Analog Circuits

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Module - 01
Lecture – 11

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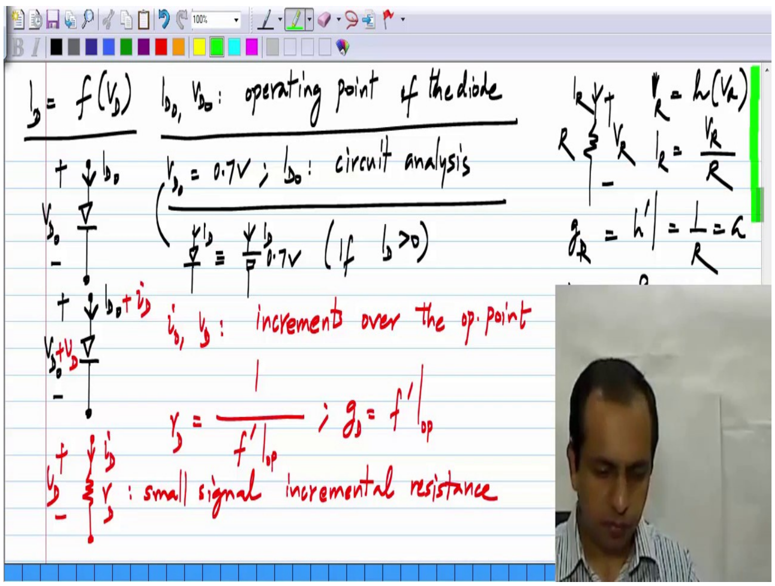

Now, we will consider the small signal incremental model of a diode that is first let us assume that the diode is at a certain operating point, and as I have mentioned before the operating point is simply let say the first solution that you calculate. And also while doing hand calculations, we will assume V_D' to be 0.7 V and I_D' is obtained from circuit analysis. Once you set V_D' to be 0.7 V, it is basically like an ideal voltage source. If there is current, so the circuit analysis is very easy, you do not have to solve any nonlinear equations. Now, if we consider the case where because of whatever reason the voltage across this changes and the current through it also changes then we can represent the new values to be increments over the operating point. So, I_D and V_D are increments over the operating point.

Now, we already know that we can define a new element which approximately shows the relationship between I_D and V_D and that element is a resistor. This is I_D and this is V_D and this is some resistance, which is known as the small signal incremental resistance. And what is the value of this r_d, this is the reciprocal of the slope of the diode characteristic at the operating point, where the surface basically the current as the function of voltage, that is the f the small signal incremental resistance is the inverse of the slope of this characteristic at the operating point and, the small signal incremental conductance is slope of this at the operating point. Now, by definition the element that depicts the relationship between a small signal increments is a linear element, and it is a resistor in case of a two terminal nonlinear element. This is because we expand the nonlinearity around the operating point in a Taylor series and neglect higher order terms that means that we are retaining only the linear dependence of the incremental current on the incremental voltage. So, it is by definition linear and this is true for any nonlinear element, not just a diode.

And just as an aside if you have a linear element to begin with you can also say that I_R is some function $h(V_R)$; of course, h itself is a straight line now. We know that I_R is V_R/R. Now in this case, the incremental conductance g_R will be the slope of this h' at the operating point, and the slope is basically 1/R everywhere, and it is equal to the conductance of the resistor. And similarly, the small signal resistance of a resistor equals the resistance itself. And this is not very surprising, the small signal conductance is the slope of the I-V characteristic at the operating point, the slope of resistor's I-V characteristic is 1/R everywhere. So, the small signal incremental equivalent of a linear element is the element itself.

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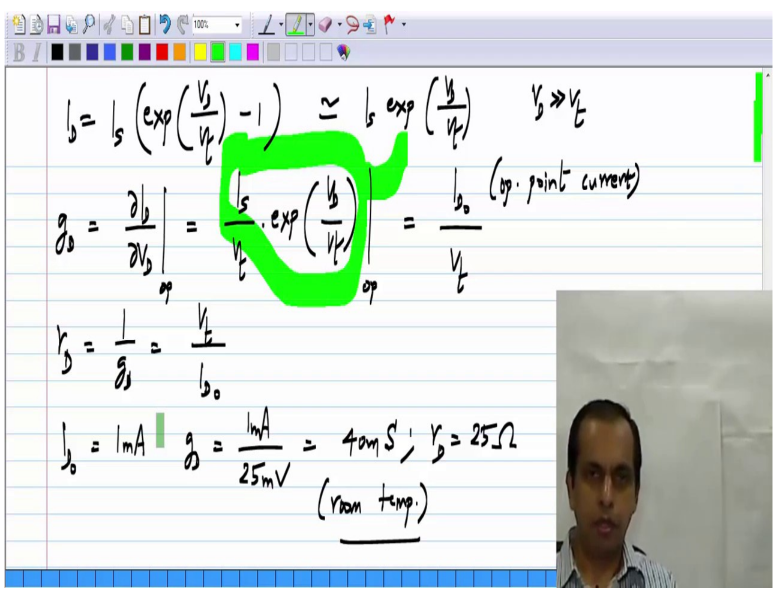

