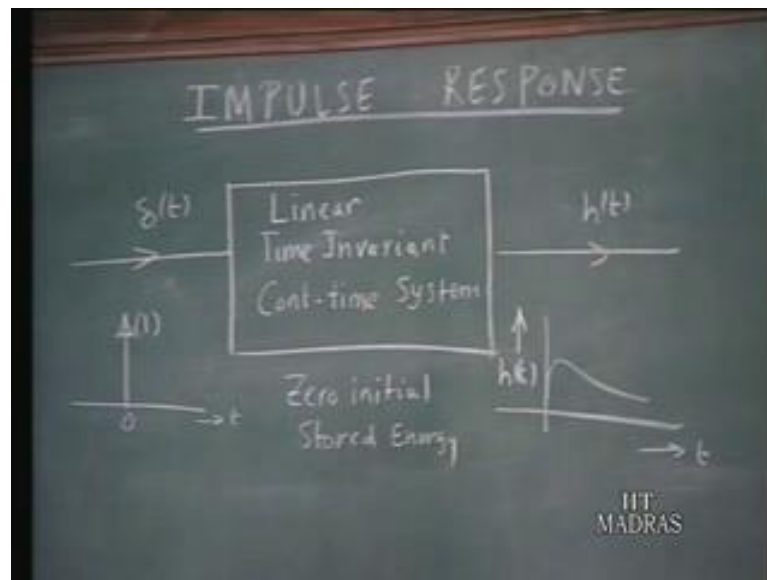


Networks and Systems
Prof V.G K.Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 06
Introductory Concepts 6

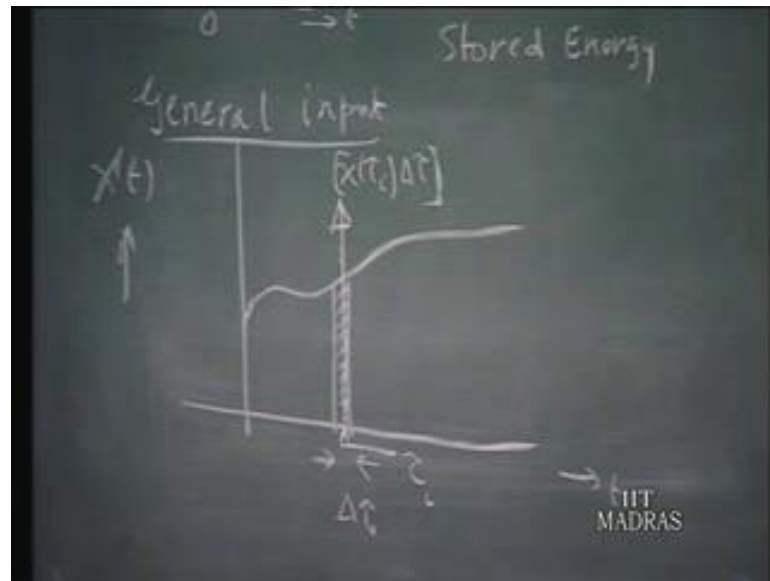
To a unit impulse will give all the information that is needed for us, to find out the response to any arbitrary input. Let us see how we go about it. This is a linear time invariant continuous-time system.

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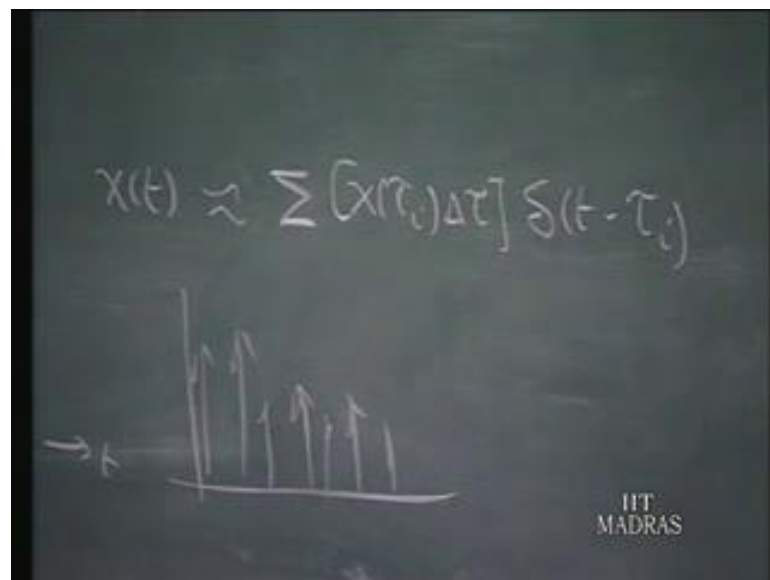
It is represented as a box like this. Suppose the input is $\delta(t)$; a unit impulse at the origin and let the corresponding response $h(t)$ is given like this. We also assume in this discussion that, if this is a network, it has no initial conditions on the capacitors and inductors or in other words in the system 0 initial stored energy. The reason is, we are going to apply the principle of superposition and whenever number of excitations are present simultaneously; the responses could be superpose only if, the system does not have any 0 any initial stored energy. Otherwise, the superposition principle is violated.

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Now, any arbitrary except t given as an input something like this; this is the general input, can always be thought of; suppose, you take a small narrow section of width Δt and then let this, at the center we will call that t_i .

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This point t_i and this section of the input; a narrow section of time, this area can be replaced by an equivalent impulse. So, if you regard this, the impulse will have essentially the same area as this narrow strip and the area of the narrow strip will be x of

$t_0 y$ times Δt . So, this will be $x(t_0) y \Delta t$ that is, the magnitude of the impulse and that impulse is situated at $t = t_0$.

So, provided you make these slices thinner and thinner, then the approximation will be so, much more accurate. So, $x(t)$ can be regarded as: approximated as number of impulses. A particular impulse which we have sketched in the figure is $x(t_0) y \Delta t$ that is, the area under the curve, that is the magnitude of the impulse and that impulse is situated at t_0 . So, this is the impulse that we are talking about. This section of the input is replaced, by an impulse and several such impulses starting from for the whole duration of the time, will be approximately will be equal to $x(t)$.

So, $x(t)$ can be regarded as summation of several impulses. So; that means, your $x(t)$ is now being regarded as several impulses. So, if we want to find out the response to this arbitrary input, we can find out the response due to the individual impulses and add them up, that will be your total response. And in the limit as Δt goes to 0 then, the response will be faithfully the 1 that will be obtained when, the actual $x(t)$ is present. That is the philosophy; that is the strategy that we adopt. Because we know the impulse response; an impulse at the origin gives rise to $h(t)$.

So, any impulse displaced by some amount will also give rise to an impulse response which, is displaced by the same amount, translated by the same amount and by additivity property; if unit impulse gives $h(t)$, an impulse of this magnitude will be this times $h(t)$.

