Analog Circuits
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Module - 05
Lecture – 08

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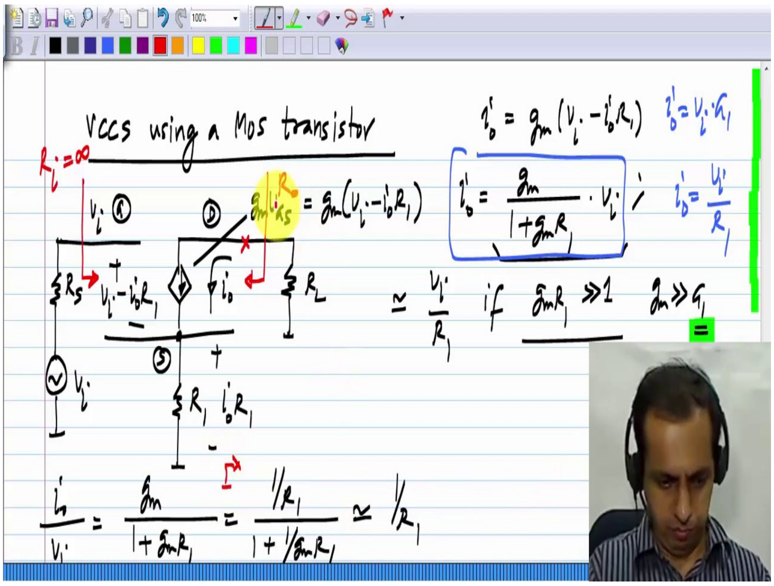

VCCS using a MOS transistor

$i_o = g_m(v_i - i_o R_1)$ $i_o = v_i \cdot G_1$

$$\boxed{i_o = \frac{g_m}{1+g_m R_1} \cdot v_i} \; ; \quad i_o = \frac{v_i}{R_1}$$

$$g_m v_{gs} = g_m(v_i - i_o R_1)$$

$$\simeq \frac{v_i}{R_1} \text{ if } \underline{g_m R_1 \gg 1} \quad g_m \gg G_1$$

$$\frac{i_o}{v_i} = \frac{g_m}{1+g_m R_1} = \frac{1/R_1}{1 + 1/g_m R_1} \simeq 1/R_1$$

We have derived the basic structure of the voltage controlled current source using a single MOS transistor. Now, we will analyze it further and see exactly how it behaves. I will assume that the load is a resistor that is connected to the drain, the input is applied to the gate. Of course, a real input source will also have a certain source resistance R_s. And we have a resistance R_1 connected between the source terminal and ground. So, what happens is whatever drain current is there, or the output current that goes through R_1 and there is a voltage i_oR_1 across R_1 and that is also the source voltage of the MOS transistor. At the gate, we have the input voltage v_i, because no current flows through the gate, and there is no voltage drop across R_s. So, the gate source voltage is $v_i - i_oR_1$.

Now, what is the actual value of i_o, you can see that this current source is $g_m v_{gs}$ or $g_m(v_i - i_oR_1)$ and that itself is the output current. So, to find the output current, all I have to do is to write this equation saying $i_o = g_m(v_i - i_oR_1)$ and solve for i_o. Solving for i_o, I get $io = \left(\frac{gm}{1+gmR1}\right) vi$

. Now what is the relationship that we wanted to have that was $i_o = \frac{vi}{R1}$. And you can easily see that this expression here reduces to that if $g_m R_l \gg 1$. So, if you choose $g_m R_l \gg 1$, the ratio of output current to input voltage in this circuit becomes independent of g_m and independent of g_m means of it is independent of temperature and so on, because the semiconductor characteristic g m etcetera is very dependent on temperature and also on other properties of the MOS transistor like the threshold voltage and so on.

Whereas the resistor could potentially be accurately be fixed. So, it is depends accurately on some resistance and it does not depend on the transistor at all; provided that the transistor provides g m which meets this condition. In other words, $g_m \gg G_l$, G_l is the trans conductance we want to achieve. If I rewrite this expression, $i_o = v_i G_1$, so that is that trans conductance we want to achieve. And if the trans conductance of the MOS transistor is much more than trans conductance we want to achieve then the resulting trans conductance is only determined by G_l, not by g_m of the transistor, so quite easy to analyze. And you can see this condition is very similar to what you got in case of the source follower, there $g_m R_L \gg 1$.

