Networks and Systems
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Lecture - 07
Fourier Series (1)

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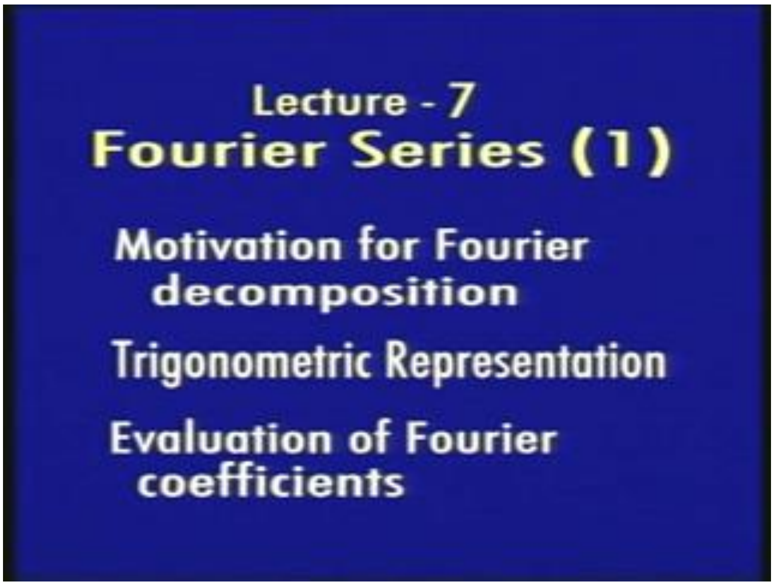

In this lecture, you will be introduced to the concept of Fourier Series and its significance in the analysis of networks and systems. You may recall that earlier we discussed, the input output relations of a linear continuous time, time invariant system with lumped parameters. Another way of describing a time invariant system is say, it is a system with constant parameters and we also take the parameters to be real.

So, if you have a linear continuous time system with lumped constant and real parameters. It can be symbolically represented as a box like this; with an input and output terminals coming out. Now we also observe that, a characteristic signal for such systems is the exponential function if indeed you have an input which is of the form f e to the power of s t then the output will be h s f e to the power of s t; that means, the input and output essentially have the same time function except that you have a proportionality function h of s.

Any signal which gives rise to that property is said to be a characteristic signal or eigen signal and in the case of the signal that, we are considering this exponential signal f e to the power of s t constitutes a characteristic signal. Now, in general f and s can be complex.

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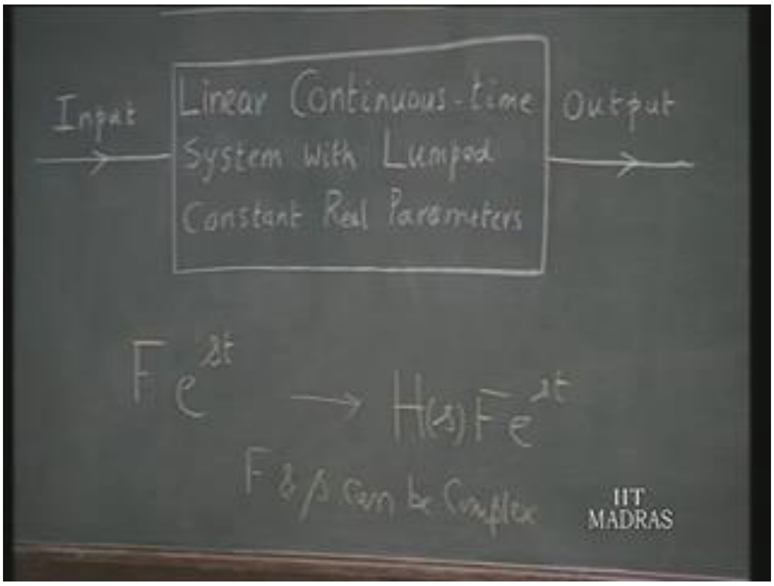

So, if s is complex in general it will be s equals sigma plus j omega therefore, if you have a signal like that, when you substitute particular values of time then the value of the signal turns out to be in general complex. So, it is only a mathematical tool that we use for the analysis of the system, in practice a complex signal like that cannot be generated in the lab. However, the sinusoidal function of time which plays such a vital role in a c circuit analysis is a very close relative of an exponential signal e to the power of s t as we will see presently.

Therefore we would like now to see how to relate this exponential signal e to the power of s t with a sinusoidal signal and if the sinusoidal signal is the input how we can obtain the output in terms of this h of s. So, let us see how it goes. We have seen just now, that f e to the power of s t gives rise to the output suppose i have a e to the power of j theta replacing f that is the constant e j omega; that means, i substitute j omega for s then; obviously, the output that i get would be h of j omega because the output s is now equal to j omega in addition you have a e to the power of j theta e to the power of j omega t this is the characteristic function.

Now, let me say that, h of j omega being a complex quantity in general will have a magnitude h j omega and an angle alpha therefore, this can as well be written as h magnitude i drop the functional notation j omega a e to the power of j theta plus alpha e to the power of j omega t. Now let me take another exponential function a e to the power of minus j theta e to the power of minus j omega t similar to this except that the multiplying constant f is replaced by a e to the power of minus j theta and instead of s equals plus j omega you take an s equals minus j omega naturally this will give rise to h of minus j omega a e to the power of minus j theta e to the power of minus j omega t.

So, the difference between this 1 and this here now instead of s equals minus j omega, instead of plus j omega you have minus j omega and instead of theta you have taken minus theta. Now, if the system parameters are real h of s turns out to be for a lumped parameter system the ratio of 2 polynomials with real coefficients and if that is. So, then a property of such a rational function with real parameters and real coefficients is h of minus j omega turns out to be the conjugate of h of j omega.

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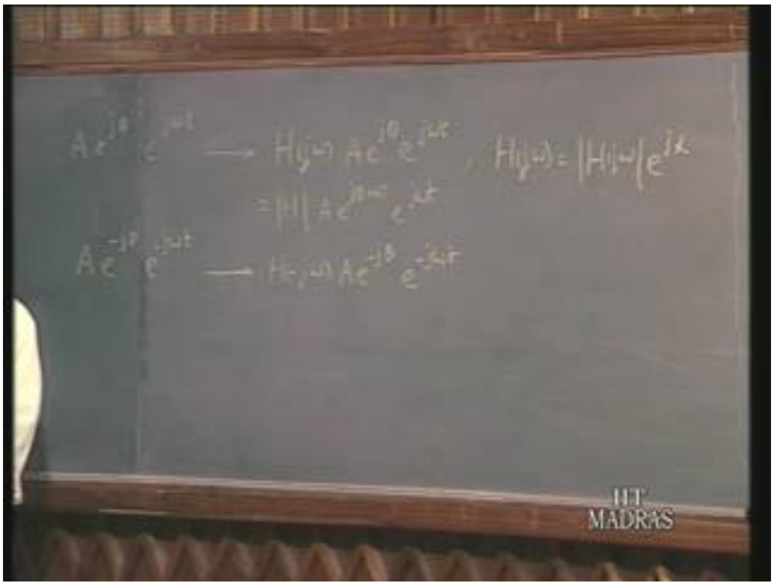

h of minus j omega will turn out to be the conjugate of h of j omega which means really that the magnitude will remain the same, but the angle will change for a real parameter system.