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The image shows a chalkboard with the following handwritten text:

$$\delta(t) \rightarrow h(t)$$

Time Invariant Property

$$\delta(t - \tau_i) \rightarrow h(t - \tau_i)$$

$$[x(\tau_i) \Delta t] \delta(t - \tau_i) \rightarrow [x(\tau_i) \Delta t] h(t - \tau_i)$$

$$\lim_{\Delta t \rightarrow 0} \sum x(\tau_i) \Delta t \delta(t - \tau_i) \rightarrow \lim_{\Delta t \rightarrow 0} \sum x(\tau_i) \Delta t h(t - \tau_i)$$

↓

$$X(t)$$

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So, that is the principle that we are going to adopt.

So now, we now say delta t gives rise to a response h of t. Therefore, any impulse which is displaced by a certain time interval will, give rise to h of t minus tau i. What is the principle which sanctions this? This is a time invariant property. Because, the network is time invariant, any shift in the input will correspond to the corresponding shift in the response. That is what we discussed earlier. And instead of an impulse of unit magnitude; suppose you have x tau y delta tau that is, the magnitude of the impulse delta t minus tau i. If, that is the input you get correspondingly x tau i delta tau h of t minus tau i.

So, a shifted impulse will give rise to shifted impulse response and an impulse magnitude increase by this amount will also; give rise to a response which is increased by the same amount. And now here we have summation of all such things; x tau i delta tau delta t minus tau i summation, limit delta tau goes to 0 that is your x of t and that correspondingly give me limit as delta tau goes to 0 of x tau i delta tau h of t minus tau i not 1 such response number of such responses, some done in various tau i's or i will put some done in various i's.

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So, in the limit, this entire thing goes to x of t . So, as you make Δt go to 0, all these impulses is equivalent to the input x of t as we have seen. From x of t we will generate this impulse. You make them closer and closer, approximately that is the same as x of t . So, x of t is regarded as the series of impulses and correspondingly this will now be; you have a number of these things Δt is an integration of x of $t - \tau$ from $-\infty$ to t . So, τ is not discrete points. Continuously it exists Δt $d\tau$, that is your y of t .

So, y of t is obtained as $\int_{-\infty}^t x(\tau) h(t - \tau) d\tau$. That is how we can obtain in the limit, the output y of t corresponding to this x of t and in general, the most general case; this integration must last for minus infinity to plus infinity, but in the more usual cases the limits can be further restricted as we will see at a later point of time. So, in other words given any input x of t ; the output can be obtained as $\int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$ integrated between minus infinity and plus infinity. This is the function of time. τ is the variable of integration that gets cancelled out, final result will be a function of time. So, this t stays and that is your y of t . And this particular type of formation is usually represented as h of t convolved with x of t . This is referred to as convolution integral.

So, if you convolve 2 functions of time; x of t and h of t the meaning is; this particular integration: $\int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$. So, let me rewrite this. We will come back to the impulse response and the calculation of y of t , but let us spend a few minutes on the meaning of convolution integral.

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The image shows a chalkboard with the following handwritten text:

$$f(t) * g(t)$$
$$= \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$$
$$= \int_{-\infty}^{\infty} g(\tau) f(t-\tau) d\tau$$

In the bottom right corner, there is a logo for IIT MADRAS.

So, convolution integral of 2 quantities; $f(t)$ convolution of $f(t)$ and $g(t)$ represented in this manner is $f(\tau) g(t-\tau) d\tau$. This is a symmetrical arrangement. We can also write this as: $g(\tau) f(t-\tau)$. Normally the integration is between minus infinity and plus infinity. This is called the convolution integral sometimes called Faltung Integral.

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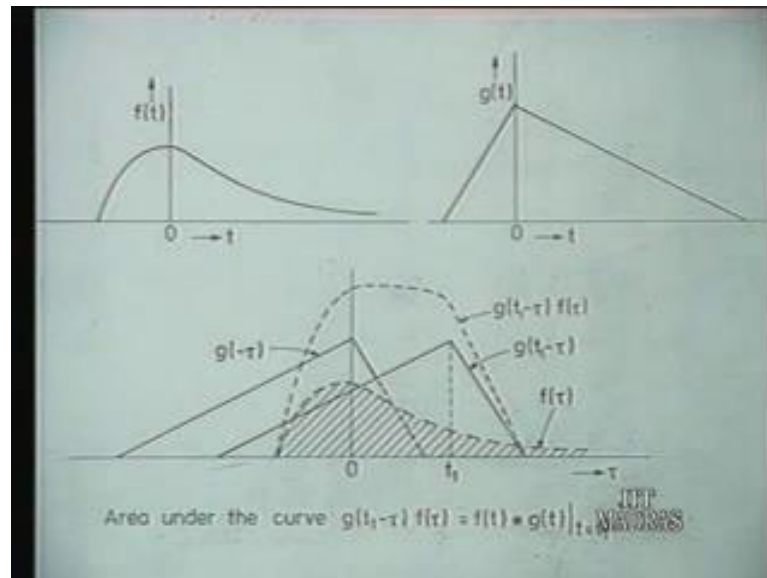
$$= \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$$
$$= \int_{-\infty}^{\infty} g(\tau) f(t-\tau) d\tau$$

Below the equations, the text "Faltung Integral" is written. To the left of this text, the word "equal" is written and underlined. In the bottom left corner, there is a partial expression $) * h(t)$. In the bottom right corner, there is a logo for IIT MADRAS.

That is the German name is Faltung Integral. So, what is involved in this integration?

It is instructive to see, physically how you interpret this integration; this convolution principle. So, let me illustrate this by means of this picture here.

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You have 2 time functions f of t and g of t . You would like to find the convolution of these 2. What you do is; you keep 1 of these f t fixed. So, let us say this is f t . This is fixed. And this g of t suppose, you reverse this time axis that means: g of t is like this. This, what we are having here is g of minus t . This is g of minus t that means, you fold it, g of t is folded along the y axis. So, the sequence of values which it takes for positive t will now, take for negative t and vice versa. So, you fold it along the y axis. This is g of minus t .

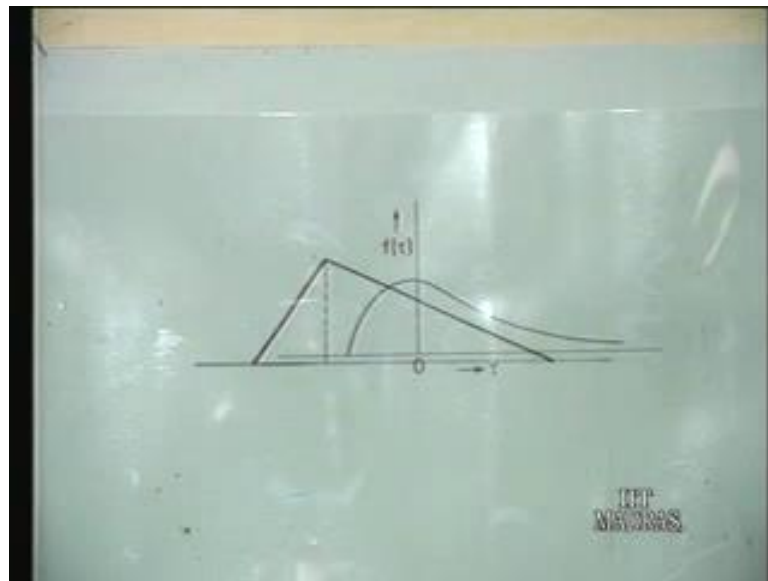
And then, suppose you want to find out the convolution integral for particular value say t_1 . Then, this g of minus t you advance it in the forward direction by an amount equal to t_1 . That mean, this is g of minus t , you advance it by an amount t_1 . This becomes g of minus of t minus t_1 that is, t of t_1 minus t . That means you shift that, you are having this. Now, we have the product of in this convolution here, f t g t minus t .