So, now what we do, we know that the diode characteristic is given by this expression, and which can be approximated to be $I_S \; e^{\frac{Vd}{Vt}}$ when $V_D >> V_t$ or basically when there is a substantial current flowing in the forward direction. So in this case, the incremental conductance of a diode is this derivative calculated at the operating point, and you can see that $I_S \; e^{\frac{Vd}{Vt}}$ is calculated at the operating point. You can also see that this quantity here is nothing but the approximate expression for the forward current in the diode. So this is nothing but I_{D0}, that is the operating point current divided by the thermal voltage V_t , so that is the incremental conductance of a diode. And the incremental resistance of course is the reciprocal of this which is V_t/ I_{D0}. And good valued remember is that when I_{D0} is 1mA that is when the operating point current in a diode is 1mA, we refer to it as the diode being biased at a current of 1mA. Then g_D is 1mA by approximate value of thermal voltage is 25mV and it turns out to be 40mS. And similarly if you calculate r_D, it turns out to be 25 Ω. All of this of course is that room temperature. Remember the value of V_t also has been approximated; it is really 25.9, but we say it is 25.

And knowing that g_D is directly proportional to the operating point current, you can easily tell that when I_{D0} is 10mA then g_D that incremental conductance will be 400mS and the incremental resistance will be 2.5Ω. So, for the incremental picture you replace the diode by a resistance of this value.

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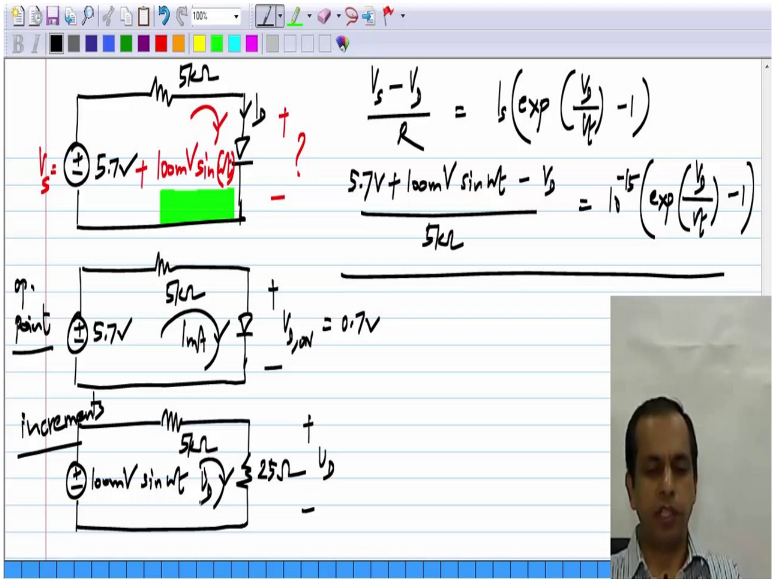

Now, what use is this, so let say again I take the same circuit. And let me assume that the so V_S this time is the time varying source, it is 5.7 V + 100mV sin(ωt). And I want to find the voltage across the diode and the current through the diode in this circuit. Of course, one possibility is to do the exact analysis again $(V_S - V_D)/R = I_S \left(e^{\frac{Vd}{Vt}} - 1\right)$, which means that (5.7 V + 100mV sin(ωt) - V_D)/5 KΩs equals whatever is on the right hand side 0^{-15}.let us say that is the value of $I_S \left(e^{\frac{Vd}{Vt}} - 1\right)$. And you can immediately see that this can be solve very easily numerically but hand calculation of this is very painful.

First of all, we have a time varying quantity here, which means that we have to choose a number of different time points and for each of those time points calculate the left hand side; and for each of those, we will get an equation, which we have to solve numerically that is we take points along the period of the sin wave, and at the each point, we solve this nonlinear equation, and this is just not possible by hand.

So, what we do is the following. First we solve for this circuit. This is typically what we do I mean what we consider as operating point is up to us, but the easiest thing is to consider the fixed part of this 5.7 V as the operating point, and solve for this. And while calculating the operating point, we take the diode model to be where in the forward direction, when current is flowing in the forward direction, the diode has the voltage of 0.7 V. So, this we already solved before, so I will just show the solution, the current here is 1mA, and the voltage here is 0.7 V. Now, the incremental part here, the extra over the 5.7 V, we calculate from the small signal incremental equivalent that is the utility of the small signal incremental equivalent. The most important thing to remember is that it is valid only over this operating point, because the small signal parameters depend on the operating point.