What do you mean by real parameter system suppose, you take circuit in an example we are saying r l and see how real values. Hence i can write this as h of magnitude a e to the power of minus j theta plus alpha e power minus j omega t. Now let me imagine that we have a composite system which is the sum of these 2, by virtue of linearity the responses can also be summed up.

So, the sum of these 2 signals will give rise to a response which is equal to the sum of these 2. Therefore if you do that then you have a e to the power of j omega t plus theta plus e power j minus omega t plus theta that is; what we are having here this will give rise to h j omega magnitude a e to the power of j omega t plus theta plus alpha plus e to the power of minus j omega t plus theta plus alpha and indeed you could now see that this is really 2 a cos omega t plus theta.

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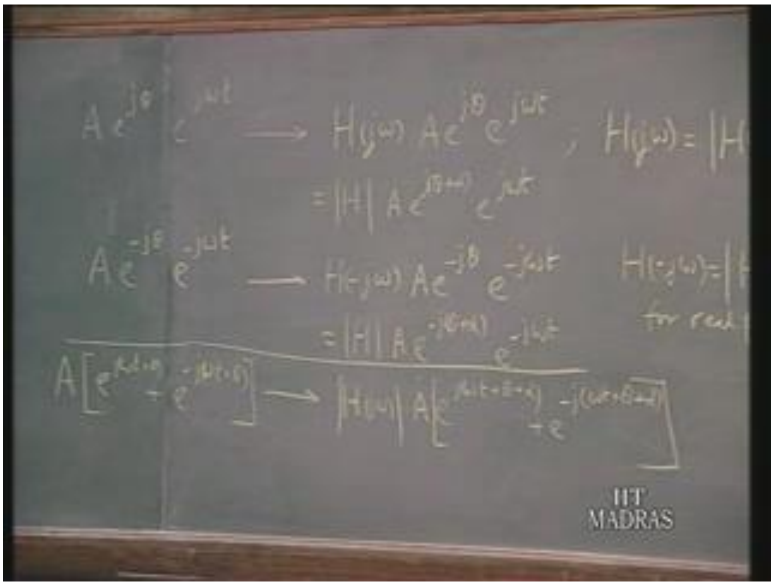

That means this is a general sinusoidal function of time and as far as the output is concerned you have h j omega magnitude 2 a cos omega t plus theta plus alpha. So, what do we have we have a sinusoidal function of time with an amplitude 2 a. The output is modified is also a sinusoidal function, but the output is magnified in magnitude by the this factor magnitude of the h of s which we call the system function as we recall.

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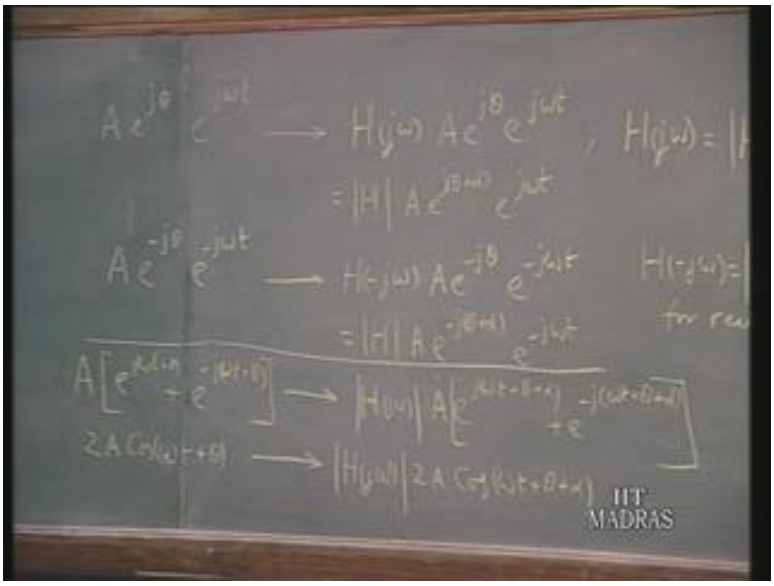

Phase is also modified by an angle alpha that alpha is the angle of h of j omega. Normally when we deal with sinusoidal functions of time in a c circuit analysis we use the phasors. Let us see, how these are related to the phasors in terms of phasors we express this forget i think now that, we have got this expression i can drop this 2 here and this 2 here and simply deal with a cos omega t plus theta because after all if this is reduced by factor 2 output also will be reduced by factor 2 the phasor for this as you recall will be written as a e to the power of j theta.

A at an angle theta and the phasor for the output is; the magnitude h j omega a e to the power of j theta plus alpha. Now h j omega times e to the power of j alpha h j omega magnitude times e to the power of j alpha equals h of j omega complex number. Therefore this can very well be written as h j omega not the magnitude, but the entire quantity times a e to the power of j theta. So, what do we have now.

The significance is that; if you have a phasor here as the input the output phasor is obtained by multiplying a e to the power of j theta by h of j omega. This is the principle that we use in a c circuit analysis.

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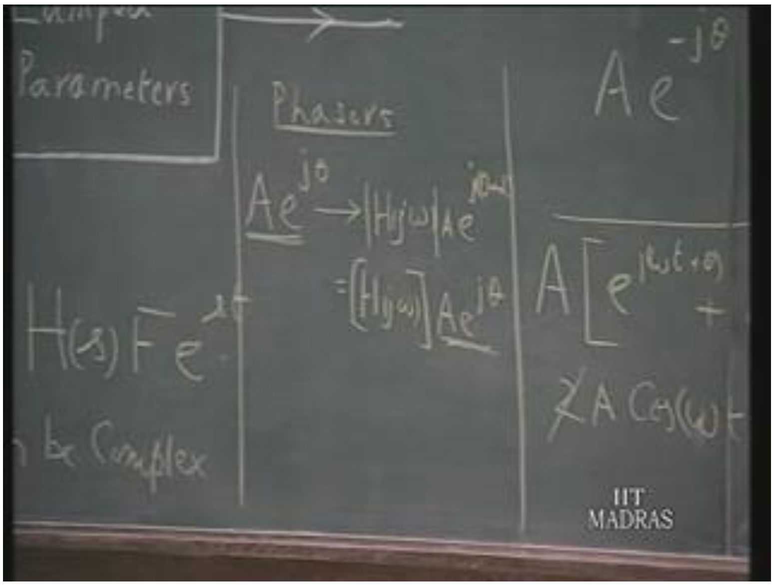

What is h of j omega in a c circuit analysis, it is the ratio of the phasors of the response quantity may be a current or a voltage to the excitation quantity which can also be a current or a voltage. So, in general h of j omega could be an impedance, it can be an impedance it could be a pure ratio. We are accustomed to doing this circuit analysis using phasor concepts, but the relation of this phasor with the system function here is quite illuminating what we are really doing is when we have a sinusoidal function of time a cos omega t plus theta you would somehow like to relate it to a characteristic signal of the linear system, this is not a characteristic system or the linear system.

