Introduction to Nonlinear Dynamics
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Module -06
Lecture-18
2-Dimensional Flows, Linear Systems, Lecture 2

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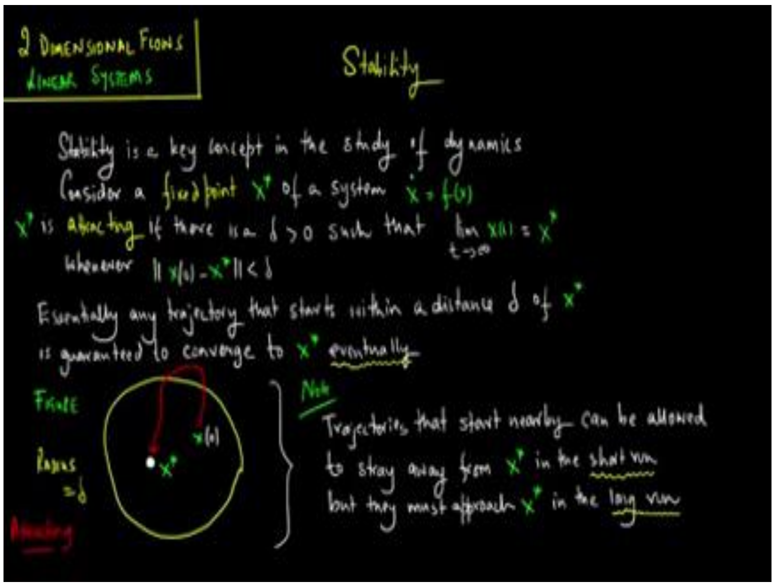

In this lecture, we focus on stability. Stability is rather a key concept in the study of dynamics. Consider a fixed point x star of a system, x dot = f of x, x star is attracting, if there is delta greater than zero, such that the limit t tends to infinity, x of t = x star. Whenever the distance between x not and x star is less than delta. Essentially any trajectory that starts within a distance delta of x star is guaranteed to converge to x star eventually.

Now let us plot a simple figure, we denote x star, you have x of 0, we have a radius of size delta and that is what trajectories look like. Note that trajectories that start nearby can be allowed to stray away from x star in the short run. But they must approach x star in the long run. So, this diagram exhibits an attracting fixed point.

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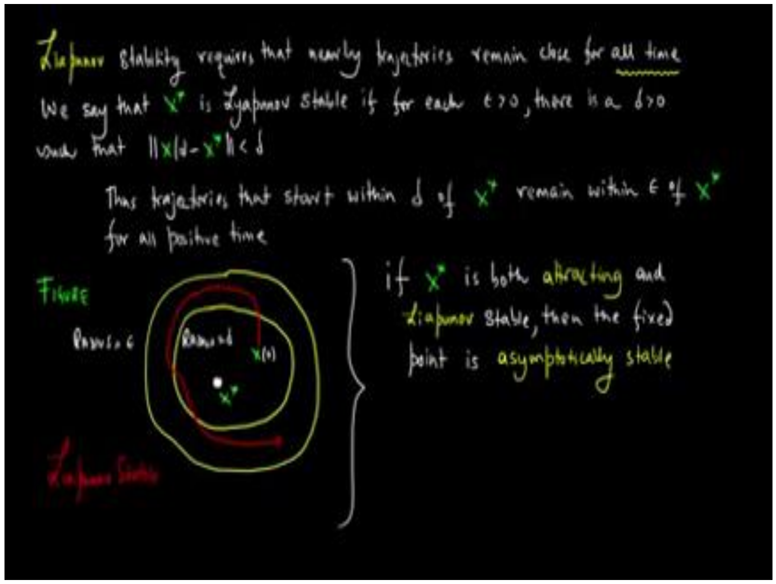

Now Liapunov stability requires that nearby trajectories remain close for all time. We say that x star is Liapunov stable, if for each epsilon greater than zero, there is delta greater than zero. Such that the distance between x not and x star is actually less than delta. Thus, trajectories that start within delta of x star remain within epsilon of x star for all positive time. Now we construct a figure to depict Liapunov stability.

