Introduction to Nonlinear Dynamics
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Module -01
Lecture - 02
Dynamics and Nonlinear systems: getting started

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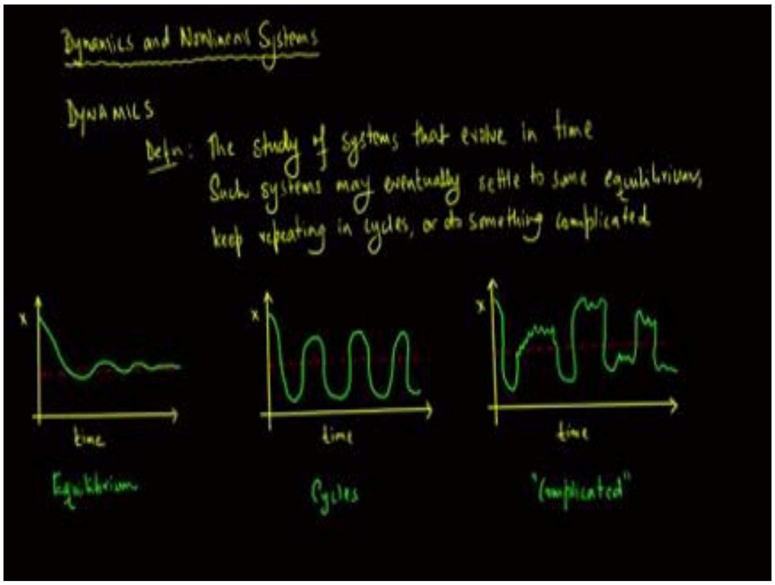

Let us start getting round up with dynamics and nonlinear systems. Now let start with the word dynamics, here is a simple definition, it is the study of systems that evolve in time. Now such systems may eventually settle down to some equilibrium or they may keep repeating in cycles or actually do something much more complicated. Let us now visualise some dynamics, x is the depended variable, time is the independent variable.

There is an equilibrium value and their trajectories settled to that equilibrium. You can have another scenario, where they do not settled equilibrium; they actually just keep repeating themselves in cycles. The third scenario is where you neither have an equilibrium nor a cycle but you keep doing something which is fairly complicated.

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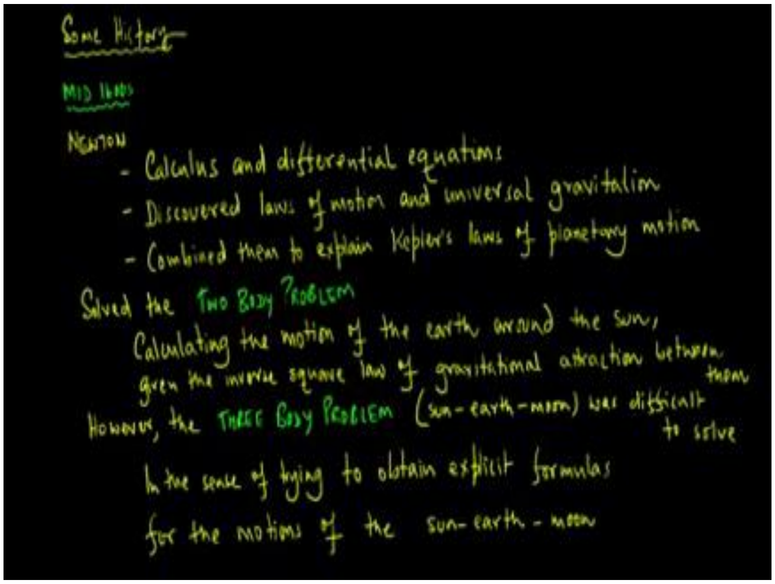

Now let us take a historical perspective on dynamics the story, really started in mid 1600s with Newton. What he did was he invented calculus and he worked on differential equations. He then discovered his laws of motion and universal gravitation and he combined them to explain Kepler's laws of planetary motion.