Therefore, what you are doing now is, this original f of x that you are having or f of t that you are having and this shifted g of t , you multiply them out and this will be your result. The product of these 2 functions will be this and you take the area under that curve that is the integration that, will give the convolution value at t equals t_1 . In other words, what we are doing is; you take 1 time function, keep 1 time function as it is. The

other time function you fold it then shift it, by the required amount then, multiply then take the area under the curve that you integrate.

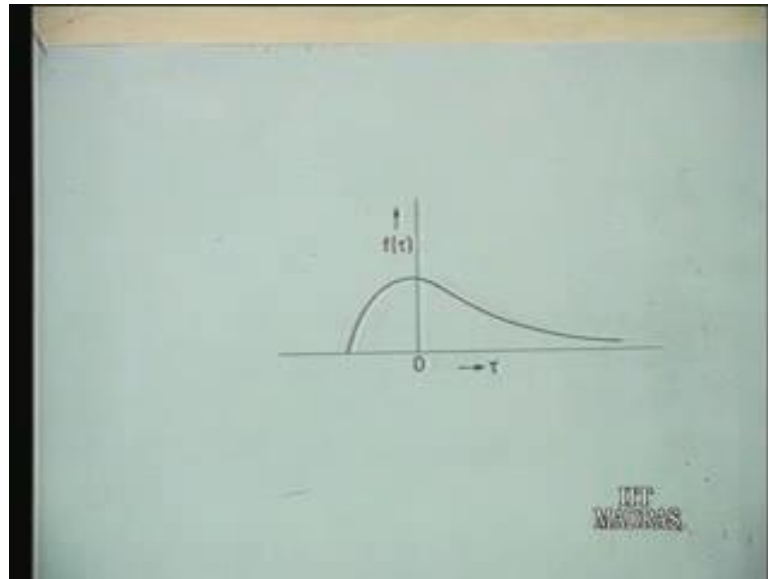
So, the keywords are: fold, shift, multiply and integrate. These are the 4 words keywords that are involved here. This will be explained a little clearly more clearly perhaps, by taking this particular thing. Suppose, this is f of t and then you are having a g of t here. This is g of t ; the triangular curve.

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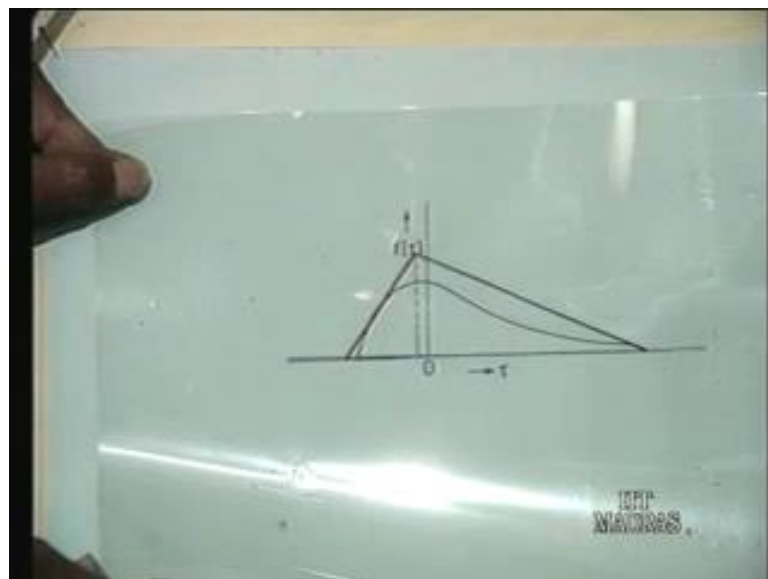
So, what we are doing now is; instead of having like this g of t you shift, you fold it like this. So, g of t originally was like this. Now, you folded it

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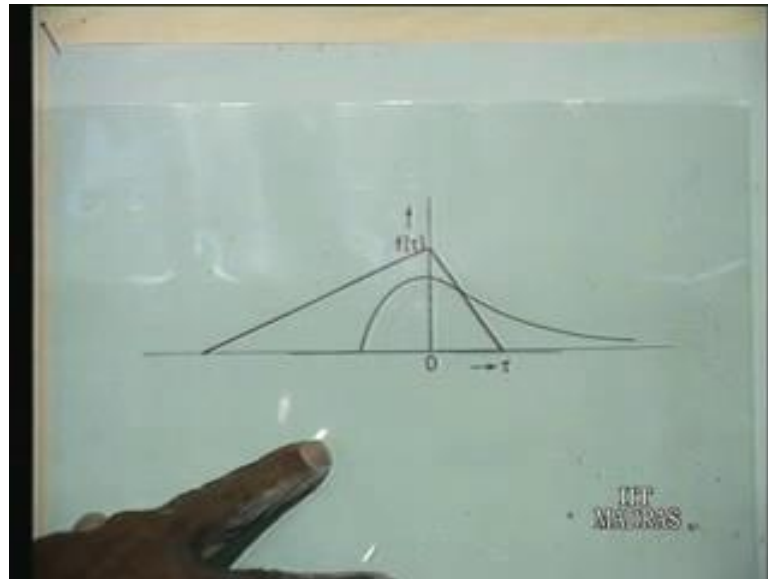
That means you made this run backwards.

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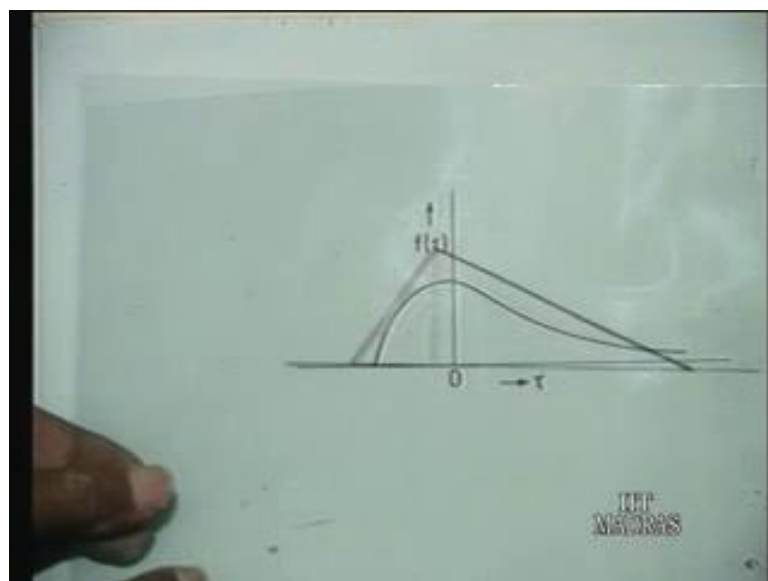


This is now g of minus t . Now, depending up on the point where you want to find the convolution, you fold, you shift it like this.