So, what is that, first we have to have a voltage source, which corresponds only to the increment, which is100mV sin(ωt). And the equivalent of a 5 KΩ linear resistor is the same a 5 KΩ linear resistor. Now the diode will be replaced by a resistor and the diode's operating point current is 1mA; it is very important that what you replace here depends on the operating point. So because the diode's current is 1mA from the previous example that I calculated you know that this is a 25 Ω resistor. And what do we get from this, this circuit is valid only for increments and only for increments that are sufficiently small, so that you can neglect higher order terms in the Taylor series. So, we get only the incremental V_D from this and the incremental I_D from there. But of course, the important thing is to note that this is the linear circuit everything is very easy to calculate.

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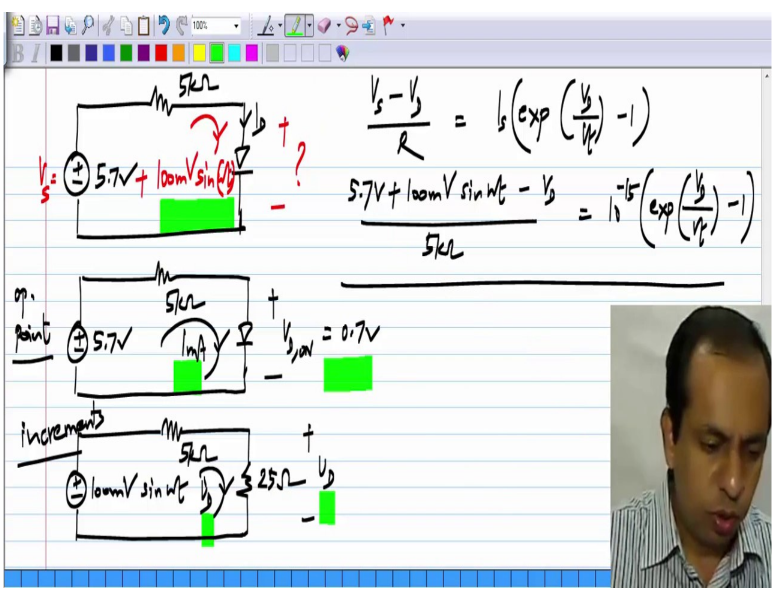

So, V_D will be calculated from the resistive divider formula; it is the $\frac{Vs*25}{25+25k}$. And I will approximate this just to do it in my head, I will neglect this 25 Ω in the denominator. So, this will turn out to be 0.5mV sin(ωt). And similarly the incremental current can be calculated to be the source voltage divided by the series combination of the two resistors again neglecting this part; we will see that this is equal to 20μA sin(ωt). So, this is the incremental voltage and this is the incremental current. And to get the total solution here, what we do is we sum the operating point quantities which are these, and the incremental quantities from here. So, what do I get from there

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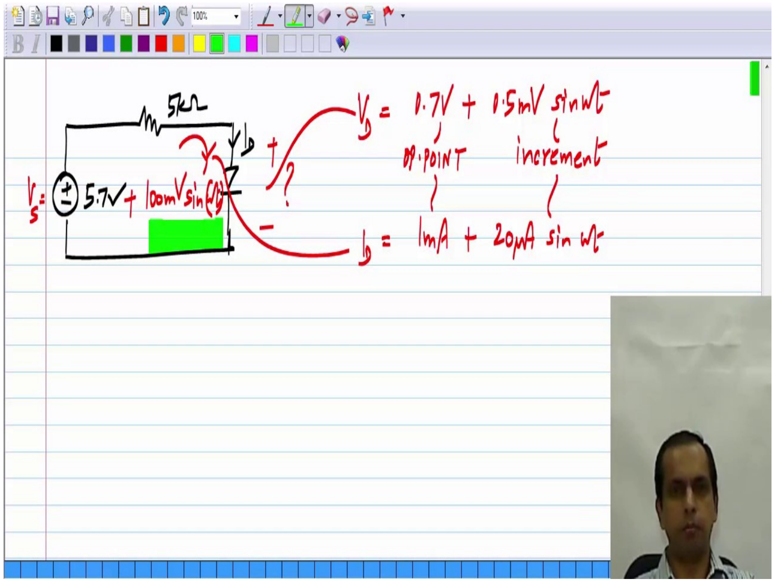

So, the total voltage here, V_D will be the operating point voltage plus the incremental voltage, and these come from separate analysis. And similarly this current here – control current will be 1mA which is at operating point plus 20μA sin(ωt). which is the increment. So, this is the utility of the small signal incremental model. We first calculate the operating point by some means may be require solving a nonlinear equation or some approximation of that. And after that we only calculate increments from the small signal equivalent picture and that is very easy, because we know linear circuit analysis. And we have the operating point to the incremental quantity to get the total solution. This is only approximate, because the increment has to be sufficiently small, but it solves our purposes in a variety of situations and it let us get some insight into nonlinear circuits which are otherwise very difficult to solve.