Introduction to Nonlinear Dynamics

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Module -05
Lecture-15
1-Dimensional Flows, Flows on the Circle, Lecture 1

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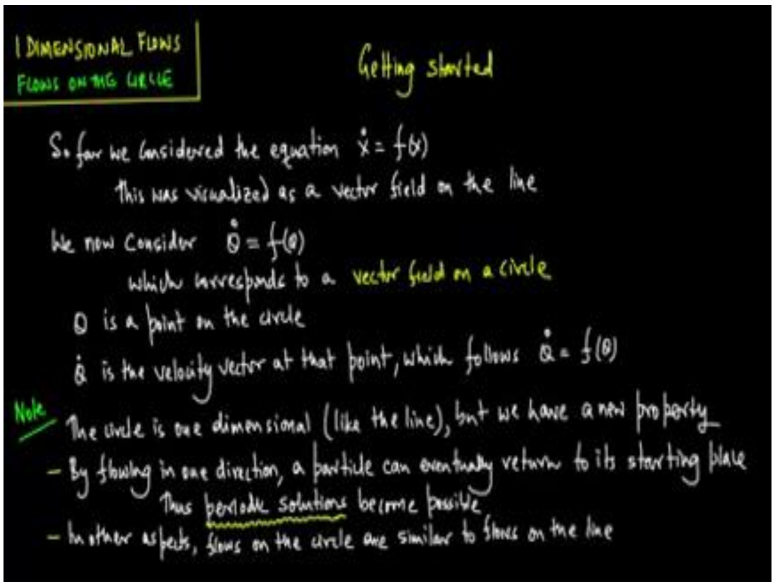

We are still dealing with one dimensional flows except that now our focus is on the circle. So, we will not be dealing with flows on the circle rather than on flows on the line. So, let us get started. So far, we have considered the equations x dot = f of x and this equation was visualised as a vector field on the line. We now consider tita dot = f of tita, which corresponds to a vector field on a circle.

So tita is a point on the circle and tita dot is the velocity vector at that point, which follows tita dot = f of tita. Here are some notes. The circle is one dimensional just like the line, but we have a new property by flowing in one direction a particle can eventually return to its starting place. Thus, in effect periodic solutions actually become possible. In other aspects, flows on the circle are actually quite similar to flows on the line.

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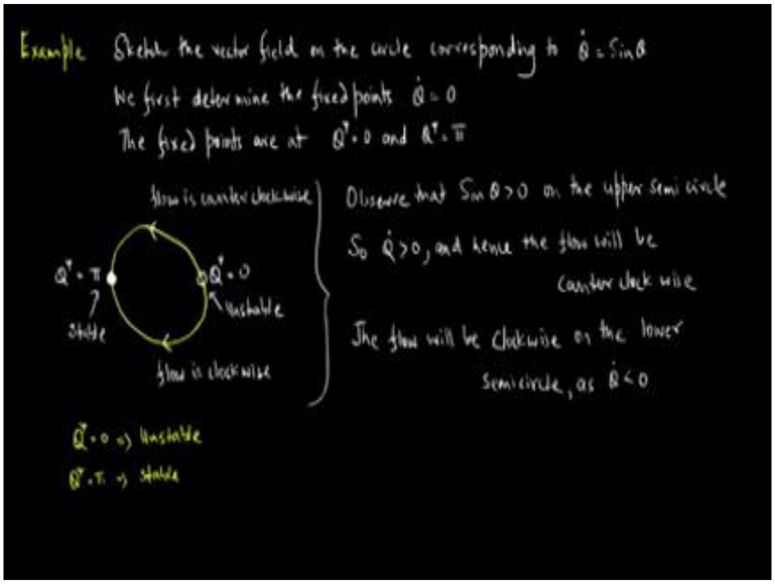

Now let us consider an example. Sketch the vector field on the circle corresponding to tita dot is equal to sine tita. We first determine the fixed points by putting tita dot =0 the fixed points are at tita star = 0 and tita star = pi. So we plot the circle and highlight tita star = 0 and tita star = pi. Now observe that sine tita is greater than zero, on the upper semi circle, so tita dot is greater than zero and hence the flow will be counter clockwise.