And in general, you will see that all negative feedback circuit behave in an ideal way if the trans conductance or in general some gain in the system becomes much more than one. Those of you were familiar with control systems know that if the loop gain is much more than one, then the negative feedback behaves in an ideal way, so this g m contributes to loop gain in this circuit and if that is much greater than something let say G_l then you will have nearly ideal behavior. So, now we have our circuit and we have also analyzed it. The exact expression for the trans conductance $\frac{i_o}{v_i} = \frac{g_m}{1 + g_m R_1} = \frac{\frac{1}{R_1}}{1 + \left(\frac{1}{g_m R_1}\right)} \quad \frac{1}{R_1}$, if $g_m R_l \gg 1$, so that is the important parameter of the voltage controlled current source, the ratio between the output current and input voltage.

We would also like the input and output resistance to meet some constraints, so in this case I would not spend much time on this. What is the input resistance looking into this, you can see that, there is no current flowing into the gate, so whatever voltage you have no current is going in, so clearly R_i is infinity. Similarly, you want to find out what the output resistance is; that means, between this point and ground, wherever the load is connected. Let us do that it is also quite easy to compute.

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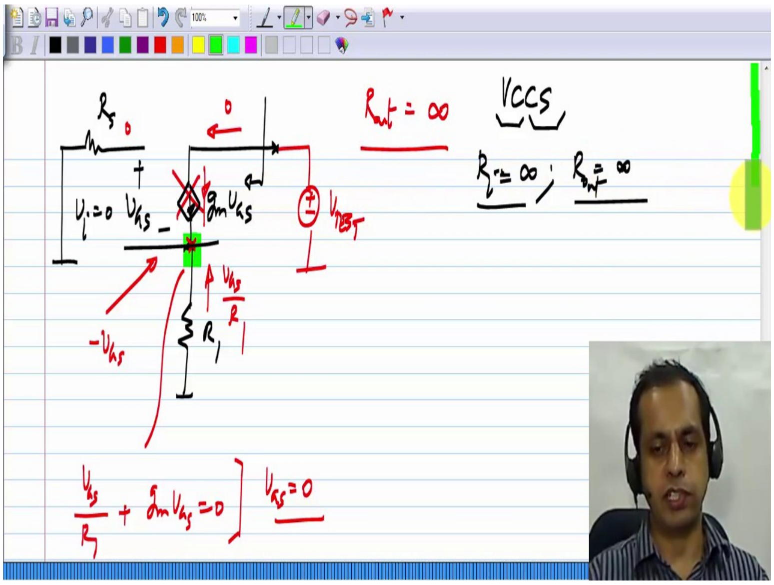

I have set the input voltage to zero; and I have R_1 here; and $g_m v_{gs}$, where this is v_{gs}. And I want to find the output resistance looking that way. So, I can connect a TEST voltage or TEST current; let me connect a TEST voltage for now. Now, it is very easy to see that the gate is at zero volts or ground. So, this is at zero. And if you apply Kirchhoff's law here, what is this, let me assume that the gate source voltage of the transistor is v_{gs}, so this is voltage is that $-v_{gs}$. So, the current in R_1 is $\frac{v_{gs}}{R_1}$ in that direction, and here we have $g_m v_{gs}$. So, applying Kirchhoff's current law here, we get $\frac{v_{gs}}{R_1} + g_m v_{gs} = 0$, so the only solution is that $v_{gs}=0$. So, both this is at zero volts and this is also at zero volts. So, this current source here is also zero, so it drops out of the picture, it becomes an open circuit. So, this becomes an open circuit, and this current is also zero.

So, because the current is zero regardless of V_{TEST}, R_{out} is also infinity. What are the desired characteristic from a voltage controlled current source, because it is voltage controlled, we would like R_i to be infinity, because then regardless of what kind of voltage source you use right that is; however, high the value R_s is, if R_i is infinity then there is no voltage drop across R_s and all of this v_i appears across the input of the circuit. So, that is why in a voltage controlled device, we would like R_i to be infinity. And it is a current source, a current source has an output resistance of infinity. Our circuit has an output resistance of infinity and it is the same as what we would like in an ideal voltage controlled current source, but of course, we have omitted something from model of the MOS transistor that is the parameter lambda corresponding to the channel length modulation.