So, we view this a cos omega t plus theta as the real part of a function like this a e to the power of j theta e to the power of j omega t. So, this is the characteristic signal whose real part is our signal which, we are using now instead of using this actual signal a cos omega t plus theta suppose, we assume the characteristic signal was present this would have been the response. But since the real part of this signal is what we are actually using the output will be the real part of that and that is what is happening.

Even though i have put these 2 steps here actually what you can see is this is the real part of this and the fact that the real and imaginary parts are the input produced independently the real and imaginary parts of the output without any mixing is a property that, comes by virtue of the real parameters of the system apart from it is linearity.

So, instead of this signal, we are using this and our phasor is related to this simply by dropping out e to the power of j omega t because in a c circuit analysis what we normally have is all sources of the same frequency single frequency excitations wherever, number of generators there may be, but all of them will have the same frequency and therefore, this omega is constant right through.

So, e to the power of j omega t is a term invariably present both in the output and the input therefore, it is a term which we can drop it is understood therefore, instead of writing every time a e to the power of j theta j omega t we may as well drop both these terms from the input side output side and say this is the phasor for the input and this is the phasor for the output that’s what we have.

So, this is how the phasor concept the impedance concept etcetera are related to the system function h of s in this discussion we had taken the magnitude of the phasor to be the peak value of the sinusoid instead of the r m s value for the sake of convenience, but the same arguments will be valid even, if you take the r m s value to be the magnitude of the phasor suppose, i have a simple circuit like this and a sinusoidal input and this is the output phasor is this r c now, if this is taken as the output of the system.

Therefore it identifies this with this system and take this as the output and take this as the input and you would like to find out this system function h of j omega in this. After all h of s is the system function h of j omega is also a system function when s equals j omega. So, all we have to do is take the ratio of the phasors v nought to v i using our familiar circuit analysis procedures, this will be by potential divider action 1 over j omega c divided by r plus 1 over j omega c which of course, is 1 by 1 plus j omega c r this is our h of j omega in this case.

Once you identify this as the output and this as the input that will be h that is our system function.

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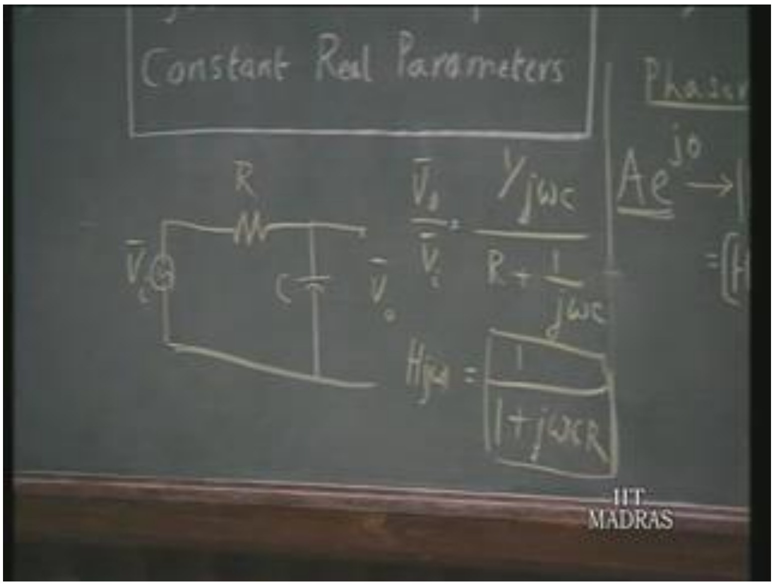

So, analysis of systems with of the type that is; described here under sinusoidal excitation becomes almost a kind of mechanical procedure algebraic procedure instead of our having to deal with differential equations. The fact that a system with arbitrary complexity involving a differential equation of high order can be analyzed in a purely algebraic fashion as here constitutes the central advantage of the phasor concept and the simplicity of a c circuit analysis methods.

Now we understand here of course, then we will talk about the output in all these in the entire discussion here that, we are talking about the particular integral solution here.

Suppose you have an input when you talk about h of s system function and. So, on and. So, forth normally the output is the particular integral solution. In the case of a c signals sinusoidal signals the common systems the complementary solution gives rise to a decaying transients and therefore, the particular integral solution is what remains forever, it is the steady state solution the complementary solution is a transient process.

The result therefore, is that whatever, we are having here is not only the particular integral solution it is also the steady state solution and if you have a system with sinusoidal inputs and you would like to observe the output and the sustained, it is periodic remains forever then it is easy for experimentation purposes. particularly the circuits you can find out the parameters of the input quantity and the output quantity using your instruments may be an oscilloscope may be a voltmeter whatever, you are having measure the output phase and output magnitude in relation to the input and calculate this system measure the system function h of j omega for different values of omega.

This is very convenient for experimentation purpose not only sinusoidal functions can easily generated in the lab, but also the input output relations can be quite conveniently measured this is not possible with exponential signals particularly when s is complex of course it is impossible because no physical signal will have that, but even if s is real and f is real then you may have a decaying exponential signal which you have to do some kind of photographic recording process you will not able to take measurements under steady state conditions.

So, this is the difficulty the upshot of all this is; that characterization of a signal in terms of h of a system in terms of h of j omega turns out to be very convenient and truthful 1 the input output relations of a system, if you express in terms of h of j omega which is called frequency responses of the system h of j omega is called the frequency response function.

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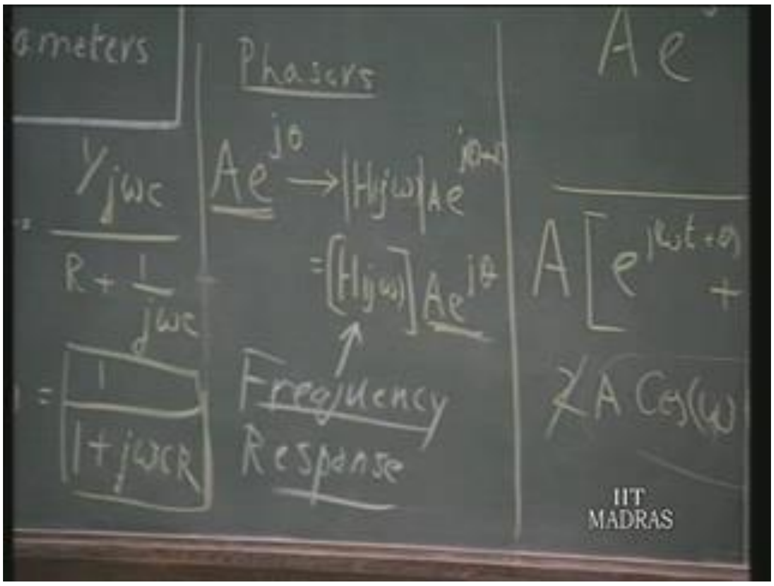

So, this is a very convenient 1 and we would like to make use of this facility to the extent that is possible and what is meant by frequency response function. It is the ratio of the output phasor to the input phasor under steady state sinusoidal conditions. However, in practice in real systems we may not always have excitation functions which are sinusoidal they may be periodic and non-sinusoidal they may not also be they may not be periodic as well they may be aperiodic.