Now essentially what he did was solved two body problem, which is calculating the motion of the earth around the Sun given the inverse square law of gravitational attraction between them. Now what proved to be rather difficult was the three body problem. That is the problem of the Sun, the Earth and the Moon, this turned out to be very difficult problem to solve, in the sense of trying to obtain explicit formulas for the motions of Sun, the Earth and the Moon.

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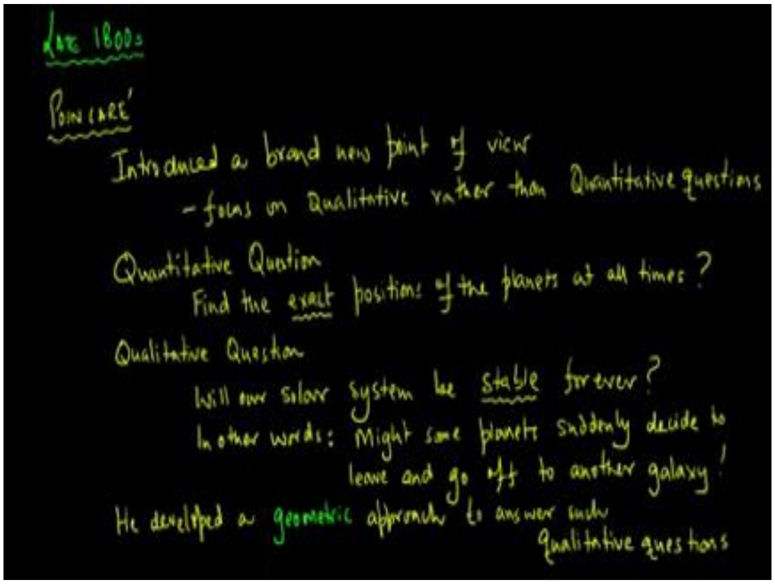

Progress then happened in late 1800s with the work of Poincare. What he really did was introduce a brand new point of view, what he said was the following: Let focus on qualitative rather than quantitative questions. Here is an example of a quantitative question, can we find the exact positions of the planets, at all times.

Here is a qualitative question, will our solar system be stable for ever, in other words might some of the planets suddenly decide to leave the solar system and go off to another galaxy. Now what Poincare did was developed a geometric approach to answer such qualitative questions.

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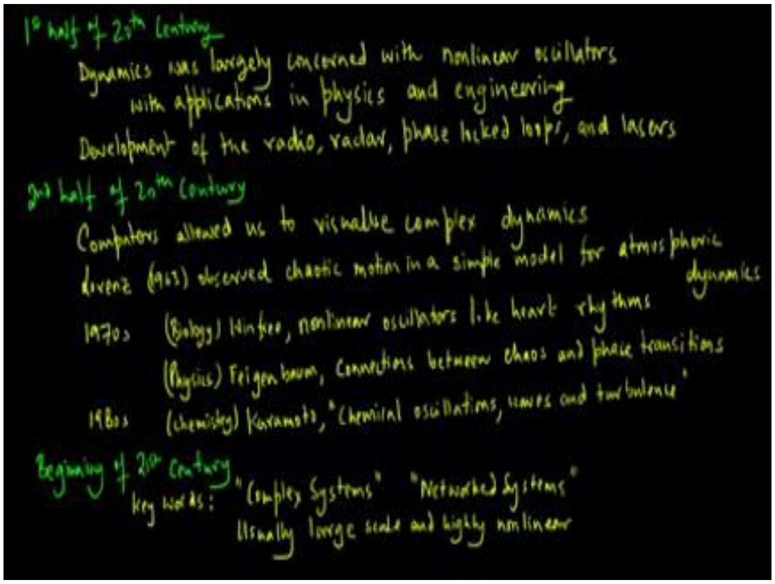

Now let us move on to the first half of 20th century. Dynamics was largely concerned with nonlinear oscillators with applications in physics and in engineering. Now this led to the development of the Radio, Radar, Phase Locked Loop and Lasers. In the second half of the 20th century computers started to allow us to visualise complex dynamics. Kepler Lorenz in his famous 1963 paper observed chaotic motion in a simple model for atmospheric dynamics.