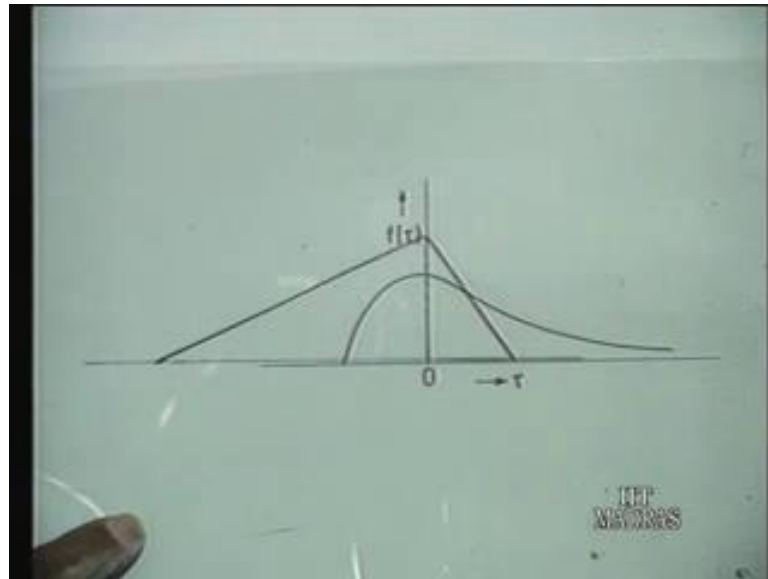
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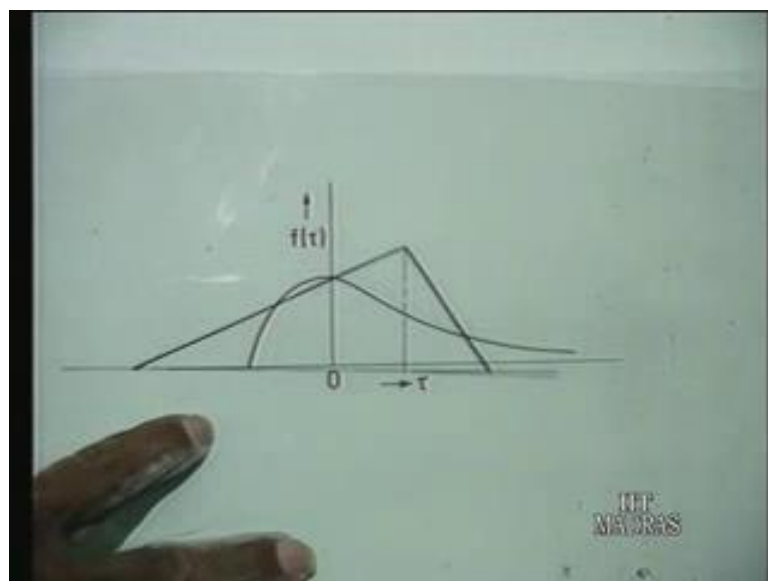
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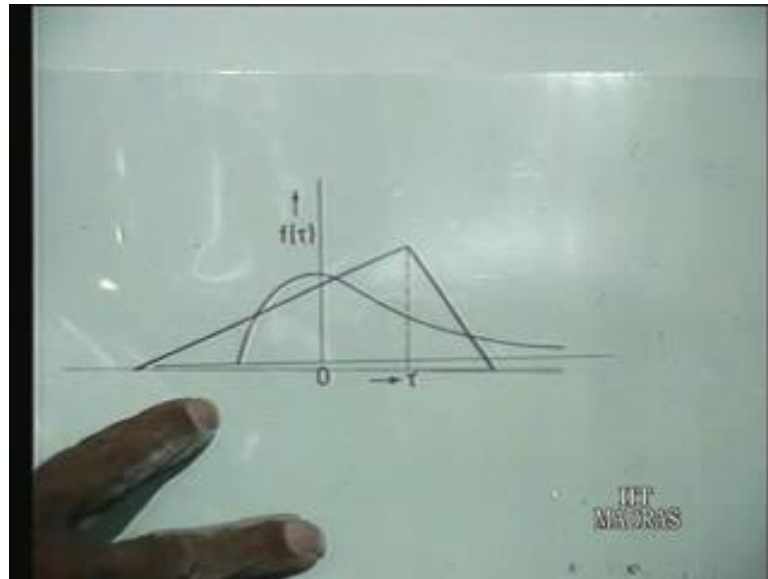
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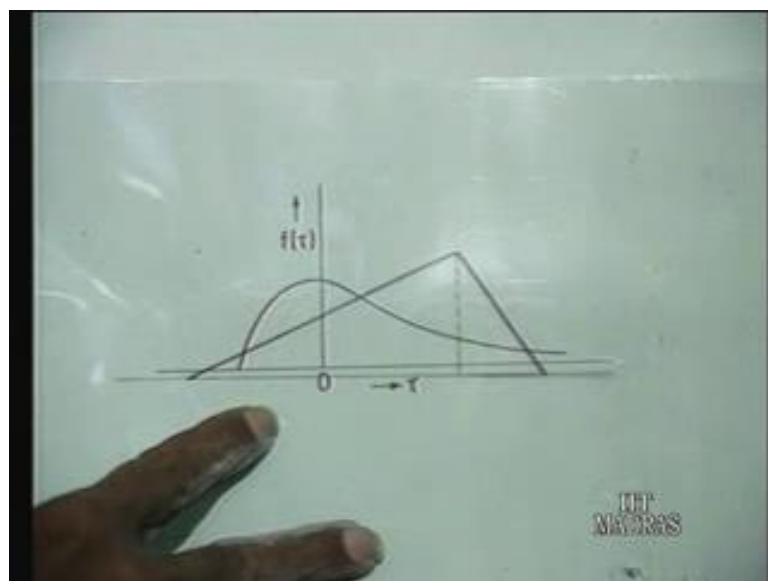
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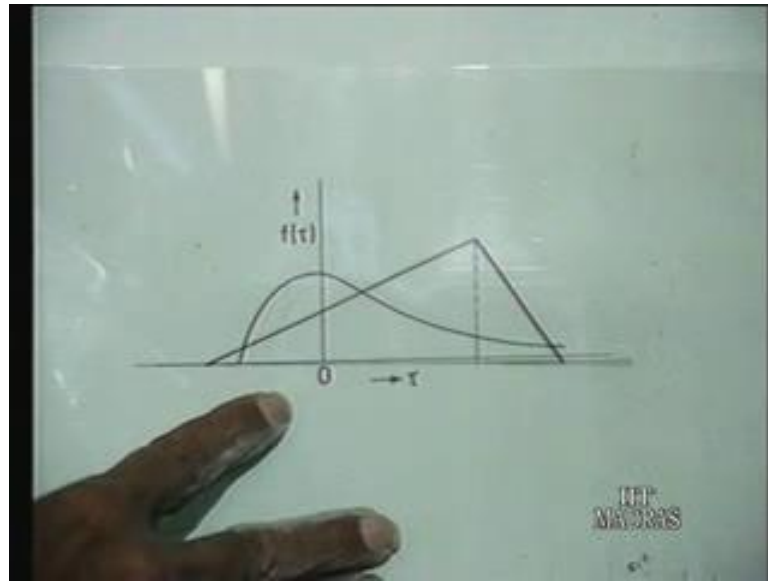
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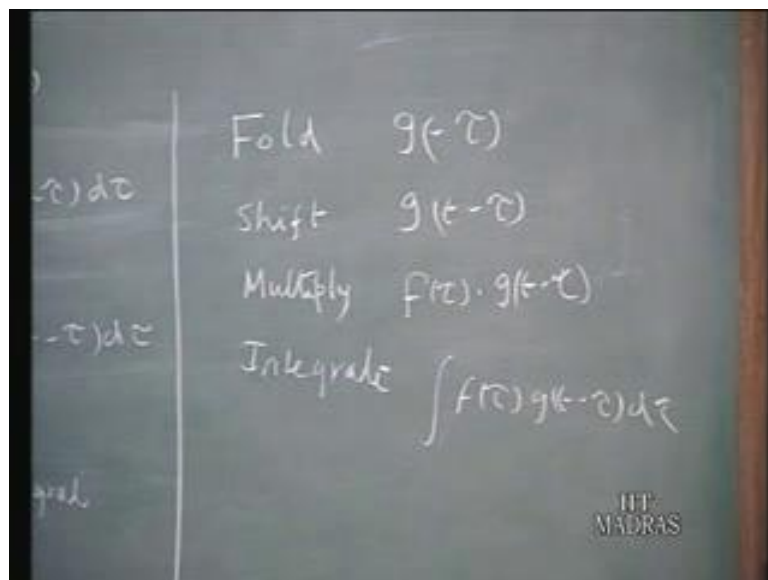


