Analog Circuits
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Module - 01
Lecture – 13

Now, we will look at how to do incremental analysis, when we have non-linear two port elements in a circuit. Now, with one port element, we have seen that we could write the equation then expand the non-linearity in a Taylor series around the operating point, neglect higher order terms and so on. But finally, that is not what we do routinely, what we did was for every non-linear component, we came up with an equivalent that is valid in the incremental picture. And for a two terminal nonlinearity that is the resistor. A two terminal non-linear element like a diode, it is the resistor. What we want to do now is to figure out what is the equivalent of a two port nonlinearity. So, in the incremental circuit, we substitute the two port by this equivalent circuit which is valid of course, only for small signal increment and go ahead with the analysis. And by definition, whatever equivalent we come up with will be linear, because we will be neglecting all the higher order terms.

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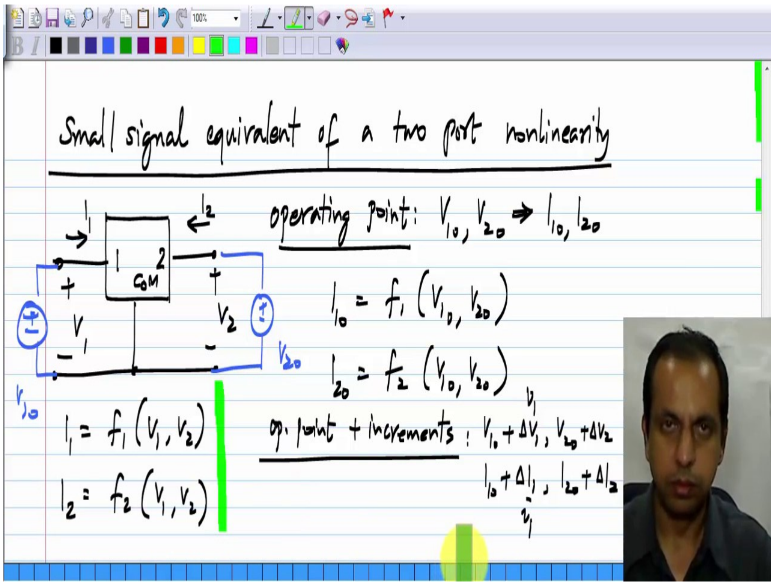

This is the two port nonlinearity, and these are the voltages and currents. Now, we know that I_1 and I_2 are some functions of V_1 and V_2. Now, let say we choose some operating point, I define the operating point as V_{10} , V_{20}, and which to current, I_{10} and I_{20}. You can think of this in many different ways. You can imagine that you have connected voltage source V_{10}, V_{20}, and these give you currents which are I_{10} and I_{20}. But, it does not have to be like that it is basically you have this non-linear element in a circuit, and at some particular point, where you choose to do the analysis, you solve the non-linear equations and found the solution that combination of voltages and currents happens to be V_{10} and V_{20} and I_{10} and I_{20}. Essentially it is the first point at which you do the analysis. But if it helps you, you can think of it as something like this. And then you can apply a increments to these voltages.

Now, by definition of these nonlinearity, I_{10} has to be equal to $f1(V_{10},V_{20})$, because these are valid for any voltages and currents, so it clearly has to be true at the operating point. And I_{20} will be $f2(V_{10},V_{20})$. Now, like I said let us imagine that there are some increments around the operating point, again it is easy if you think of V_{10} being change to $V_{10} + \Delta V_1$; V_{20} being change to $V_{20} + \Delta V_2$ and so on. You can think of it that way, but it does not have to be like that. It does not have be voltage source applied here. Any change in condition because of whatever reason, you will have a new set of voltages and currents which are $V_{10} + \Delta V_1$ or sometimes you use lower case v_1. And $V_{20} + \Delta V_2$, and the currents will be $I_{10} + \Delta I_1$ and again sometimes you see lower case i1 used here. And $I_{20} + \Delta I_2$. So, all it says is that it could be change from the operating point. And of course, again these voltages and currents have to satisfy these relationships which define the nonlinearity.