Later in the 1970s, working in mathematical biology, Winfree worked on nonlinear oscillators like heart rhythms. In physics Feigen Baum went on to establish connection between chaos and phase transitions. And in the 1980s there was a famous book by Kuramoto called chemical oscillations, waves and turbulence. Now moving on to the beginning of the 21st century, the key words that one often hear are complex systems and network systems.

Now usually such systems are large scale and highly nonlinear and their applications all over science and engineering.

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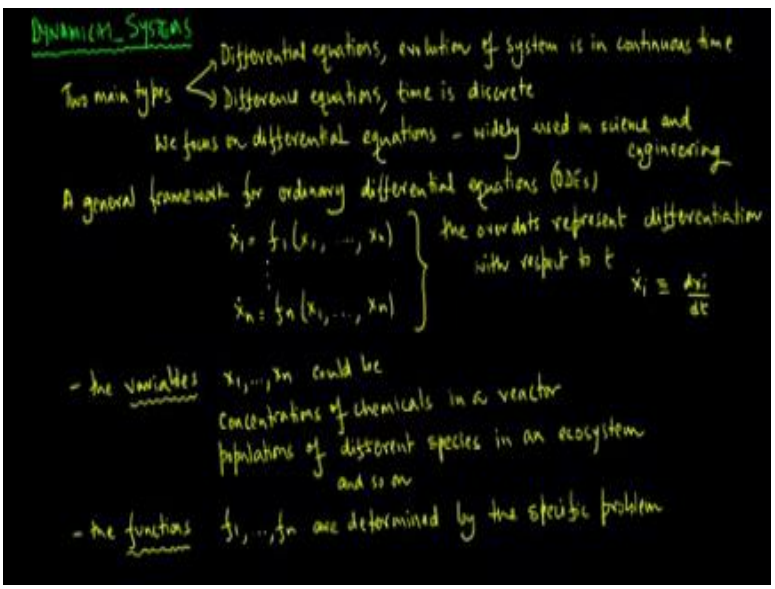

Now we start talking about dynamical systems. There are two main types of dynamical systems, one is differential equations where the evolution of the system happens in continuous time. The other variant is difference equations, where time is discreet. We focus on differential equations, as these are widely used in science and engineering. Very general frame work for ordinary differential equations abbreviated as ODEs is x1 dot is equal to f1 x1 to x n all the way up to x n dot is equal to fn x1 to x n the over dot's represent differentiation with the respect to time t so the xi dot is equal to dxi dt.

The variables x1 to x n could be concentrations of chemicals within a reactor or there could be populations of different species within an ecosystem and so on. The functions f1 to fn are determined by the specific problem that had.

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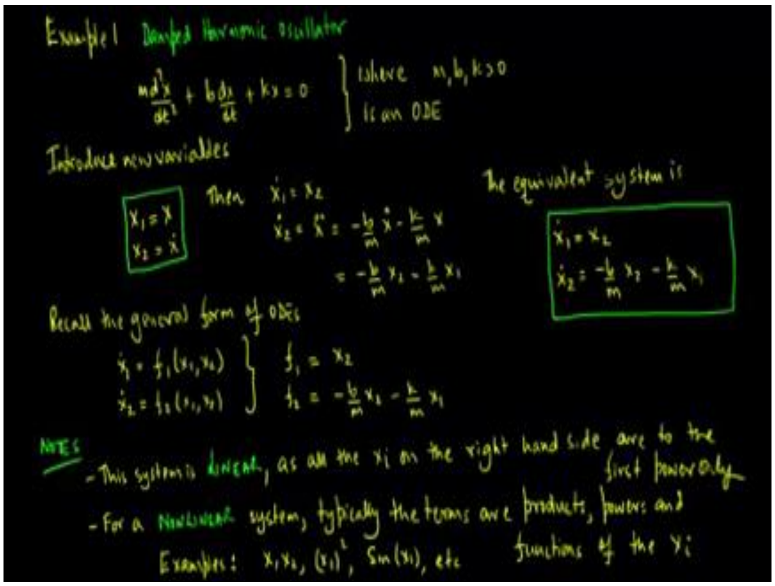

