Introduction to Nonlinear Dynamics
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Module -06
Lecture-20
2-Dimensional Flows, Linear Systems, Lecture 4

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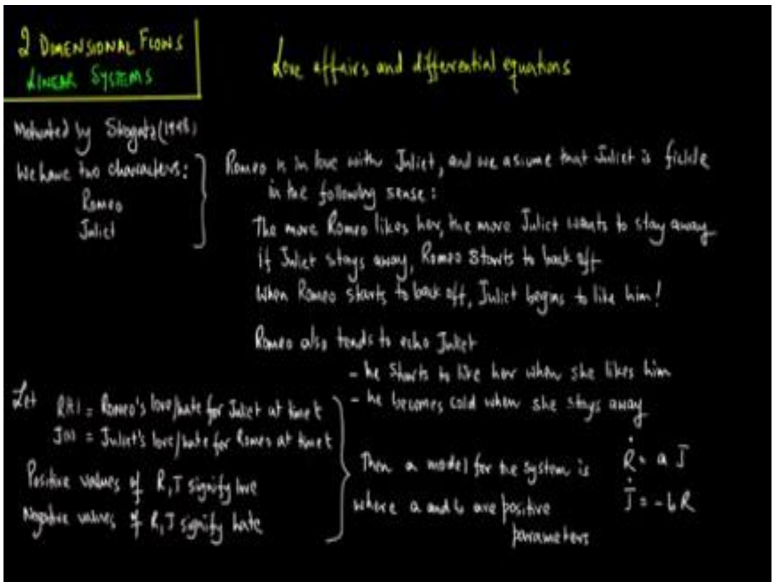

This lecture model's love affairs with differential equations, this is motivated by an excellent article by Stogate in the year 98. We have two characters in the plot one is Romeo and other is Juliet. Romeo, we find is in love with Juliet and we assume that Juliet is fickle in the following sense. The more Romeo likes her, more the Juliet wants to stay away. If Juliet stays away Romeo starts to pack off, when Romeo starts to pack off, Juliet actually begins to like him.

Romeo also tends to echo Juliet, so he starts to like her when she likes him. And he becomes cold when she stays away. So, let us R of t be Romeo's love slash hate, for Juliet at time t, J of t be the Juliet love slash hate, for Romeo at time t. Positive values of R and J signify love and negative of R and J signify hate. Then a model for the system is R dot = aJ and J dot = -bR where a and b are positive parameters.

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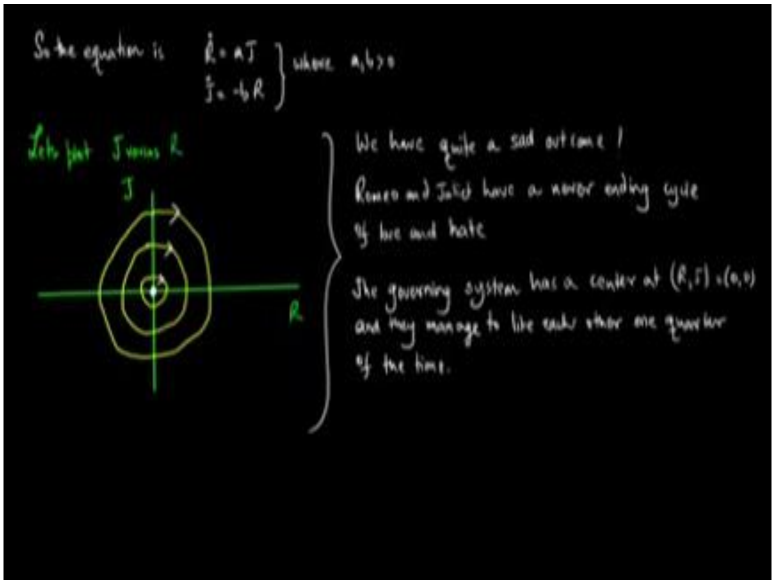

So, the equation is R dot = aJ and J dot = - bR where a and b are greater than zero. So, let us plot J versus R, so when we plot J versus R, we find that we get a bunch of closed orbits. So, we have quite a sad outcome because of the close orbits; Romeo and Juliet have a never-ending cycle of love and hate. So, the governing system has a center at RJ = 0 0 and they manage to like each other about one quarter of the time.