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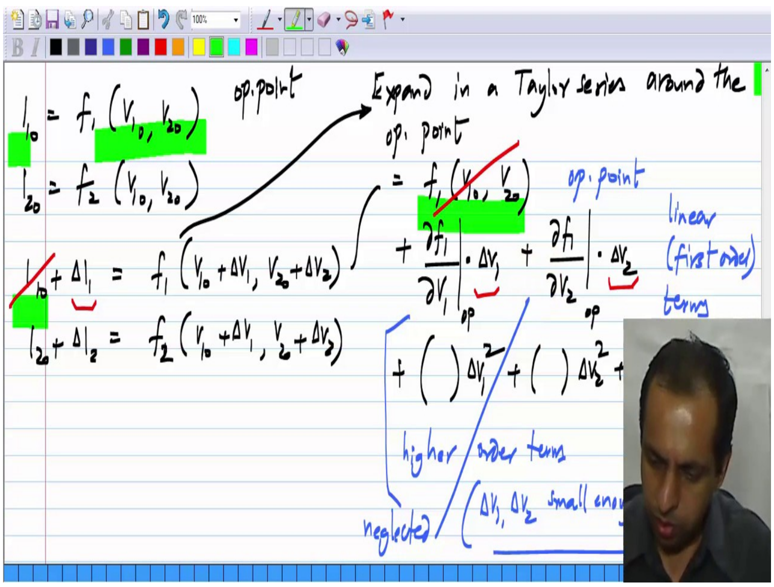

So, we have the operating point conditions here and we will have conditions with the increments also. Now, the modest operant is very simple, so far we haven't insert anything new, all we did was to express new voltages and currents as the original operating point voltages and currents plus some incremental quantities that by itself is not a significant step. Now, we will expand these relationships in a Taylor series; this will of course, be Taylor series of two variables and they are very useful by their own write, but for us we do not need to go into too many details. So, all we need to know is that we expand let say I will take f1 as the example the same hold for f2 in a Taylor series around or about the operating point. So, what will we get, this will turn out to be this is the operating point right. So, let me write it like that.

Taylor series is nothing but, we will have constant terms you will have a first order dependents on the variables, and second order dependents on the variables and so on. Now, we have two variables, so the first order terms will be these which have partial derivatives with respect to each variable that is partial derivative of this function with respect to V_1 times ΔV_1, and this partial derivative has to be evaluated at the operating point, because the derivative itself changes depending on the voltages and currents. And you will also have the partial derivative with respect to V_2 also evaluated at the operating point times ΔV_2. And these are linear or first order terms, these are linear in terms of variable, which is the incremental voltage ΔV_1, or the incremental voltage ΔV_2.

And then you will have higher order terms, we would not have to worry about them, you will have terms which are proportional to $(\Delta V_1)^2$ and something with $(\Delta V_2)^2$ and also something will $\Delta V_1 \Delta V_2$ and so on. So, all of these are basically higher order terms. Of course, we would not have to worry about them, because what we do with this, we just neglect them. We assume that you can always find a small enough value of ΔV_1 that these are even smaller, because the idea is the following, if ΔV_1 is very small, compared to something $(\Delta V_1)^2$ is even smaller. Now, it is started to see with the dimension variable, but you can think of a dimensionless quantity right, as I make let say a quantity x go from 1 to 0, x^2 able also go from 1 to 0, but much more rapidly, because when x is $\frac{1}{2}$, x^2 is only 0.25; and x is one-tenth, x square is already one by hundredth and so on. And x cube will go down even more rapidly and so on that is all.

So, you can always put some limits on the size of ΔV_1 and ΔV_2 so that these higher order terms are negligible. And that is the only way to define the limits for this incremental small signal analysis, that depends very much on the nonlinearity and the operating point you choose and so on. And when I say neglected what I really mean is ΔV_1 and ΔV_2 are small enough so that these can be neglected. Now, what do I do after neglecting them, I recognize that this I_{10} is nothing, but $f1(V_{10}, V_{20})$ because that is comes from here. So, that part cancel from the two sides, and as before as with one port element I have relationship between the incremental quantities ΔI_1, ΔV_1 and ΔV_2.

And what kind of relationship do I have, that is between ΔI_1 and ΔV_1 and ΔV_2, it is linear combination; ΔI_1 is nothing, but the linear combination of ΔV_1 and ΔV_2. And exactly the same things happens for ΔI_2, I do not have to repeat that. It is exact same expression except that instead of f1 over here, you will have f 2, you have to partially differentiate f2 with respect to V_1 and with respect V_2 and find the values of partial derivatives at the operating points.