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So, for various shifts you find out the product of these 2 curves and find out the area under that curve. And this is how, 1 can calculate the convolution values, integral values for elementary time products.

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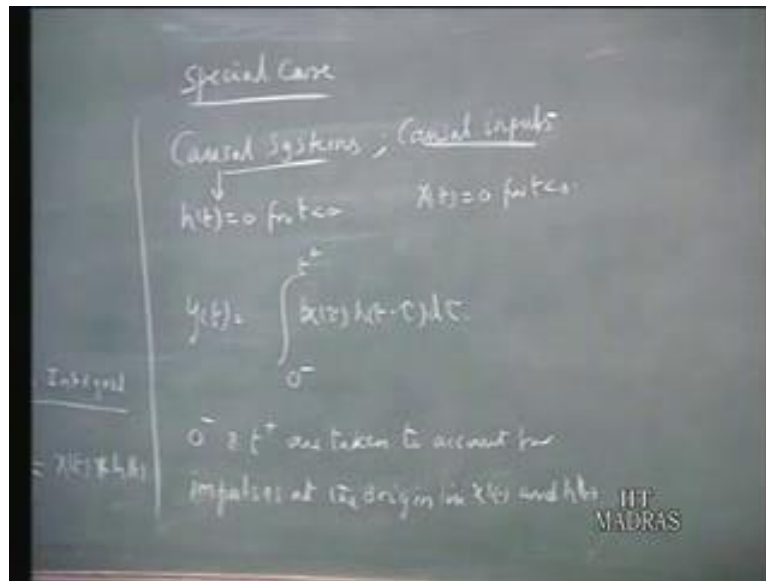


So, the key words here are: fold, shift, multiply and integrate. So, you are having here f of t , is a time function, g of minus t is what is obtained by folding. Shift it; so g of t minus τ . You shift it by the amount at which you want to calculate the integral. At t equals, t_1 you want to find out the product of this and t_1 you shift it by the amount t_1 .

You have general t g of t minus t 1. Multiply; so f t is multiplied by g of t minus t and then integrate. That means you find the area under this curve.

So, these are the steps that are involved. If you keep this at the back of your mind, we can find out the convolution of simple functions quite easily, without going for the mathematics, from physical graphical approach, you can find out the convolution of different functions very easily. Now, let us look at these limits.

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This is the general expression for the response y t for any given x t ou, any given x t . So, how are, when we talk about causal systems; special case of causal systems and causal signals or causal inputs. If you take that then, what are the consequences? If the system is causal then, an impulse x δ t ou given here cannot have an impulse response for negative values of time. That means, h t will be 0 for t less than 0.

So, if the system is causal; the impulse response must be 0 for negative values of time. So, the consequence of this causal system is: h t is 0 for t less than 0. If the input is a causal; causal input; that means, x t is 0 for t less than 0. Because of these 2 restrictions; this impulse convolution integral that you are having here minus infinity to plus infinity, these limits can be now simplified. Because x t ou is going to be 0 for negative values of t ou, we need not start the integration from minus infinity. You can start from 0.

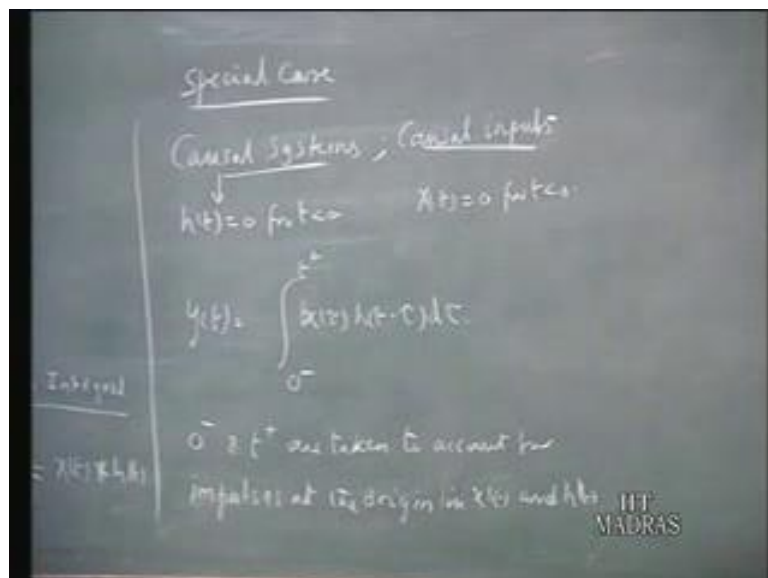
Similarly, τ is a running variable. When τ exceeds t then, this h of t minus τ ; the argument becomes negative.

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Therefore, that is going to be 0. Therefore, this integration needs to be carried out only, for up to τ equals t instead of infinity.