The flow will be clockwise on the lower semicircle as tita dot is less than zero. So, the flow is counter clockwise on the upper semicircle and the flow is clockwise on the lower semicircle, so highlighting the fixed points one is unstable and the other is stable. So tita star = 0 is unstable and tita star = pi is stable.

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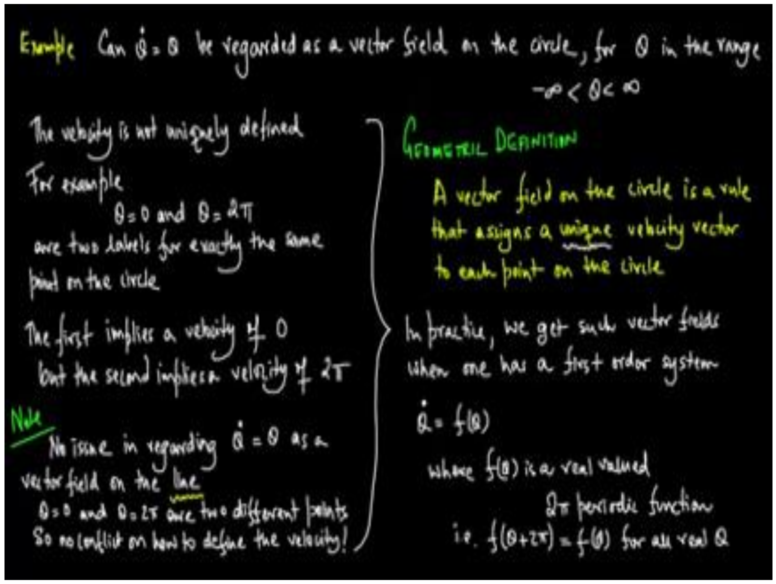

Now let us consider another example. Can tita dot = tita, we regarded as a vector field on the circle for tita in the range, where tita is less than infinite, greater than minus infinity. The power is that the velocity is not uniquely defined. For example, tita = 0 and tita = 2pi are two labels, for exactly the same point on the circle. The first implies a velocity of zero, the second implies a velocity of 2pi. Now note that there is actually no issue in regarding tita dot = tita as the vector field on the line, tita = 0 and tita =2pi are two different points.

So, there is actually no conflict on how to define the velocity. So, we now go ahead and offer a geometric definition. A vector field on the circle is a rule that assigns a unique velocity vector to each point on the circle in practice, we get such vector fields when one has a first order system tita dot = f of tita. Where f of tita is a real valued 2pi periodic function ie f of tita + 2pi = f of tita for all real tita.

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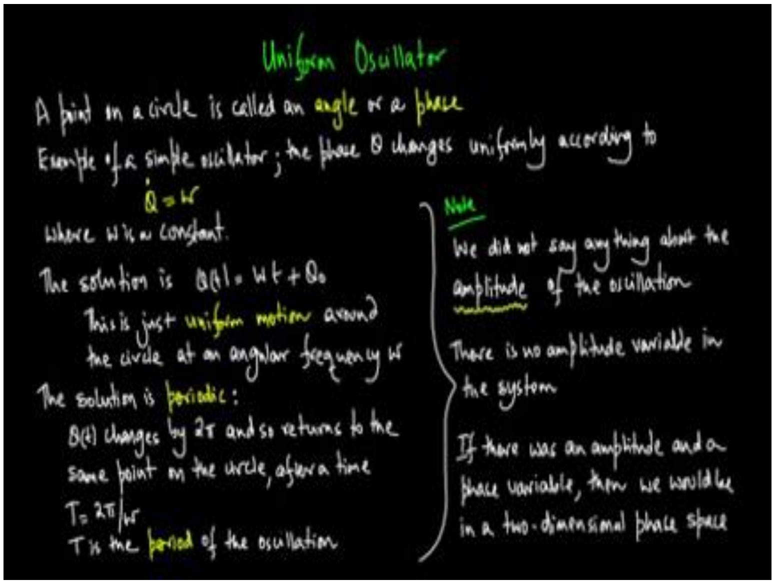

Let us consider a uniform oscillator; a point on a circle is called an angle or a phase. So, example of a simple oscillator is as follows, the phase tita changes uniformly according to tita dot = omega where omega is a constant. The solution is tita = omega t + tita not. Now this is just uniform motion around the circle at an angular of frequency of omega. Note that the solution is periodic tita t changes by 2pi and so returns to the same point on the circle after a time capital T = 2pi by omega capital T is the period of the oscillation.