However, we would like to make you use of the simplicity of sinusoidal circuit analysis to such situations as well and in this task it is the Fourier methods which come as a blessing as a very convenient tool for analyzing system with these types of excitations essentially using sinusoidal circuit methods, sinusoidal steady state methods and it is this particular aspect that we will now, take up to the form of the Fourier Series. So, let me now introduce the concept of Fourier Series with this background.

Let us now take, a look at the form of the Fourier Series for a periodic function in terms of sinusoidal functions of time you recall that a periodic function is 1 which has this characteristic f of t equals, f of t plus t nought for all t, for all small t in other words, if i have a basic variation over 1 period a same variation continues indefinitely and so, on and so on forth on both directions in defining this value of capital t nought which is the period we take the value of t nought such that, it is the smallest value that satisfies this equation.

For example, it is possible for us to take this as the basic period because whatever, sequence of values it takes from 0 to 2 t nought will also repeat, but; however, the particular value which the smallest such value is taken as t nought. So, this is the period we also know that f nought which is 1 over t nought is called the frequency of this signal and in line with what we do in the sinusoidal steady state analysis methods we use omega nought 2 pi f nought 2 pi over t nought as the angular frequency. These are terms which you are already familiar with.

We will make use of them here as well according to Fourier any periodic function of this type can be expressed as, the sum of a d c component with plus and various sinusoidal functions it goes like this f t in general can be expressed as a nought plus a1 cos omega nought t a2 cos 2 omega nought t like that a general term will be a n cos n omega nought t and so, on up to an infinite number of terms plus sine terms sine omega nought t b2 sine 2 omega nought t a general term b n sine n omega nought t etcetera.

More compactly this can be written as a nought a n cos n omega nought t n ranging from 1 to infinity plus b n sine omega nought t n ranging from 1 to infinity (noise). So, we have a whole infinite set of terms whole lot of them. It is also sometimes convenient to combine a cos term and a sine term together put them together because after all a cosine theta and sine theta terms can be combined and therefore, we can put them together and have an alternative notation alternative way of expressing this where when you combine a and cos n omega nought t and v

And sine n omega nought t combine them together the amplitude of the resultant will of course, be square root of a n square plus b n square and that we call 2 c n why this 2 comes we will explain in a moment: the angle here is r tangent of minus b m over a m.

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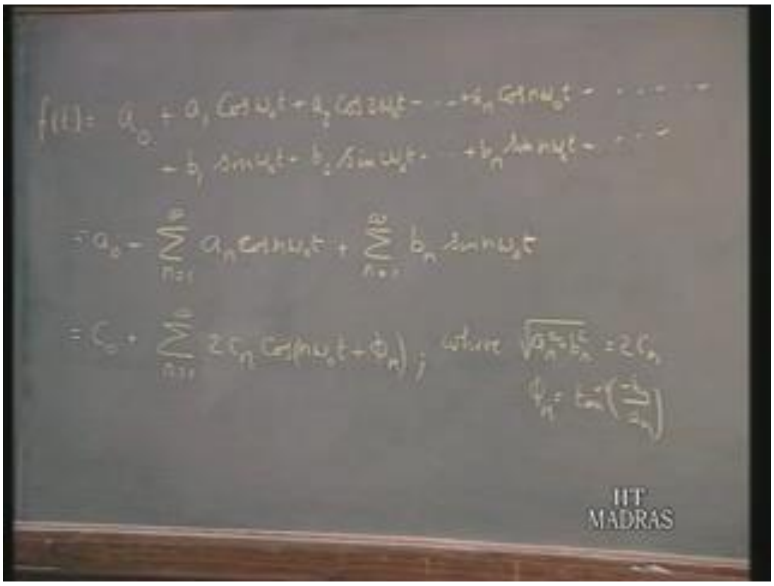

We would like to use c nought in 2 c n because later on we will see another way of expressing Fourier Series in terms of exponential functions at that time we will use the coefficients of the exponential functions as c’s and c’s that you get there we would like to relate to these 2 c’s and if you express this as 2c and instead of cn, it will be found convenient in the context of expressing this in terms of exponential functions.

So, you will see why we use 2cn, when we come to that. Now let us see, what it means; F of t contains the import of the Fourier Series is that a periodic function f of t can be expressed as a sum of a constant quantity plus sinusoidal terms of various frequencies. What are the frequencies present, omega nought2 omega nought3 omega nought4 omega nought and so, on they are all the integral multiples or the basic frequency omega nought.

So, they are called harmonic frequencies a nought which is also equal to nought is called the d c component because it is a steady value d c component of the signal. a1 cos omega nought t plus b 1 sine omega nought t which of course, is 2c1 cos omega nought t plus p 1 is called the; fundamental component and a general term an cos n omega nought t plus bn sine n omega nought t which will be 2cn cos n omega nought t plus p n is called the n'th harmonic component.

So, you have fundamental second harmonic, third harmonic, fourth harmonic and so, on an infinite number of harmonic components and since, we are calling this n'th harmonic component you may as well also call this first harmonic component, but that is normally we refer to this as the fundamental component, the fact that any arbitrary periodic function of time like this can be expressed as sum of sinusoidal components like this is an aspect which when fourier introduced is a series gave rise to some kind of controversy.

As a historical note Fourier is a french mathematician cum administrator, the french probably pronounce his name as Fouier, but in common with the usage in English speaking countries we refer to them as Fourier. So, Fourier when he introduced this notation many eminent mathematicians of his day apparently were not convinced of the validity of this their argument is this fourier said this could be extended to functions like this as well.

So, when you have functions of time which are discontinuous exhibit how can you express this; in terms of functions which are continuous and smooth all sine functions have infinite derivatives, first derivative, second derivative on had infinite terms. So, how can the sum of smooth functions like this which are continuous and which have continuous derivatives how can they add up to a function which exhibits discontinuities.

So, this is the problem which had to be resolved and because of this when Fourier submitted a paper based up on his work apparently it could not be accepted in a scientific journal because 1 very eminent mathematician, who was a referee seriously objected to it, in the event Fourier finally, published his work in the form of a book and as they say the rest is history because, the Fourier methods, today find a central role in many aspects of analysis in, science and engineering and recorded Fourier methods is 1 of diverse and growing applications in various branches of science and engineering.