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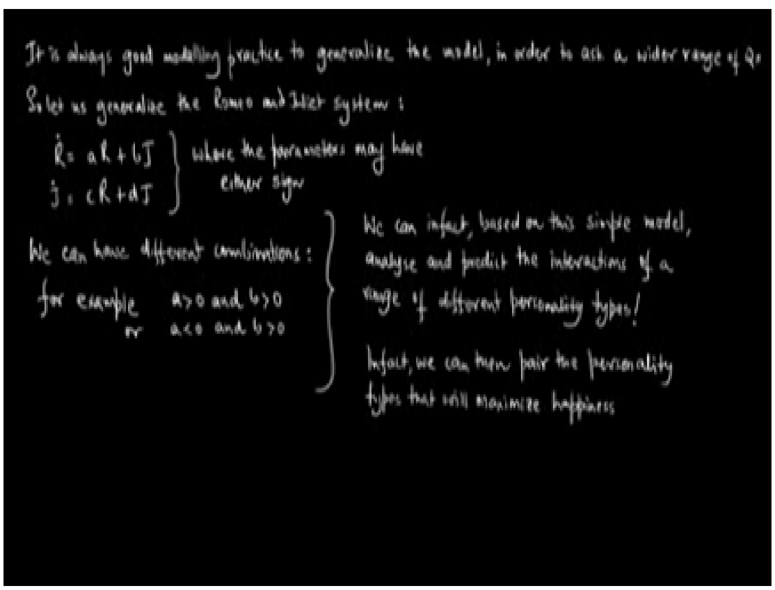

It is always good modelling practice to generalise the model in order to ask a wider range of questions. So let us generalise the Romeo and Juliet system, R dot = aR + bJ, J dot = cR + bJ, where the parameters may have either sign. We can have different combinations, for example a greater than zero and b greater than zero or a less than zero and b greater than zero. We can in fact based on the simple model analyse and predict the interactions of a range of different personality types. In fact, we can then pair the personality types that will in theory maximise happiness.

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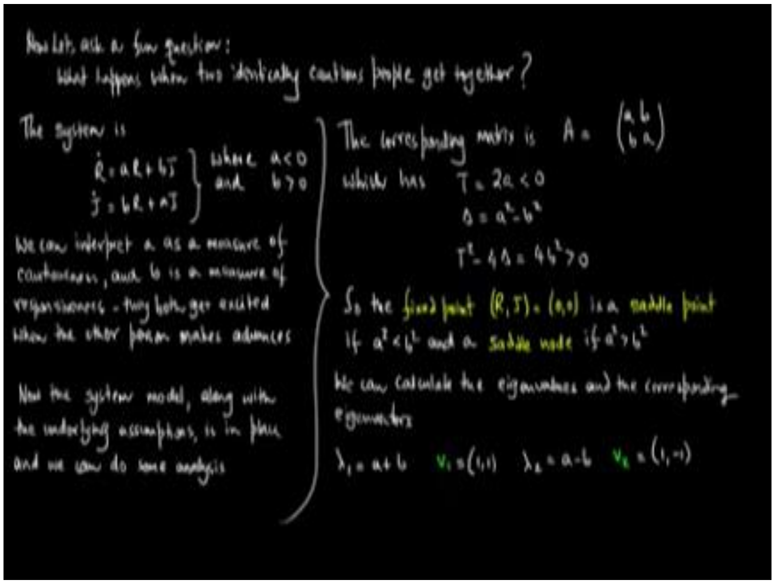

Now let us ask a fun question, what happens when two identically cautious people get together. The system is R dot = aR + bJ and J dot = bR plus aJ, where a is less than zero and b is greater than zero. We can interpret a as a measure of cautiousness and b is a measure of responsiveness. So they both get excited when the other person makes advances. Now the system model along with the underlying assumptions is in place and we can do some analysis.

The corresponding matrix is A = a b b a, which has the trace T = 2a which is less than zero, the determinant is a squared - b squared and t squared -4delta =4b squared which is greater than zero. So the fixed point RJ = 0 0, is a saddle point. If a squared is less than b squared and a saddle node if a squared greater than b squared, we can calculate the Eigen values and the corresponding value Eigen vectors. So lambda1 is a +b, v1 = 1 and 1, lambda2 = a -b and v2=1 -1.