Note that we actually did not say anything about the amplitude of the oscillation. There is actually no amplitude variable in the system, if there were amplitude and a phase variable then we would be in a two-dimensional phase space.

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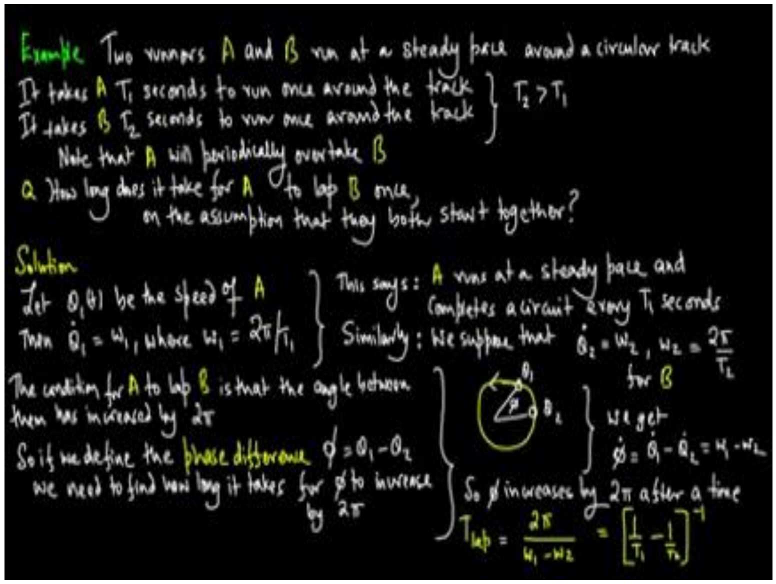

Now let us consider an example two runners A and B run at a steady pace around a circular track. It takes A, T1 seconds to run once around the track and it takes B, T2 seconds to run once around the same track, assuming that T2 is greater thanT1. So, note that A will in fact periodically overtake B. So, the question is how long does it take for A to lap B once, on the assumption that they actually both start together?

So here is the solutions let tita one of t be the speed of A, when tita one dot = omega1 where omega1 =2pi / T1. This says A runs at a steady pace and completes a circuit every T1 seconds. Similarly, we suppose that tita 2 dot = omega2, where omega2 = 2pi / T2 for B. The condition for A to lap B is that the angle between them has to increase by 2pi. So, if you define the phase angle, the phase difference fy = tita1 - tita2.

We need to find how long it takes for fy to increase by 2pi. So that is the simple schematic of the circular track. We note down tita1, tita2 and phace difference fy. So, we get fy dot = tita1 dot – tita2 dot which is equal to omega1 – omega2. So, the phase difference fy actually increases by 2pi after a time Tlap = 2pi / omega1 – omega2, which is equal to 1 upon T1 - 1 upon T2 to the -1.

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This was a short lecture, which got a started-on flows on the circle, we are still dealing with one dimensional flows. However, we have moved away from flows of the line to flows on the circle. So, we now deal with the equation of the form tita dot = f of tita where tita is point on the circle and tita dot is the velocity vector at that point. And there is one key difference between flows on the line and flows on the circle and that as follows, say imagine that you are on a circle rather than a line and if you keep going around.

Essentially what will happen is that after a point a time you come back to the same point, so what will happen is that in flows on the circle periodic solutions actually becomes possible and that is the key differentiator between the flows on the line and flows on the circle. And in this lecture, we started off by introducing very simple uniform oscillator of the form tita dot = omega.