Now, at this stage, we have to perhaps a few doubts come up in our minds 1 is what type do all periodic functions have Fourier Series does every periodic function expressible or can it be resolved in terms of a d c and harmonically related sinusoidal functions. Second question is; if we have an infinite terms to grapple with will it be of any great use to us.

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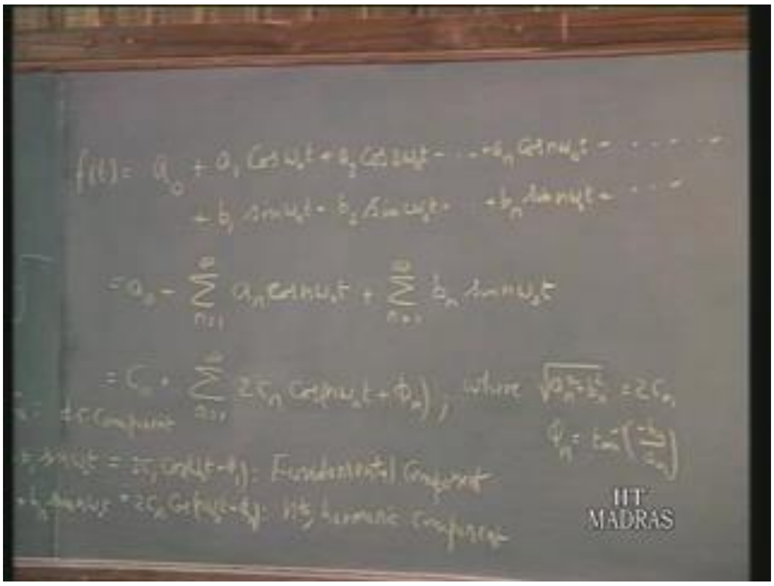

Thirdly, how do we find these a and b or c coefficients as far as the first question is concerned the necessary and sufficient conditions for the existence of Fourier Series is a problem which mathematicians are still wrestling with certain sufficiency conditions are known, necessary conditions are known, but a complete set is not known, but as far as we are concerned, all types of functions that, we are likely to encounter all periodic functions, we can safely say that they admit the Fourier Series description.

So, we need not be bothered about this and can proceed with a clean conscience in our further work as far as the question of there being an infinite number of terms, we can say that i mean; essentially the idea of Fourier Series is that, if the input function excitation can be expressed as number of sinusoids we can find the response to each 1 of the sinusoids and superpose all the responses to get the total response.

That is the basic idea of the Fourier Series as far as circuit analysis is concerned or system analysis is concerned a given composite function periodic function can be resolved in terms of various sinusoids and we know, if there is a sinusoidal driving force we can find out easily the steady state response and therefore, we have a large number of terms. For each sinusoid, we find the response and add all these responses to get the total response.

So, it is in this context that, the question that arises, if we have an infinite number of terms how are we going to grapple with this the answer to this is; that in general we can develop a expression a formula for the n’th harmonic response and you have a series which in many cases can be combined and we can obtain a closed form expression, but even otherwise it turns out that; for most of the common functions that we are likely to deal with the magnitude the amplitude of the n’th harmonic term goes down decays and increases progressively it decreases.

That means; beyond a certain n then the Fourier the various harmonic components will be almost negligible. So, it is permissible for us to truncate the series at certain point depending upon, the accuracy which we would like to have may be when n equals 5 6 may be 29 whatever, it is and then stop at that stage and take only those 2 terms those terms and if you have to calculate the value of the response numerically for a particular value of time you can take the truncated series and the add them up.

The third aspect of how we calculate a and b coefficients which we will take up next, but before that let me, illustrate this how a given periodic function is resolved into various harmonic components and how as we go on adding up the various harmonic components the series the truncated series comes closer and closer to the given f of t, i will refer to this figure here.

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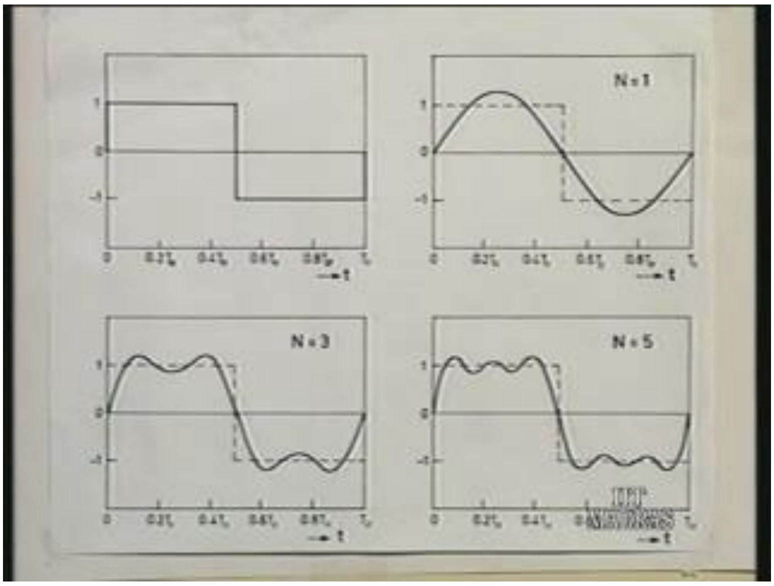

Now, this is a periodic function which is a square wave which is depicted here for 1 period. Now, if you take in this particular case the d c term is offset because the average of this function is 0 this is the fundamental component n equals 1 that is a pure sine wave of course, If you take the third harmonic also into account and add the fundamental and third harmonic this heat becomes this.

This is certainly as you can see closer to this the next harmonic that is present in this particular wave form is the fifth harmonic. So, if you add the fifth harmonic this is the wave that results which is very close to this. Now, if you want to go further we will go to this next chart if you go up to the forty ninth harmonic you observe that it is almost close to this square wave. So, a very small error exists difference exists between the square wave and the series truncated at n equals 49.