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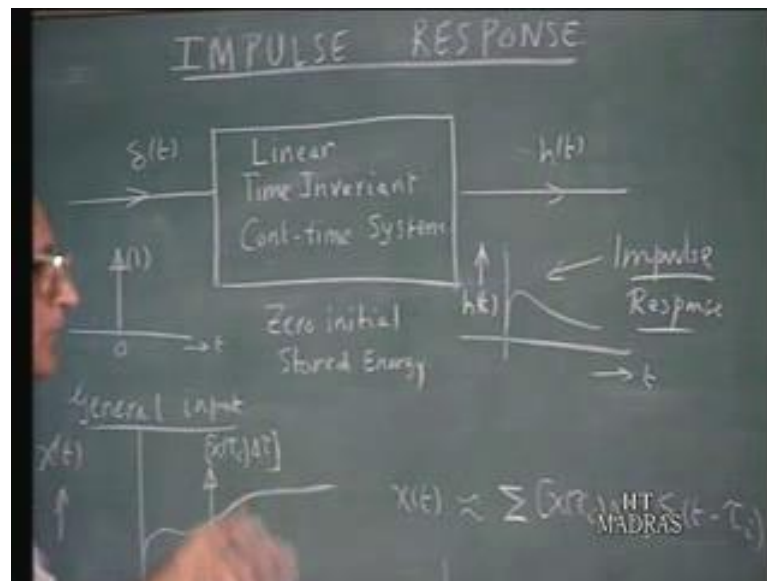
So, this lower limit and upper limit can be reduced now to 0 and t . Then we can write x of τ h of t minus τ . So, for the special case of a causal system with a causal input, we need to carry out this integration only between the limits 0 and t , not between minus infinity and

plus infinity. In particular we, if $x(t)$ for example, may have an impulse at the origin. So, we must take that effect to the impulse to the account. Therefore, it would be advisable for us to start the integration from 0 minus so that, the impulse is fully taken into account in our calculation.

Similarly, the impulse response may have an impulse at the origin. It may be possible that if you have an, you give an impulse then, there may be impulse present in the response also. That means: $h(0)$ may also have an impulse. Therefore, to take that into account you can take this t plus. So, 0 minus and t plus are taken to account for impulses at the origin in, $x(t)$ on the 1 hand and $h(t)$ on the other.

If there are no such impulses then, you can always take 0 and t . But, if you have impulses present it is advisable to take from 0 minus to t plus. That is how it goes.

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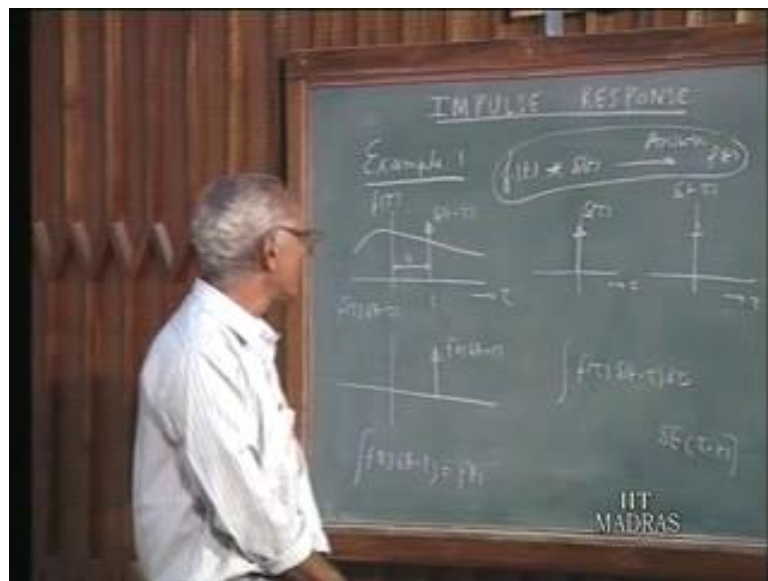


So, to summarize this discussion up to this point is; given the response of a linear time invariant continuous system to an impulse, we describe this $h(t)$; $h(t)$ is called an impulse response. This gives complete information about the system to enable us to, calculate the response to any input. So, any given input $x(t)$ can be regarded as the summation of several impulses, as we have taken like this and each impulse gives rise to corresponding response, which can be calculated in terms of the impulse response. Putting all of them together, we come to the integration; $y(t)$ is given by this integral which is called the convolution integral and the meaning of convolution we have seen.

And in particular when we are talking about causal systems and causal inputs, the limits need not be minus infinity to plus infinity as it would be in a general case. But, in the particular case of these pipe systems from 0 to t, in particular we may have impulses at the origin both in h of t and x of t. To take care of them, you must take the integration from 0 minus to t plus. Otherwise, only half the impulse will be taken into account.

Now, let us work out a few examples to illustrate these ideas.

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Let us take an example to illustrate the idea of convolution. Let us take a general function f of t. Try to convolve it into delta t and see what happens. So, you have 2 time functions and since, the final convolution is a function of time and you are doing some integration, it would be advisable for us to involve; another variable which gets integrated in the process.

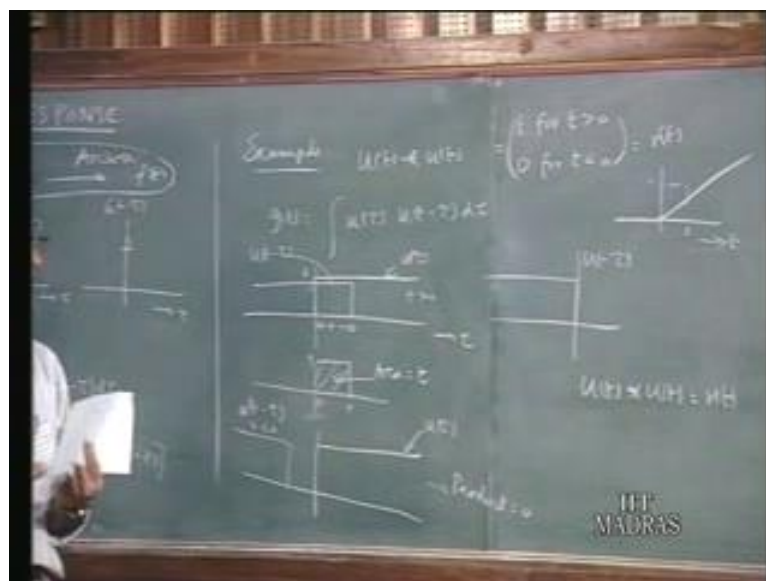
So, let me say that this is tau and this is f of tau. Now, delta tau is this. So, delta minus tau is also the same thing because, it is an even function. So, if you fold it becomes it becomes the same thing delta tau. So; that means, delta tau is now here; delta minus tau and in order to, what is it that we want to do? We want to take f tau delta t minus tau d tau and you want to integrate that. So, instead of delta tau you must take delta t minus tau. That means; you must take delta minus of t minus tau tau minus t. That means; you must advance this delta by an amount equal to t.

So, if you want to calculate the product or the convolution at a value t , you shift it by an amount. So, this will be $\delta(t - \tau)$, where this is the value of t . So, this shift depends upon the particular value t at which you want to evaluate. For 1 second you shift it by 1 second. If, you want to evaluate it at 2 seconds it shift it by 2 seconds and so on and so forth.

Now, you have folded this signal, then shifted this signal, then multiply this 2 out. If you multiply this 2 out, you have $f(\tau)$ multiplied by $\delta(t - \tau)$ what you get is; because δ is 0 everywhere else except at this point, you have a small δ here whose value is; f at the value $t = \tau$. And when you integrate that, $f(\tau) \delta(t - \tau)$ that will be equal to, the area under the curve is $f(t)$. So, the result is that, $f(t)$ convolved with $\delta(t)$ is $f(t)$ itself. Answer is $f(t)$ itself.