We got(but) x star you have x not, we have a radius of size delta and a radius of size epsilon and that is your trajectory. So, this is the visual representation for Liapunov stability. If x star is both attracting and Liapunov stable then the fixed point is asymptotically stable.

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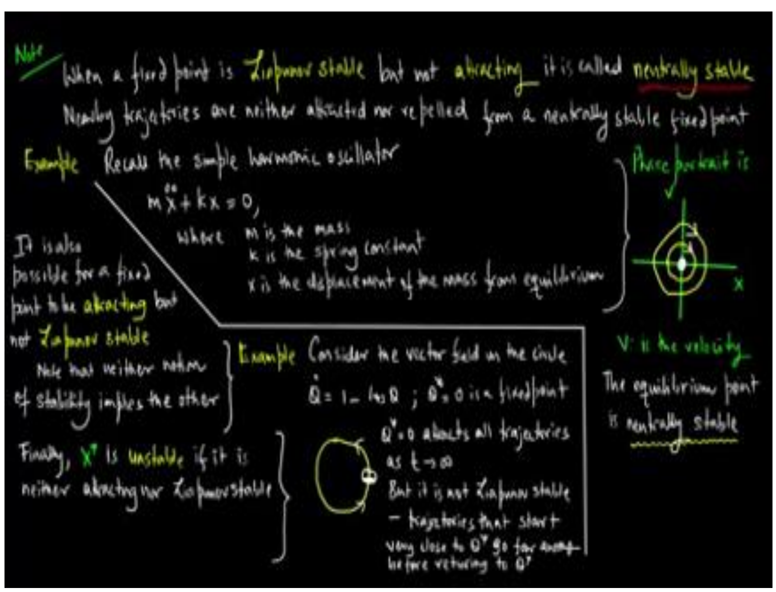

And here is a note, when a fixed point is Liapunov stable but not attracting is actually called neutrally stable, nearby trajectories are neither attracted nor repelled from a neutrally stable fixed point. Let us consider an example, recall the simple harmonic oscillator mx double dot + kx =0, where m is mass, k is the spring constant and x is the displacement of the mass from equilibrium, the phase portrait for this particular system is the following; where v is velocity.

Note that the equilibrium point is mutually stable. It is also possible for a fixed point to be attracting, but not Liapunov stable, note that neither motion of stability implies the other. Let us consider an example, consider the vector field on the circle tita dot = 1- cos tita. So tita star = 0 is a fixed point. We plot the vector field and so tita star = 0, attract all trajectories as t tends to infinity.

But it is not Liapunov stable trajectories that start very close to tita star actually go far away before returning to tita star. So finally, x star is unstable if it is neither attracting nor Liapunov stable

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In this lecture, we introduced the motion of stability. Stability is a really key and a really important topic in the study of dynamics and we start with by introducing motion of an attracting fixed point. So, consider you have a fixed point which is at top of my finger. And you have trajectory which starts closely with initial condition that starts close to it. If as time goes to infinity the trajectory eventually converges to the fixed point.

We call the fixed point attracting, we should note that even though the trajectory starts close to the fixed points there is no requirement that you should always remain close to the fixed point. So you can start close, you can go far away, you can come back and then you can go hit the equilibrium point, if that is the case than the fixed point is called attracting fixed point. The second motion was that of Liapunov stability.

So, a fixed point is called Liapunov stable, if trajectories that start close to the fixed point actually remain fairly close to the fixed point for all time. So, they cannot say that is the fixed point. And you are actually starting close to it, you cannot go very far away from it. So, if you are close then for all time, you kind of remain close to that equilibrium point.

So, if the fixed point is Liapunov stable and is attracting is called asymptotically stable. We can also have a scenario were the fixed point is Liapunov stable, but not attracting, if that the case it is referred to as neutrally stable. So essentially what happens is that nearby trajectories will neither get attracted to the fixed point nor get repelled from fixed point. So, in such a scenario the fixed point is called neutrally stable.