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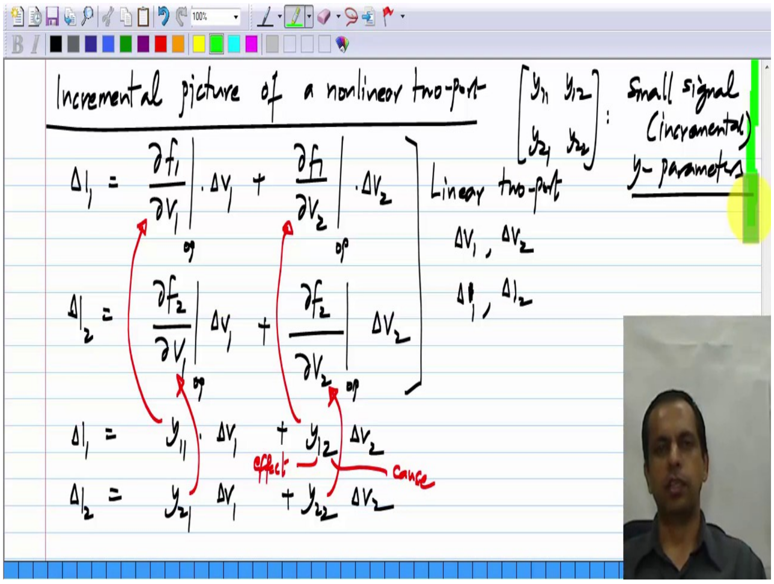

So, the relationship between the incremental variables of non-linear two port this is the following. So, ΔI_1 will be partial derivatives of f1 with respect to V_1 at the operating point times ΔV_1 partial derivatives of f1 with respect to V_2 at the operating point times ΔV_2. And similarly, this is the partial derivatives of f2 with respect to V_1 at the operating point. So, ΔI_1 is the linear combination of ΔV_1 and ΔV_2; ΔI_2 is also linear combination of ΔV_1 and ΔV_2. Now, we choose V_1 and V_2 to be independent variables while expressing the nonlinearity; it does not have to be like that. I_1 and I_2 can be independent variables or it can be a mixture I_1 and V_2 or V_1 and I_2. Then these partial derivatives will have different dimensions that is all that is there to it, but for our purposes we will keep this form.

Even if there is different representation of the nonlinearity, you should be able to easily calculate the small signal representation. Now, of course, I am sure all of you are familiar with this set of equations. What is this after all this whole thing is a linear two port with voltages ΔV_1 and ΔV_2, and port currents ΔI_1 and ΔI_2. And when you talk about linear two ports, these would have been expressed as $y_{11} \Delta V_1 + y_{12} \Delta V_2$; and ΔI_2 is $y_{21} \Delta V_1 + y_{22} \Delta V_2$. So, when you have a voltages as independent variables and currents as dependent variables, the set of parameters that relate them, it is known as y parameters, sometimes you express them in metrix form as well. And the notation is y_{11}, y_{12}, y_{21} and y_{22}, where the first number denotes the effect that is the ΔI_1; and the second one denotes the cause. So, y_{12} relates the effect of ΔV_2 on ΔI_1. And clearly, you can see the correspondence between these y parameters and this partial derivatives.

So, y_{11} clearly is partial derivative of f1 with respect to V_1 at the operating point. And similarly, y_{12} is partial derivative of f1 with respect to V_2; and similarly, for these two as well. So, what we get finally, is the linear two port network. In case of a two terminal element or a one port element, we got a resistor. Now, we have a linear two port network and the circuit equivalent is also the same as what we used for a linear two port network. Now, these parameters are known as small signal y parameters. For obvious reason, because these tell you the linear relationship between small signal increments, it is only for increments that are sufficiently small that the linear relationship holds..

So, what we do is the following, when we encounter a non-linear two port in a circuit, first we obtain the operating point by the usual painful way of solving any non-linear equations. Once we have the operating point things are easy; all we do is substitute the non-linear two port with a linear two port and what are the parameters of the two port, they are nothing, but partial derivatives of the large signal characteristic at the operating point.