As an example, let us consider Damped Harmonic oscillator here is the differential equations that is m d square x dt squared plus b dx dt plus kx is equal to zero, where m b and k are greater than zero. Now this is an example of an ordinary differential equation. What we do is we introduce new variables x1 is equal to x and x2 is equal to x dot. Then x1 dot is equal to x2 and x2 dot is equal to x double dot which is equal to minus b on m x dot minus k on m x which is minus b on m x2 minus k on m x1.

The equivalent system when turns out be x1 dot is equal to x2 and x2 dot is equal to minus b on m x2 minus on k on m x1. Now recall the general form of the ordinary differential equation that is x1 dot is equal to f1is the function of x1 and x2 and x2 dot is equal to f2 is the function of x1 and x2 . So if f1 is x2 and f2 is minus b on m x2 minus k on m x1 then we essentially just turn the Damped Harmonic oscillator into general form of the ODEs that we had earlier.

And here are some notes the above system is linear, as all the xi's on the right hand side are to the first power only. For a nonlinear system, typically the terms are products, powers and functions of the xi's. For example, x1 x2, x1 squared sine of x1 etc are all examples of nonlinear terms.

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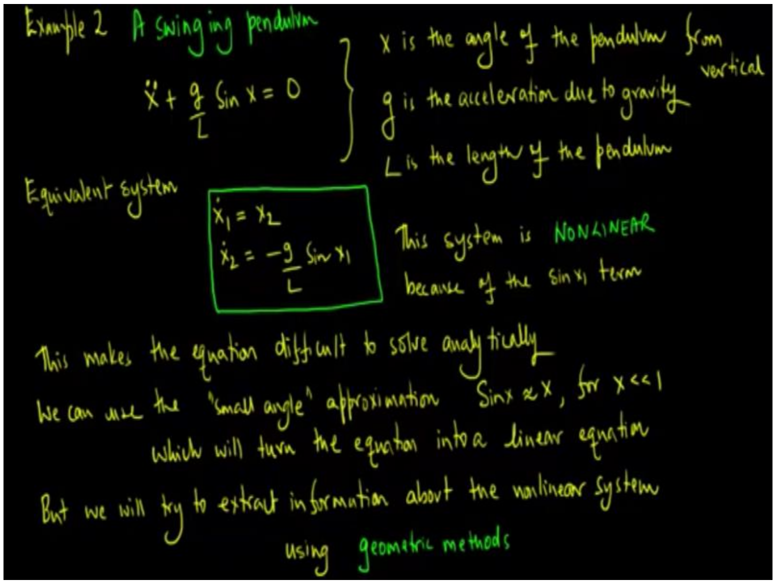

Now let us consider the example of a swinging pendulum the differential equation is x double dot plus g on L sine x is equal to zero. Where x is the angle of the pendulum from vertical, g is the acceleration due to gravity and L is the length of the pendulum. The equivalent system is x1 dot is equal to x2 and x2 dot is equal to minus g on L sine of x1. Now we get to this equivalent system using exactly the same trick as the previous example.

This system is nonlinear because of the sine x1 term. This makes the equation very difficult to solve analytically. Now we can use the small angle approximation that is sine x is approximately equal to x for x much less than one, which will turn the equation into linear one. But we will also try to extract information about the original nonlinear system using geometric methods.