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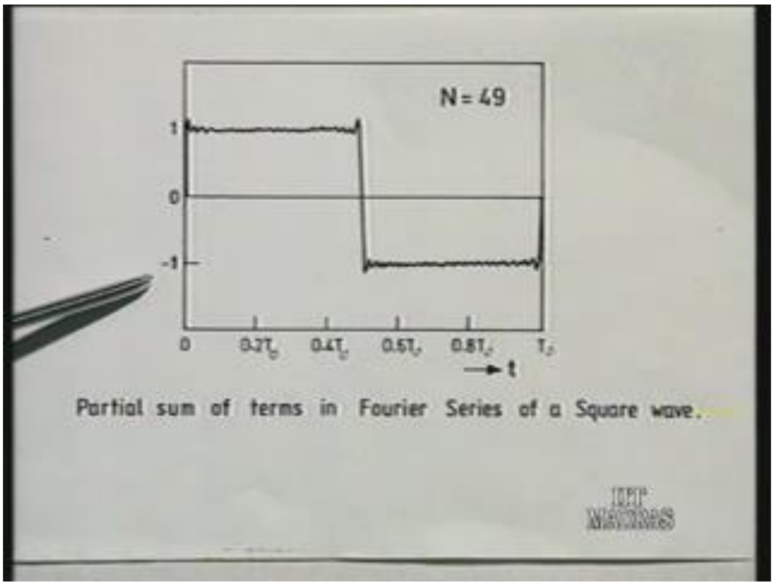

So, you can see as you go on increasing the terms then the series the truncated series approach is the original f of t very closely, i would like to show another figure which shows how different harmonic components add up to build a composite wave.

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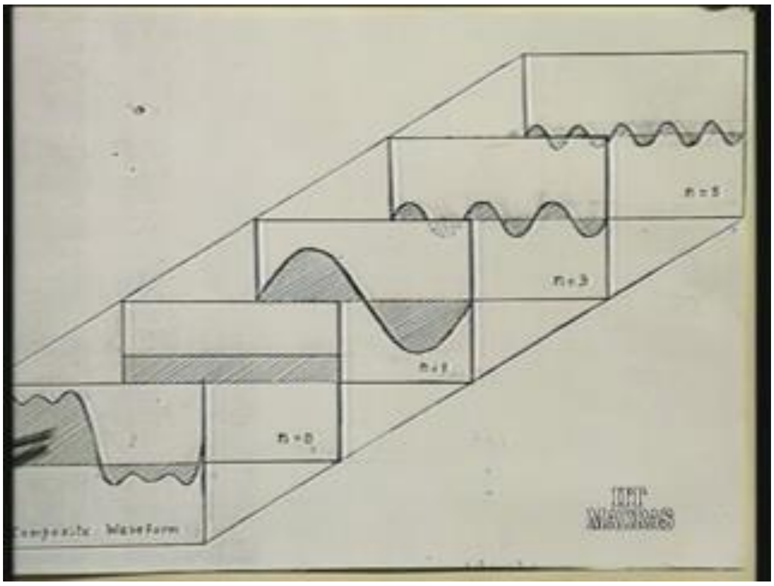

Suppose, this is a composite wave form that, we would like to have in this case there is d c term there's a fundamental there's a third harmonic and there's a fifth harmonic. The composite wave that, i have drawn here is 1 which is built from this. Therefore it really means that; there is only 4 terms in this Fourier Series expansion and the 4 terms in the

Fourier Series expansion are the d c term the fundamental the third harmonic and the fifth harmonic you add up all these curves 1 2 3 4 it will add up to 6. Now let us, get back to the main discussion and see how we can calculate the various Fourier coefficients, before we set up expressions for the evaluation of the Fourier coefficients we need some background information.

We have set up a series of integrals and show their values and we assume that, all these integrals the range of integration in the following integrals is for 1 full period starting from any arbitrary instant that is say t 1 to t 1 plus t nought. Now over such an integral such a range we know that, cos n omega nought t d t over 1 full period goes to 0 because in this period 1 period of fundamental you have n complete cycles of the cosine wave there will be as many positive loops or negative loops.

So, the average will become 0. Similarly, we also have for example, this could be t1 for t1 plus t nought it could be from 0 to t nought as well similarly, sine n omega t nought t will also we likewise be 0. Now, if i have a product of a sine function and a cosine function sine m omega nought t cos n omega nought t d t this, can be expressed as half of over the same range sine m plus n omega nought t plus sine m minus n omega nought t d t again each of these terms will produce a 0 integral because integration is over a complete number of cycles integral number of cycles.

So, both these terms go to 0. So, this will be 0 for all m and n. On the other hand, if i take the product of 2 sine terms sine m omega nought t sine n omega nought t d t this will be half of the integral of cosine m minus n omega nought t minus cosine m plus n omega nought t d t. Now as far as m plus n omega nought t is concerned that again is an integral over a complete number of cycles that goes to 0 no problem over that as for the first term is concerned.

If m minus n is an integer which is not 0 then once again it goes to 0. On the other hand if m equals n this is the constant and therefore, over the complete period it gives the value t plus, it gives the value t nought. So, the upshot is we can say this will be 0, if m is not equal to n and this is equal to t nought by 2 if m equals n.

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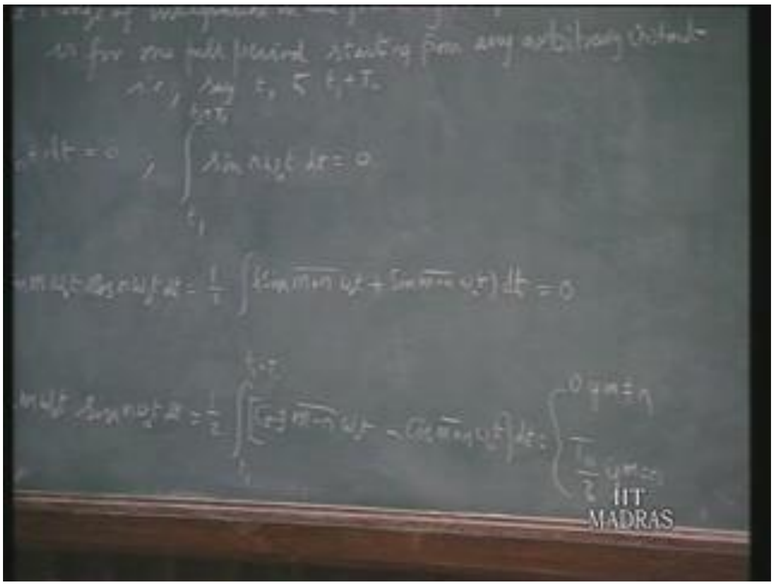

In the same manner, we can also write over a complete period cos m omega nought t cos n omega nought t d t is 0, if m is not equal to n equals t nought by 2 say t1 to t1 minus t nought t nought by 2 if m is equal to n.

