Analog Circuits
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Module - 02
Lecture – 01

We now know that in order to have incremental power gain, we need to have a nonlinear two port. What we will now do is look at the nonlinear two port, specifically its incremental linear equivalent and establish the condition for maximizing the gain. So far, we have talked about power gain, but for simplicity I will consider voltage gain, but you will easily see that maximizing this the way I do it also maximizes the power gain.

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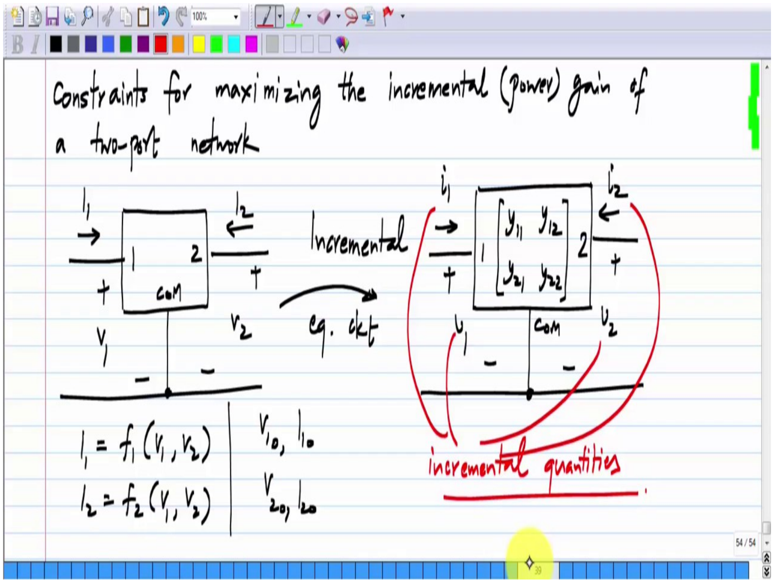

Constraints for maximizing the incremental (power) gain of a two-port network

$I_1 = f_1(V_1, V_2)$

$I_2 = f_2(V_1, V_2)$

V_{10}, I_{10}

V_{20}, I_{20}

Let us assume that we have a three terminal nonlinear two port for simplicity. And we have the port one voltage, and port one current, port two voltage, and port two current, the nonlinearity is given by currents as functions of voltages. Like I said earlier other representations are possible. Now, let us also say that there is some operating point, we will later worry about what exactly the operating point is that we must choose. So, at the operating point, we have the port one and port two currents. Now, if we take the incremental equivalent circuits around the operating point, what we have we know that it is going to be a linear two port and because we have here currents as function of voltages. In this case, we will have y parameter description. And the port one and port two incremental voltages and currents are given here. This and that and that and that all are incremental quantities.

What is it that we do, we connect a source – a signal source to port one of this incremental equivalent, and a load to port two of this incremental equivalent and evaluate the gain. This is still the incremental equivalent, we will get some constraints for maximizing the gain. Then we look at those constraints and see what it implies for the large signal the total characteristics f1 and f 2 of the nonlinear two port.

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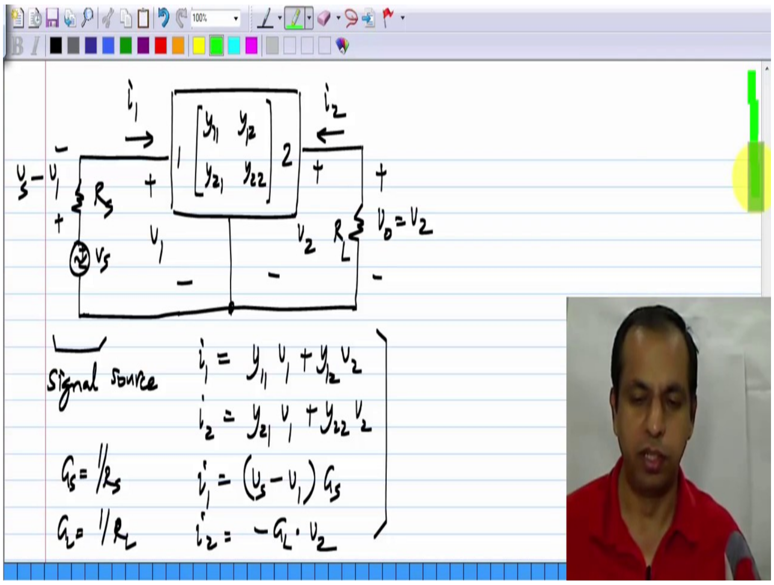

$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

$G_s = 1/R_s$ $\quad i_1 = (v_s - v_1) G_s$

$G_L = 1/R_L$ $\quad i_2 = -G_L \cdot v_2$

So, we will now assume that we have a signal source-V_S with a resistance-R_S. This is the representation of any source. Signals can comes from some sensors, let say the microphone into which I am speaking and it can be from some other stage, let say another amplifier output or it can be anything. The bottom line is it always can be in general represented by a voltage source in series with a resistance. Now, we connect that to a linear two port, which we know is the incremental equivalent of the nonlinear two port, because we already establish that we need a nonlinear two port to have incremental power gain. So, this is the incremental equivalent of a nonlinear two port. We will later see when we go to the specific device how to come up with the circuit which has this has the incremental equivalent. We will assume that the incremental equivalent of our circuit is such that the signal source is connected to port one and the load is connected to port two. So this part is pretty simple.