We assume that the incremental output conductance is zero, but it really not zero. Now, in case of the common source amplifier and the common drain amplifier or the source follower, the output conductance, it changed the result little bit, but it did not make any qualitative changes. Whereas in this case, you will see that the output resistance is infinity if you assume that there is no output conductance, but if the transistor has some output conductance, the output resistance will not be infinity of the voltage controlled current source. So, it will make a qualitative difference to the result. We can analyze and see how much is the output resistance is.

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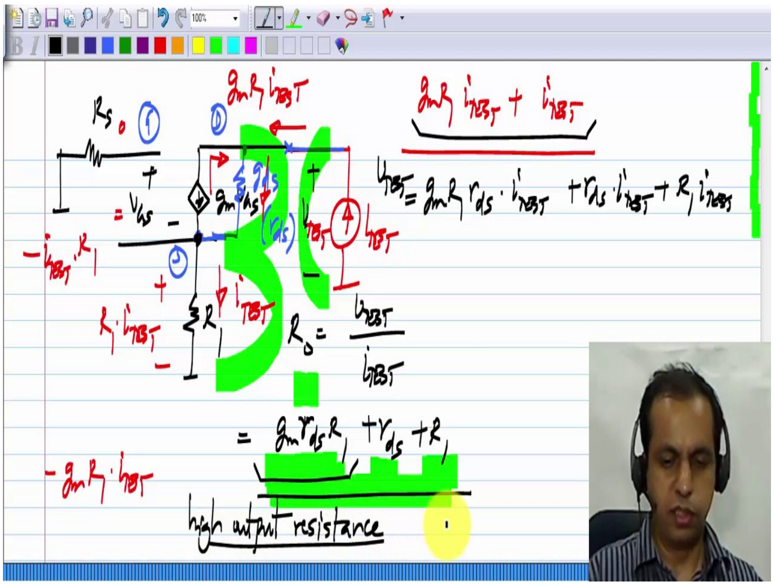

So, in this case, it is important to model the output resistance of the MOS transistor. Again I have MOS transistor with v_{gs}, and $g_m v_{gs}$. I have the resistor R_1 over there. And we need to have the output conductance of the MOS transistor g_{ds} or the output resistance r_{ds}. Now, again one of the mistakes frequently made by student is to connect it between the drain and ground. It should be connected between drain and source. In this case, the source is not grounded; in case of the common source amplifier, it is grounded and it is appears between output and ground, whereas, in this case it does not. Now, to find the output resistance, we can connect a voltage source and find the current or connect a current source and find the voltage. In this particular case, it turns out it easier to connect a current source and find the voltage. And also we have been using the voltage source for far too long, so let us now connected a current source and find the voltage that develops across it.

Now, the easy thing about using the current source is that when you have number of series path, the same current flows through all of them and that can lead to some simplification. Now, we do not know how much current is flowing and how much current is flowing there. But whatever goes into this, has to come out here, so it should be very easy to see that the current here is simply equal to I_{TEST}. Again you can imagine that by writing the Kirchhoff's current law around that whole circuit, you know that Kirchhoff's current is valid not only just at a node, but any closed circuit that could be some other components inside, because charges conserved and there is no local accumulation of charge. So, the total current going into a closed surface should be equal to zero. So, the current here is equal to I_{TEST} that is simply this thing, the voltage across it is R_1 times I_{TEST}. The voltage at the gate is zero.

And again because this R_s is connected to ground and no current flows in the gate, so this $v_{gs}= - I_{TEST}R_1$. And this current source $g_m v_{gs}$ will be $-g_m R_1 I_{TEST}$, that is what it is flowing downwards, so what is flowing upwards like this is $g_m R_1 I_{TEST}$. And I_{TEST} is flowing that way, obviously, the current through g_{ds} will be $g_m R_1 I_{TEST} + I_{TEST}$. So, now we can write down the voltage drop V_{TEST} in a straight forward way, it is equal to the voltage drop across r_{ds} plus voltage drop across R_1. Now, voltage drop across r_{ds} is this current times r_{ds}. So, it is $g_m R_1 r_{ds} I_{TEST} + r_{ds} I_{TEST}$. The voltage drop across R_1 is I_{TEST} times R_1, we have already calculated that plus R_1 times I_{TEST}, this whole things equals V_{TEST}.