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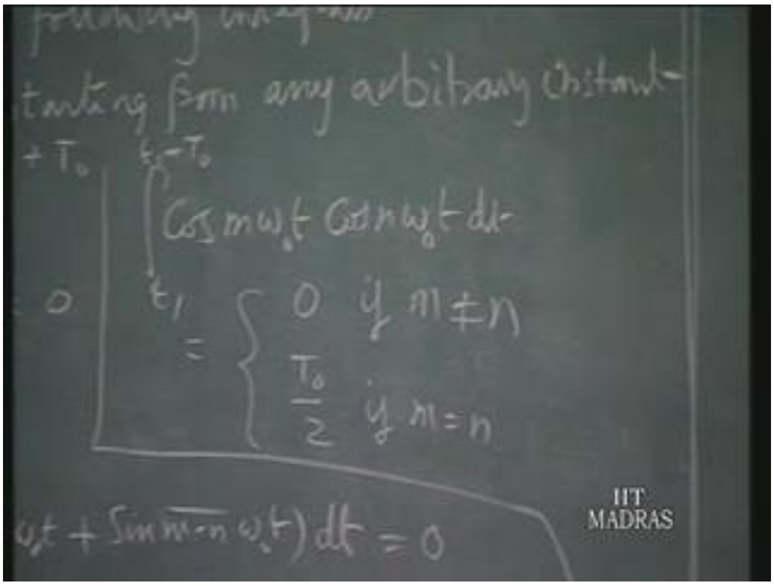

So, these are the background results we need for setting up formulas for evaluating the Fourier coefficients you recall that, the formula for we said that, f of t can be expanded as a nought plus a n cos n omega nought t plus b n sine n omega nought t suppose, you multiply both sides no let us, first find out a nought. Suppose, you integrate both sides and assume that as for the right hand side is concerned you can carry out the integration term by term. Then we have f of t d t, once again the integration can be from t nought t1 to t1 plus t nought or for the sake of convenience let me, take it as 0 to t nought it could be form any arbitrary small t 1 to small t 1 plus t nought.

Now, similarly on this side if you carry out the integration for each 1 of these terms by virtue of the results that, we already established each 1 of them will have f 0 average value over a period fundamental period. Each 1 of these terms will have a 0 average value over a fundamental period. So, it is only this term which give rise to a contribution therefore, i can write this as 0 to t nought a nought d t plus terms which become 0 as a result we have this is a nought into t nought sorry a nought times t nought.

This immediately gives us a way of evaluating a nought you have a nought equals 1 over t nought f t d t say 0 to t nought or t 1 to t 1 plus t nought likewise, if i multiply the left hand side by say cos n omega nought t and then integrate the product f t cos n omega nought t over, a complete period each 1 of these terms must be multiplied by cos n omega nought t and integrated over a complete period. Obviously, a nought multiplied by cos n omega nought t over a complete period will vanish and every 1 of these terms will also vanish except for the term which involves index n because we have seen earlier that, if you take the product cos m omega nought t cos n omega nought t and integrate this will yield non 0 only if m equals n.

Therefore, if you are multiplying this whole set of terms by cos n omega nought t only when the index here happens to be n it will give rise to a non 0 product.

Therefore i can write this therefore, 0 to t nought a n cos n omega nought t multiplied by cos n omega nought t d t plus other terms which go to 0 other terms which are 0 .

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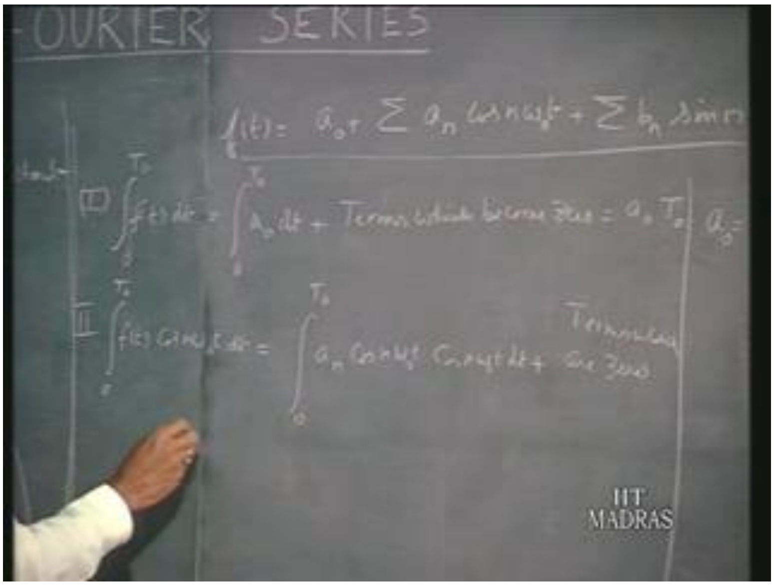

So, this will now be a n times t nought by 2 because when both these indices agree this is t nought by 2 a n t nought by 2. So, you can write this as a n equals 2 up on t nought 0 to t nought f of t cos n omega nought t d t similarly, it can be shown that by multiplying this by sine n omega nought t d t you get b n times t nought over 2.

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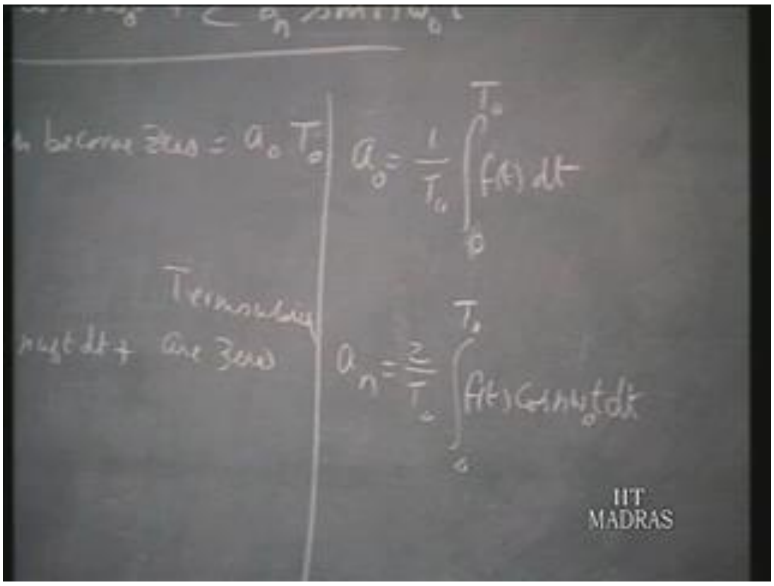

So, b n will be 2 up on t nought 0 to t nought f of t sine n omega nought t d t. So, these are 3 formulas which can be used to evaluate these various integrals.

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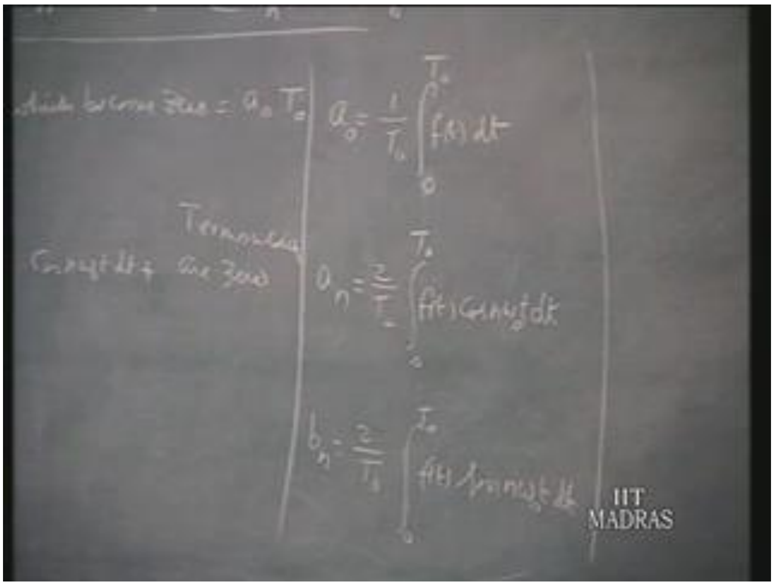

It is easier to remember these as saying a nought equals the average value of f of t. Whenever, I am talking about average, it is average over 1 complete period and a n is 2 times the average value of the product f of t cos n omega nought t and likewise b n is 2 times the average value of f of t sine n omega nought t. So, these are the 3 integrals that 1 makes use of to evaluate the various Fourier coefficients.

