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> Module - 06 Lecture – 03

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We have examined the current control voltage source and found out the conditions under which it does behave like a current control voltage source with trans resistance or trans impedance of  $R_m$ . Other important aspects of a controlled source are the input and output resistances; we will evaluate them now. The input resistance, of course is the resistance looking into this part, that is between the two terminals across which the input source is connected, it is influenced by  $R_L$ . And similarly the output resistance is the resistance looking that way between these two terminals, this point and ground across which the load resistance is connected. As usual you can evaluate them by connecting a voltage source and finding the resulting current or connecting a current source and finding the resultant voltage.

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Now to calculate the input resistance, I connect a voltage source  $V_{TEST}$  between this point and ground. I evaluate the current  $I_{TEST}$  that goes in there. The ratio  $V_{TEST}$  by  $I_{TEST}$  will give me the resistance looking into it. Now,  $V_{GS}$  equals  $V_{TEST}$  here right, so this current source here is  $g_m V_{TEST}$ ; and this voltage is of course, something we do not know. Let me call that  $V_0$ . The current here is

 $\frac{VO}{RL}$ ; and this I<sub>TEST</sub> simply comes that way. The current I<sub>TEST</sub> is whatever current is going through R<sub>m</sub>. So this voltage here is I<sub>TEST</sub>R<sub>m</sub>. So, this voltage V<sub>0</sub> is nothing but V<sub>GS</sub> or V<sub>TEST</sub> minus I<sub>TEST</sub> times R<sub>m</sub>.  $V_O = V_{TEST} - (I_{TEST}R_m)$ . And writing Kirchhoff's law here, we get

$$I_{TEST} = \left( \boldsymbol{g}_{m} \boldsymbol{V}_{TEST} \right) + \left( \frac{\boldsymbol{V}_{O}}{\boldsymbol{R}_{L}} \right) \quad .$$

So, substituting for  $V_0$  from there, we get an expression relating  $V_{\text{TEST}}$  and  $I_{\text{TEST}}$ . And it turns out

that the input resistance, which is  $\left(\frac{V_{TEST}}{I_{TEST}}\right) = \frac{R_L + R_m}{g_m R_L + 1}$ . So, this is the resistance looking into

the input. Now what happens here, the ideally we would like the input resistance of a voltage control current source to be zero.



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That is because if  $R_{in}$  is zero regardless of the value of  $R_s$  all of this  $i_i$  will flow into the circuit. So, this current here will be equal to  $i_i$  if  $R_{in}$  is zero, so that is why we want zero input resistance in a current controlled voltage source. Ideally of course, we would not get zero, but we get a small number. (Refer Slide Time: 04:45)



And here you can see that this number turns to zero, if  $g_m$  tends to infinity. So, the key to realizing an ideal current controlled voltage source is making  $g_m$  infinity. This again you have seen repeatedly for all controlled sources if  $g_m$  tends to infinity the control source becomes ideal that is it follows the relationship that is required, and also its output and input resistances will be exactly as we want them. Again we want the product  $g_m R_L$  to be very large, if this has to behave like a good current controlled voltage source.

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For calculating the output resistance, the input current source is set to zero, so that that means it becomes open circuit, and I connect a test voltage source between these two points, and find the current that flows. You can see that the current that flows here equals this voltage divided by the

total resistance, so this current will be 
$$\frac{V_{TEST}}{R_m + R_s}$$
.  $V_{GS} = V_{TEST} \left( \frac{R_s}{R_m + R_s} \right)$ . So, this current will

be  $g_m$  times this number. So, now you can work out that  $R_{out}$  which will  $\frac{V_{TEST}}{I_{TEST}}$ .

In this case, you can find I<sub>TEST</sub> by summing this current and that current and that will be equal to

 $\frac{R_s + R_m}{g_m R_s + 1}$  . As before as  $g_m$  tends to infinity this output resistance becomes zero. It is a current

control voltage source, so the output resistance should ideally be zero and it will be zero if  $g_m$  becomes infinite. So, we want this  $g_m R_s$  product be very large, so that this resistance becomes quite small. You can also observe the symmetry between this expression and what we got for the input resistance, they are quite similar. We have  $R_s$ , wherever we have  $R_s$  in this case, we have  $R_L$ 

in the other case. So, our control current controlled voltage source does give you a small input resistance and a small output resistance as desired provided that the value of  $g_m$  is very large.