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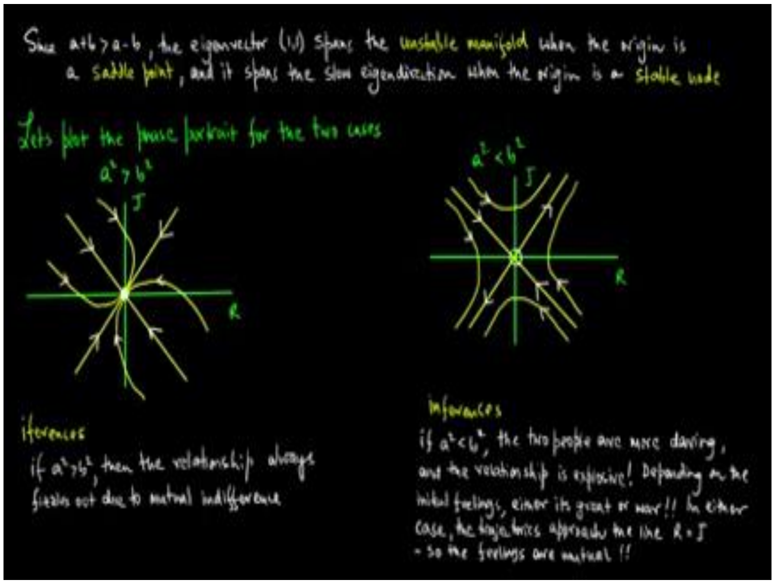

Since a + b is greater than a - b, the Eigen vector 1 1 spans the unstable manifold, when the origin is a saddle point and its spans the slow Eigen direction, when the origin is in stable node. So let us plot the phase portrait for the two cases. First consider a squared greater than b squared, where we plot J versus R, that is the stable node, so now let us make some inferences, if a squared is greater than b squared then the relationship actually always fizzles out due to mutual indifference.

Now consider a squared less than b squared and plot J versus R, this turns out to be much more interesting case, very good, now we got the full diagram. So now let us make inferences on this case as well. So a squared is less than b squared, the two people are infact much more daring and the relationship turns out to be explosive. Depending on the initial feeling either it is absolutely great or complete war. In either case the trajectories approach the line R = J and so the feelings are absolute mutual.

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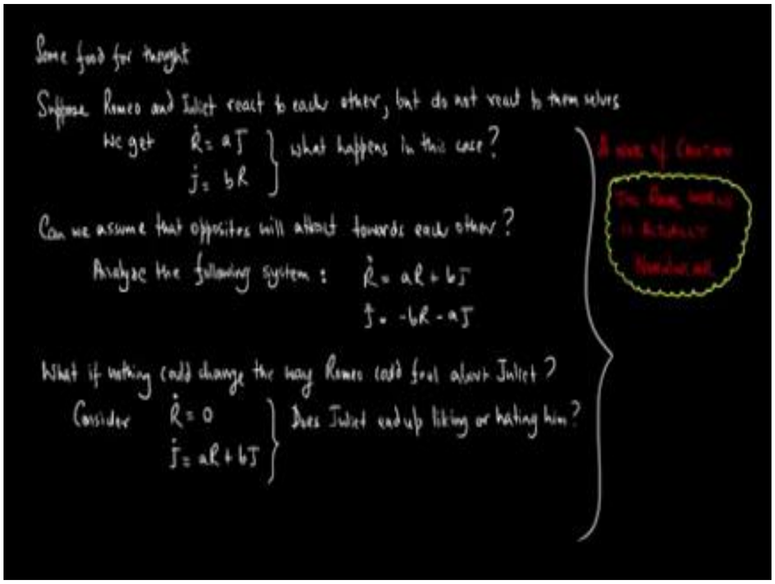

Let us provide some food for thought. Suppose that Romeo and Juliet react to each other, but actually do not react to themselves. So, we will get R dot = aJ, J dot = bR. So, what really happens in this particular case? Can we assume that opposites will attract towards each other? So, we can analyse the following system now, R dot = aR +bJ, J dot = -bR - aJ, what if nothing could change the way Romeo could feel about Juliet.

We have to consider R dot = 0 and J dot = aR + bJ. So, does Juliet and end up liking or hating him. We have to take in lot of caution before you taking this moral seriously, that is that the real world is actually highly nonlinear. So, while these are good starting point, we have to note that these are only linear systems. But they themselves lead to interesting conclusions.

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So this was a short and fun lecture on exploring the relationship between love affairs and differential equations. The motivation comes from William Shakespeare's Romeo and Juliet. But an abstraction the question is roughly the following. You have two characters in the plot and let us assume that the two characters at this particular case is Romeo and Juliet, have let say slightly different personality types or they might have very different personality types ie they might react to situations in different ways.

And the question that you really want to ask is well, what happens, as times tends to infinity, are they going to spend large fraction of the time being happy or will they spend large fraction of the time fighting with each other and being unhappy. So, you can ask question of the form, while if you had opposites, then those opposite attracts or will they actually repel each other in the long term. So this was a toy example of course and we try to model this through linear two dimensional systems.

But when even if it was a toy model we noticed that some elements or some crude element of reality could be crept into the model. So, there is some element of, this is not absolutely ridicules to think of the interaction of the two human beings being model through differential equations, having said that I should put an caveat that human beings very rarely behave like linear objects and the real world is almost certainly nonlinear.