This is a very interesting result that; the delta function when it convolves with any time function yields the same time function identically without any change. It is something like what of course, you may not have δ in priority, but if you have a narrow slit for example, instead of a delta function, suppose you are having a small slit like this and you do the same convolution then, you get an output which is very closely resembling this $f(t)$.

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Something like you're for example, the sound track on a film passing through a sensing head or a sound signal on a tape passing through a head. Then if you have a narrow slit

then, you pass it through this and if our response is something like similar to the product and then integration over the small area then, you get more or less essentially the same signal as the output, something similar to that.

Now, let us take a second example. Let us see what happens when, a step function is convolved with another step function. u of t convolved with u of t . So, the result is suppose result if, this is called g of t . According to the formula this will be u of $t - \tau$ that is the second signal $d\tau$ and you integrate it over the limits which are appropriate to this. Let us see what they are.

Now, you take first of all u of τ . This is u of τ . This is 0 here and this is equal to 1. Now, u of $t - \tau$ to take care to picturize this; u of τ is like this, u of $t - \tau$ will be something like this.

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This is u of $t - \tau$. But then, you do not want u of $t - \tau$. You must shift it by an amount equal to τ . u of $t - \tau$ would be something like this. That will be u of $t - \tau$ where, this amount is equal to τ . So, you multiply these 2. This is also equal to 1. This is multiplied by this. You will get this curve 0 to t .

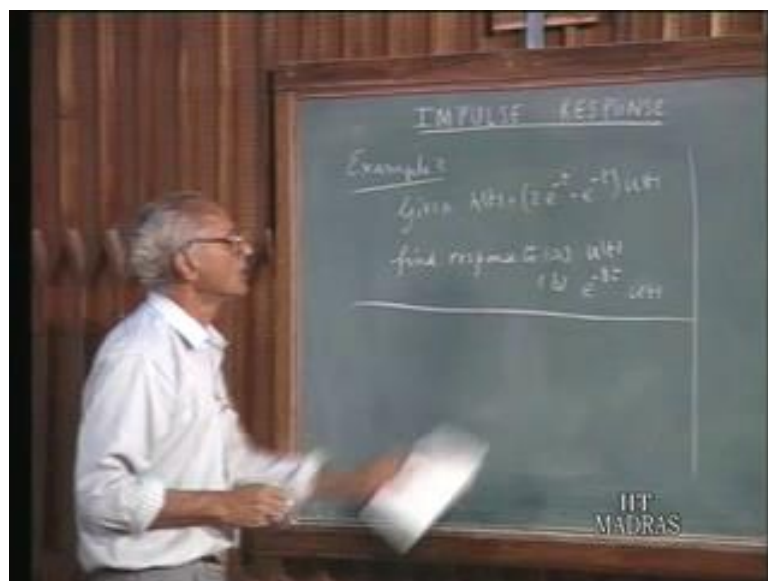
So, the area under this curve, after all when you want to integrate this it means; you are finding out the area under this curve. The area under this curve is t units. Therefore, the result of u of t star u star with u of t or the convolution of that is going to be t for positive t . So,

we can say; this is t for t greater than 0 because we have shifted in the forward direction. Now, let us imagine what happens for negative t . This is t what you have pictured here, is for t greater than 0. What happens when t is less than 0?

So, this is $u(-t)$. This is u of minus t . For negative t you shift it in the other direction. Therefore, you have a curve like this that will be $u(t)$ minus t for t less than 0. So, when you make the product of these 2 it will be 0 because, when this is non 0 the other is 0. When this is non 0 this function is 0. Therefore, the product is 0. Product of $u(t)$ times $u(t - t)$ is going to be 0 identically. Therefore, area is going to be 0. Therefore, the convolution product is 0. So, the product is described by $r(t)$. That is a ramp function. That means; $u(t)$ it will be like this. If it is 1 this is 1.

So, the conclusion is $u(t) * u(t)$ convolved with $u(t)$ equals ramp function. So, the idea of convolution is relatively easily seen in simple cases, by plotting these graphs in this manner.

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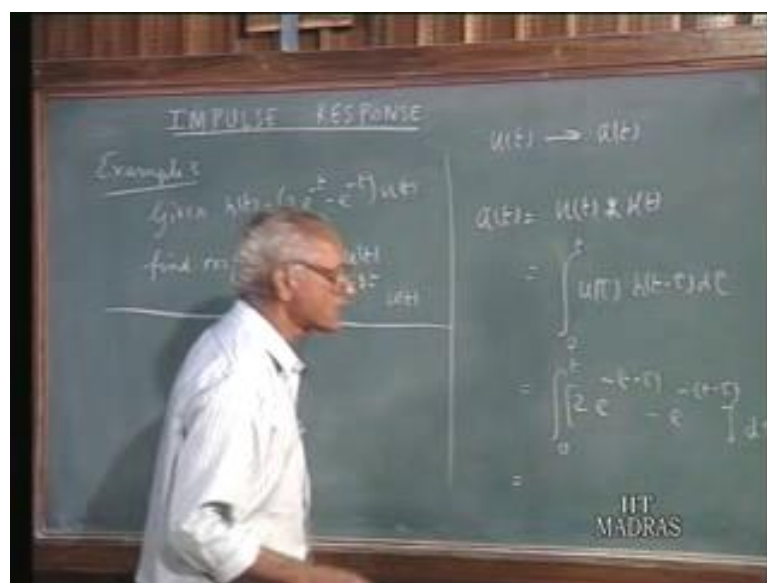


again key words are: fold ,shift, multiply and integrate. Integrate means; finding out the area under the curve. For simple geometries the area to the curve can be easily found out without doing integration and you can picture what is the final result it is going to be using these concepts.

Now, 1 more example. Given: $h(t)$ for a type of system that we are talking about is $2e^{-t}$ for an initially relaxed linear time invariant continuous time system. Find response to a $u(t)$ to the power of minus 3 t . So; that means, we are asked to find out using this impulse response; the response to 2 arbitrary inputs.

Now, let us see the first 1. If $u(t)$ gives rise to a response $a(t)$ then, $a(t)$ can be found out by convolving $u(t)$ with the $h(t)$.

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So, I will write this as; the input now is $u(t)$ and that is convolved with the impulse response $h(t)$. So, the meaning of this would be both now this is a causal system. The impulse response is 0 for negative values of time and the input is also causal. Therefore, we can straight away say this is 0 to t .

So, I will call it $u(t) * h(t)$ minus t . Then, this can be regarded as 0 to t . In the interval 0 to t when t takes from positive values, this is going to be 1 anyway. Therefore, we can disregard this. So, $h(t)$ we can write this $2e^{-t}$ minus t minus t . Again $u(t)$ is there, $u(t)$ minus t is there; $u(t)$ minus t is going to be 1 in the interval from 0 to t because, as long as t is less than t then, t minus t is going to be positive. Therefore, both these $u(t)$ here and $u(t)$ minus t that is coming here are going to be 1 in the range of integration. Therefore they can be omitted.