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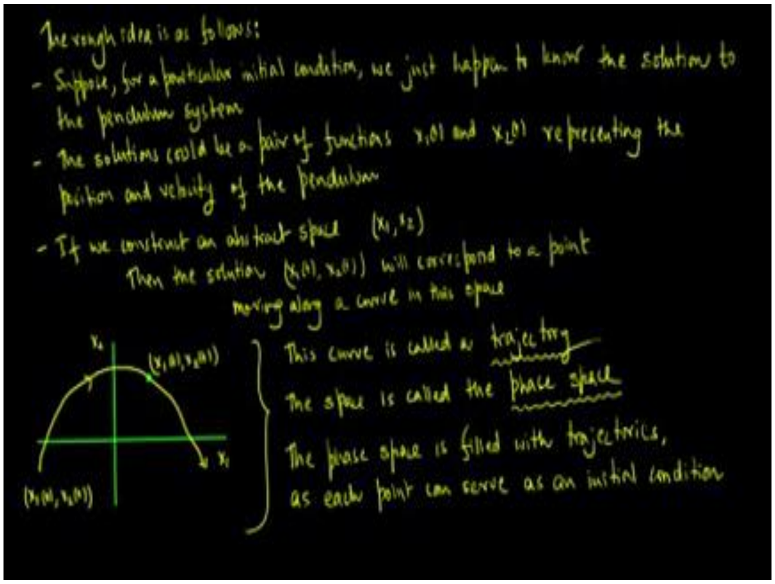

The rough idea is as follows now suppose that for a particular initial condition, we just happen to know the solution to the pendulum system. The solution could be a pair of functions x1 and x2 representing the position and velocity of the pendulum. Now if we construct an abstract space x1, x2   then the solution x1 t and x2   t will correspond to a point moving along the curve in this space.

Now let just go ahead and plot this we have an initial condition x1 zero and x2   zero we go ahead and plot curve. Now this curve is called a trajectory. Space is called the Phase Space. Now the Phase Space is actually filled with trajectories, as each point can actually serve as an initial condition.

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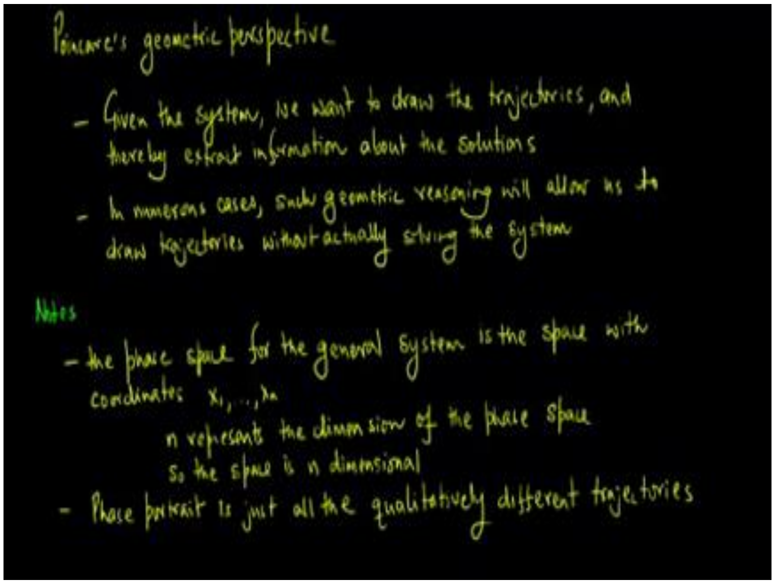

Now we will outline the Poincare's geometric perspective. Now given the system we want to draw the trajectories and there by extract information about the solutions. Now in numerous cases such geometric reasoning actually allows us to draw trajectories without actually solving the system. Here are some notes the phase space for the general system is the space with coordinates x1 to xn.