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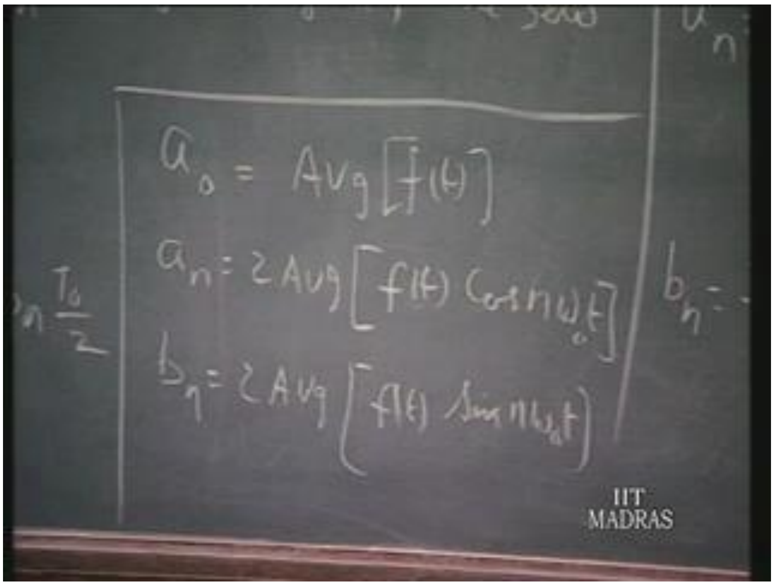

It easy to remember this and this form or certainly these are equivalent to this. Now, as far as the integration is concerned, if you have got n little expressions for f of t you can carry them out analytically or you can also do them numerically, if f of t is given for example, in a graphical form a wave form is recorded and you would like to perform the Fourier series analysis of that, then you can carry out these integrations in by numerical methods.

To sum up them in this lecture, we started with the system function of a linear time invariant system h of s and how it relates the output and input and then we saw that, if the input is a sinusoidal function of time, how the output can be easily computed, by multiplying the input phasor with h of j omega which is a system function under sinusoidal conditions and obtain the output phasor.

This advantage of phasor notation and the associated analysis of the sinusoidal inputs we would like to carry over to functions which are not sinusoidal as well. And as a first step we took up how a periodic function can be decomposed into various harmonic components of the basic frequency as well as a d c term and we started with this expression for periodic function in terms of a d c plus various harmonic components and once, we obtain the response to each 1 of these harmonic components we can using these phasor methods, we can obtain the total response by superposing the various responses and

For this purpose, we arrived at the formulas for the d c component coefficient for the cosine terms and the coefficient for the sine terms and these are the 3 expressions which we would like to remember in calculating this a and b coefficients.

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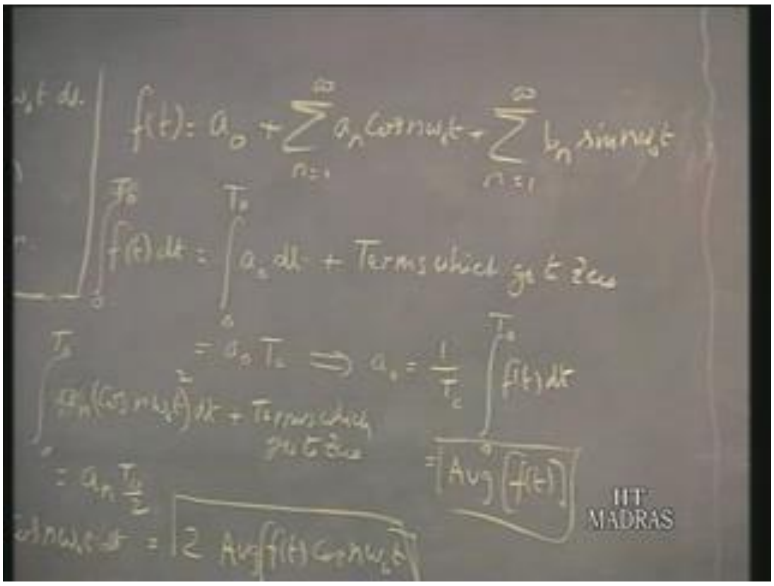

In particular, the d c the average value of f of t over a complete period. A n is twice the average value of the product of f of t at cos n omega nought t and b n is twice the average of f of t sine n omega nought t.

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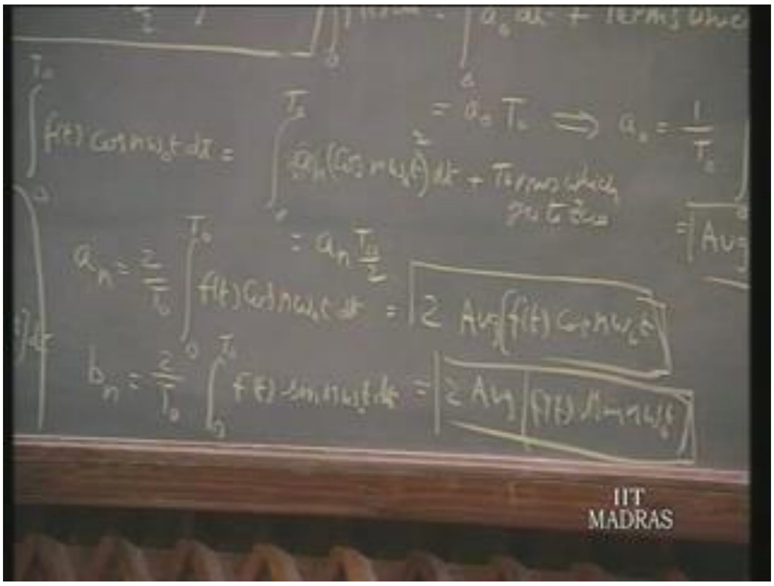

For every function that, we have we do not have to really calculate a nought a n and b n it turns out that, there are symmetry conditions which enable us to guess even before hand, what parameters what coefficients are present and what are absent, these come under the symmetry conditions the topic, we will take up in the next lecture.