And what is the output resistance, it is the ratio of $\frac{V_{TEST}}{I_{TEST}} = g_m r_{ds} R_1 + r_{ds} + R_1$. So, this is the output resistance of our voltage controlled current source. Now, what can you see from this expression is it going to be very large, like we want from a current source or small, and it is not really useful as a current source. So, it is easy to see that it is going to be quite large, because we have this number here, $g_m r_{ds} R_1$. Now, $g_m r_{ds}$ is the inherent gain of the MOS transistor, so this value of R_1 is multiplied by the inherent gain of the MOS transistor, which is likely to be quite large number. Another way to think about it is that $g_m R_1$ has to be much more than one that is for our negative feedback circuit to work properly.

So, here we see that for this expression to approximate $\frac{V_i}{R_1}$, $g_m R_1$ has to be much more than one. So, this number r_{ds} here is getting multiplied by a number much more than one, so the output resistance is going to be much higher than what you see in a single transistor. The output resistance of just a single transistor is r_{ds}. The output resistance here is $g_m R_1 r_{ds}$, and there are some extra terms here $r_{ds} + R_1$, but you can see that this number here is much more than either this one or that one. In most reasonable cases, the transistor of course, is assumed to be in saturation region. So, this gives you a very high output resistance.

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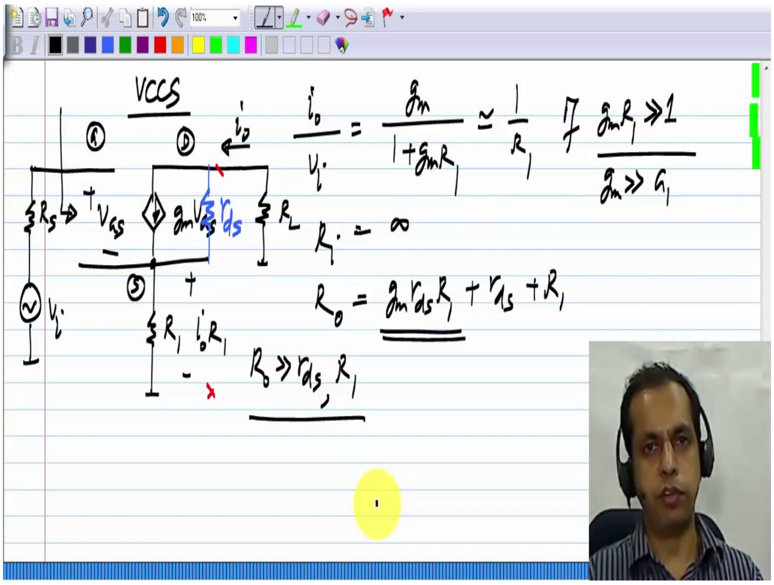

So, in summary, our voltage controlled current source, this is our implementation of the voltage controlled current source. Again, we have implemented the small signal incremental picture, then we will combine this with some biasing arrangement. What do we have here, the ratio of output current to the input voltage is $\frac{g_m}{1+g_m R_1}$, which reduces to $\frac{1}{R_1}$, if $g_m R_l >> 1$ and in fact, this $g_m R_l$ being much more than one is a criterion that you use to choose g m.

If you have a certain value of R_1, certain trans conductance G_l that you want to realize then you make g_m much more than that trans conductance G_l, $g_m >> G_l$. And R_i looking here is in fact, infinity because no current flows into the gate. Of course when we add biasing arrangements to this that may change, and the output resistance which can be usefully modeled only with the output conductance of the MOS transistor or output resistance rds is given by $g_m r_{ds} R_1 + r_{ds} + R_1$. The most dominant term is this, so what this says is R_o is much greater than r_{ds}, and it is also much greater than R_l. So, it does provide a very high output resistance, but just not infinity.