So n represents the dimension of the phase space, so the space is n dimensional. Now phase portrait is collection of all the qualitatively different trajectories in the system.

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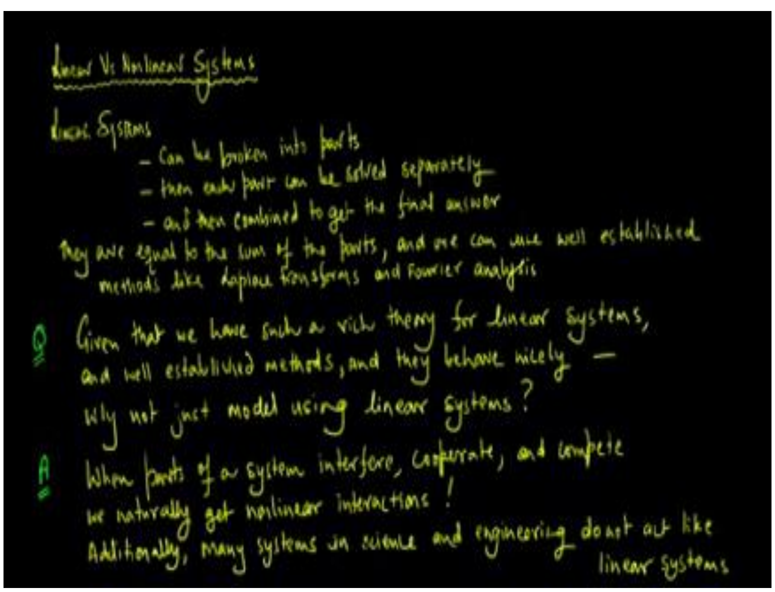

Now let us have brief discussion on linear versus nonlinear systems. When you look at linear systems they can be broken into parts then each part can be solved separately and then recombined to get to final answer. Now they are equal to the sum of their parts and utilise methods like Laplace transforms and Fourier analysis. So natural question to ask is given that we have such rich theory for linear systems and very well established methods and they behave nicely then why not just model using linear systems.

The answer as follows when parts of the system interfere, cooperate and compete. We very naturally get nonlinear interactions. Additionally, most systems in science and engineering do not really act like linear systems.

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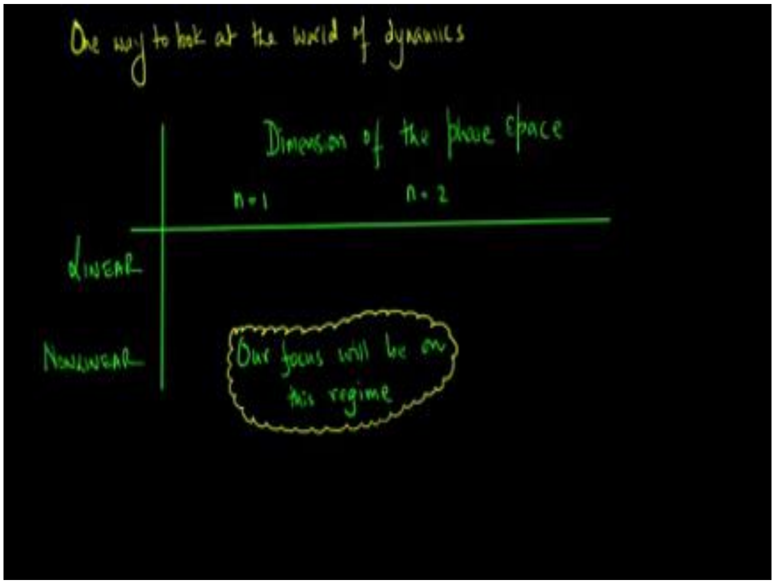

Now here is one way to look at world of dynamics. On one axis we write down the dimension of the phase space and on the other axis we classify system as linear or nonlinear. Then our current focus will be on this regime where we will be dealing with one two dimensional nonlinear systems.