Now we have to solve for the output voltage V_o which you see it is exactly the same as this V_2.So, essentially we have to solve for V_2.So, first of all we have relationship impose by the two port, this must be familiar to you from regular circuit analysis; every element in the circuit imposes some constraints between its voltages and currents. And you take all of the constraints and solve for the circuit. So, this nonlinear two port by the way I could also draw the small signal equivalent of this using conductance and control sources that is the circuit equivalent of the y parameters. So, it will be the same thing; y_{11} V_1 + y_{12} V_2, and i_2 will be y_{21} V_1 + y_{22} V_2, in addition to this, we have a relationship from this side. We know that the voltage across this resistor is V_S - V_1. So, the current through this which is equal to i_1 has to be(V_S - V_1)times 1/ R_S or G_S which is the conductance.

And on this side, i_2 is the current flowing in R L, but from bottom to top. So, this imposes a constraints between its voltage which is V_2 and its current which is i_2.So, what do we have, i_2 will be equal to - G L times V_2.This comes about because i_2 is flowing in the upward direction. So, now we have these four equations and we have four variables, so we have to solve for i_1, V_1, i_2 and V_2. And it is specifically its V_2 that we are interested in; we have to find a relationship between V_2 and the source voltage V_S. So, at this point, I would ask you to pause the video and solve these simultaneous equations by yourself. I hope you are able to solve for it yourself ;it is a simple system of linear equations.

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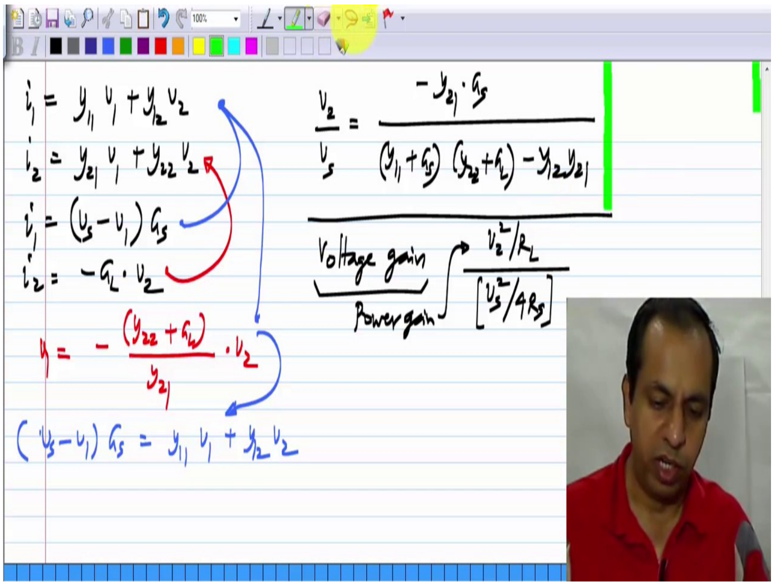

$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

$$i_1 = (v_s - v_1) G_s$$

$$i_2 = -G_L \cdot v_2$$

$$v_1 = -\frac{(y_{22} + G_L)}{y_{21}} \cdot v_2$$

$$(v_s - v_1) G_s = y_{11} v_1 + y_{12} v_2$$

$$\frac{v_2}{v_s} = \frac{-y_{21} \cdot G_s}{(y_{11} + G_s)(y_{22} + G_L) - y_{12} y_{21}}$$

$$\underbrace{\text{Voltage gain}}_{\text{Power gain}} \rightarrow \frac{v_2^2/R_L}{[v_s^2/4R_s]}$$

So, let us see, I would not show every step of the calculation, but show you how it is done. First of all, you can substitute this into that; from these things, we have to eliminate i_1, i_2 and v_1.So, by substituting this into that one, we will get a relationship between v_1 and v_2.So, we will see that we will get v_1 to be equal to - $(y_{22} + G_L)/ y_{21} V_2$. Then, we can take this one and this one and put it in here, so when we substitute this into that we will get $(v_S - v\ 1)\ G_S = y_{11} v_1 + y_{12} v_2$ and we can substitute this one into this to completely eliminate v_1. And if you obtain the final expression, you will see that $v\ 2 / v\ s$ will be equal to $-(y_{21} G_S)/ (y_{11} + G_S y_{22} + G_L - y_{12} y_{21})$.So, this is the expression for the voltage gain. And you can also from this make an expression for power gain, which will be related, but of course little more complicated than this. The output power will be $(V_2)^2/R_L$, and the input power – the available power from the source which has a source voltage of V_S and a source resistance of R_S, this is the maximum available power which you know from maximum power transfer is $(V_S)^{2/}4\ R_S$.

You can calculate this, this is the power gain. Now this expression will be lot more messy than this, so I will just maximize this though our intension is to have a large power gain. It turns out that the conclusion we draw from maximizing this will be exactly the same as what we get from maximizing that one. So I will stick to the simple expression for now, but you can as an exercise try to do this and you will find that you will get the same answer.

So, why did we calculate this, we have some gain and of course we would like to have as large a gain as possible mainly because we are greedy I mean we have a small signal and we want to amplify it as much as possible that is our aim is to realizes amplifiers and amplifying by a large amount has lots of benefits. Now, one obvious thing is so that you can make small signal large, but you also know from preliminaries of negative feedback that you have to have a large gain in order to realize an effective negative feedback system. So, large gains are very useful so that is what we will try to do.