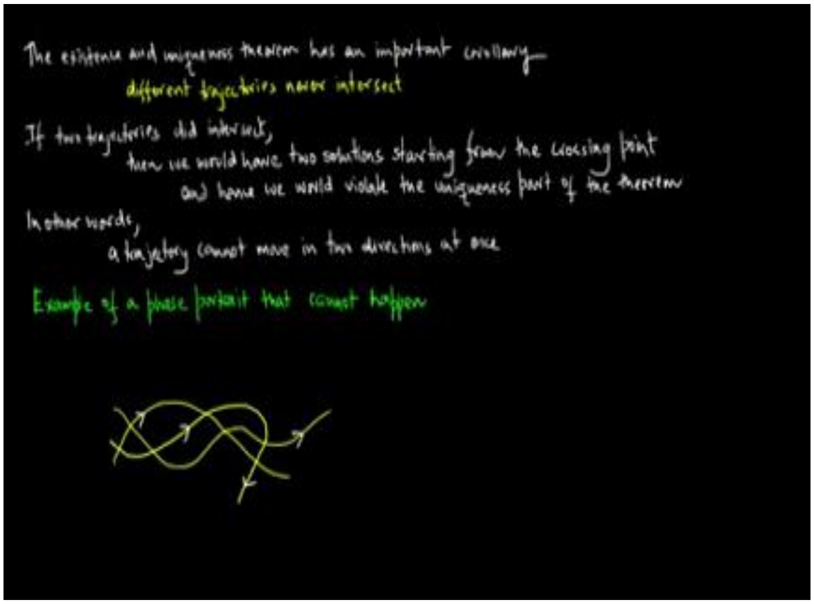

Now note that the existence and the uniqueness theorem has an important corollary that is that different trajectories never intersect. So if two trajectories in did intersects, then we would have two solutions starting from the crossing point. And hence we would actually violate the uniqueness part of the theorem in other words a trajectory cannot move in two directions at once.

So we give an example of a phase portrait that actually cannot happen, so this is an example of a phase portrait that actually will not happen because it will violate existence and uniqueness theorem in particular it will violate the uniqueness part of the theorem.

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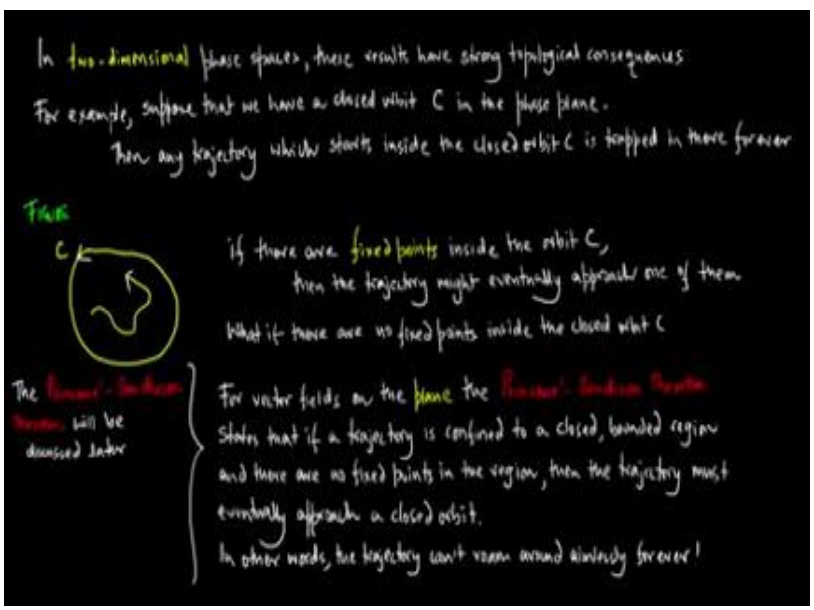

In two dimensional phase spaces these results actually have very strong topological consequences. For example, suppose that we have a closed orbit C in the phase plane, then any trajectory which starts inside the close orbit C is trapped in there forever. So let us plot a figure so that is the close orbit C and that is the trajectory which is inside closed orbit. Now if there are fixed points inside the orbit C then the trajectory might eventually approach one of them.

Now what happens if there are no fixed points inside the closed orbit C, so vector fields on the plane, the Poincare Bendison theorem states that if a trajectory is confined to a closed bounded region and there are no fixed points in the region then the trajectory must eventually approach a closed orbit. In other words, the trajectory just cannot roam around aimlessly forever; the Poincare Bendison theorem will be discussed later.

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In this lecture we discussed existence, uniqueness and some topological consequences. When we start working with the differential equation or when the system of differential equation it is always use full to know under what conditions the solutions would exists and be unique. So, we stated a theorem for n dimensional systems x dot = to f of x in vector notation. And essentially that is how was that as long as f was continuously differentiable, the solution were exist and would be unique.

There was an interesting corollary to the existence and uniqueness theorem which essentially said that the trajectories would not intercept. In other words, the trajectory cannot move in two different directions at the same time. But we also highlighted rather interesting topological consequence that would be there in two dimensional flows. As considering the following scenario, consider that you have a closed orbit and a trajectory starts inside the closed orbit.

Now in the case that there is fixed point inside the closed orbit then it is possible that this trajectory would approach the fixed point eventually. But what happens if there was no fixed point inside the closed orbit. If that is the case that there is a theorem known as Poincare Bendison theorem which says that this trajectory which start inside the closed orbit would eventually approach a closed orbit itself. And so, this theorem is known as the Poincare Bendison theorem, which we will deal with in some detail later.