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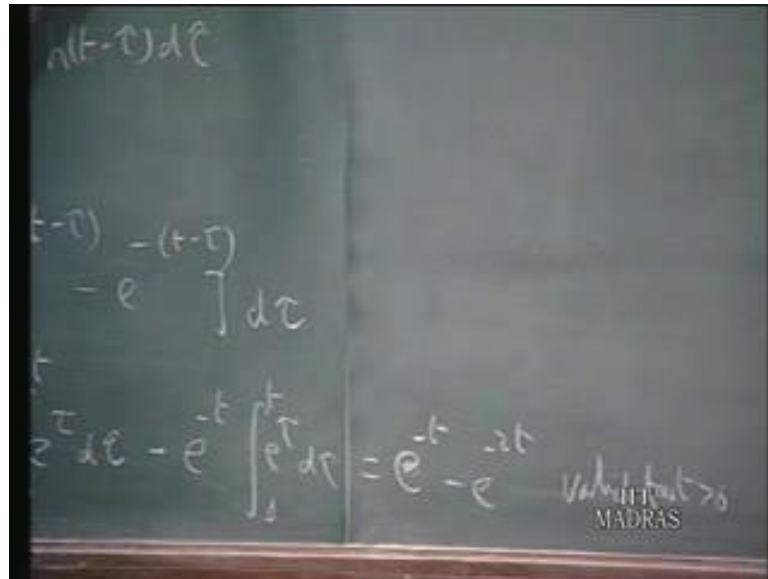
$$\begin{aligned} &= \int_0^t u(\tau) h(t-\tau) d\tau \\ &= \int_0^t \left[z e^{-2\tau} - e^{-(t-\tau)} \right] d\tau \\ &= z e^{-2t} \int_0^t e^{\tau} d\tau - e^{-t} \int_0^t e^{\tau} d\tau \end{aligned}$$

In the bottom right corner of the chalkboard, there is a logo for "IIT MADRAS".

So, this will be what you are having here and this can be written as; u in the integration e to the power of minus t comes out with the first term. Therefore, you will have $2 e$ to the power of minus t and then when you are integrating e to the power of, I should have put this minus $2 t$. Kindly note this. That is what this is minus of 2 of t minus t . Otherwise, there is no meaning here writing $2 t$. 2 minus t minus t .

Therefore, you have $2 e$ to the power of minus $2 t$ and 0 to $t e$ to the power of t $d t$. That is $1 t$ and then minus e to the power of minus t that is, the common factor that comes out.

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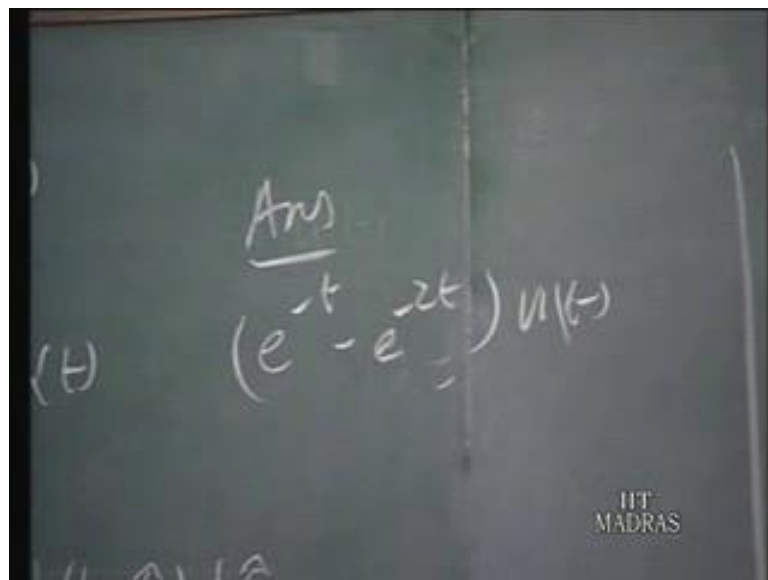
The chalkboard shows the following work:

$$\int_0^t (t-\tau) e^{-\tau} d\tau$$
$$= \int_0^t (t-\tau) e^{-\tau} d\tau$$
$$= e^{-t} \int_0^t e^{\tau} d\tau = e^{-t} (e^{\tau}) \Big|_0^t = e^{-t} (e^t - 1) = e^{-t} e^t - e^{-t} = 1 - e^{-t}$$

There is a watermark in the bottom right corner that reads "IIT MADRAS".

e to the power of tau d tau 0 to t. You can carry out these 2 integrations and final result will be e to the power of minus t and minus e to the power of minus 2t.

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The chalkboard shows the final answer:

Ans

$$(1 - e^{-t}) u(t)$$

There is a watermark in the bottom right corner that reads "IIT MADRAS".

This is valid for t greater than 0. So, a of t is now obtained as; answer is e to the power of minus t minus e to the power of minus 2t. That is the answer as far as the first part is concerned. The second part the y t is now 0 to t again.

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$$\begin{aligned}
 (b) \quad y(t) &= \int_0^t e^{-3\tau} [2e^{-2(t-\tau)} - e^{-(t-\tau)}] d\tau \\
 &= 2e^{-2t} \int_0^t e^{-\tau} d\tau - e^{-t} \int_0^t e^{-2\tau} d\tau \\
 &= -\frac{3}{2} e^{-3t} + 2e^{-2t} - \frac{e^{-t}}{2} \quad \text{for } t > 0 \\
 &= 0 \quad \text{for } t < 0
 \end{aligned}$$

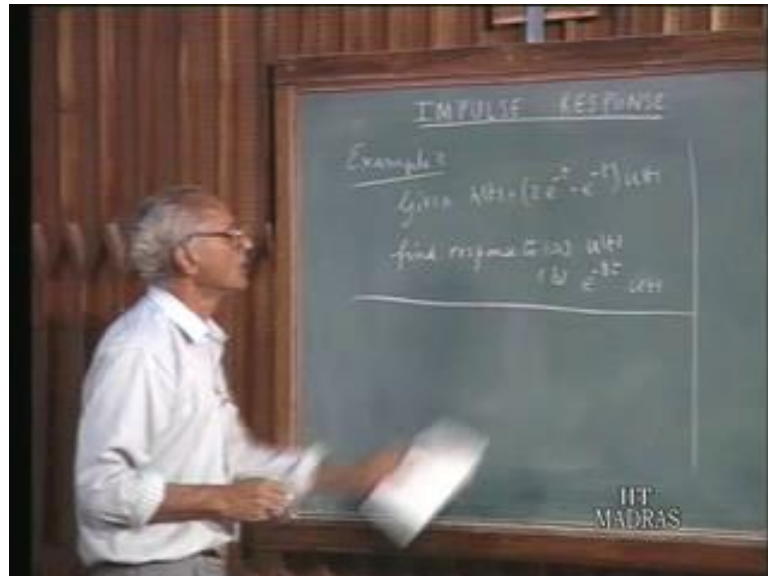
u tou not u tou. This time it is e to the power of minus 3 tou that is, the input. This is going to be 1 for the range of integration. h t minus tou that is 2 e to the power of minus 2t minus tou minus e to the power of minus t minus tou. That is the integration that is to be done and working out as before, this is integration with respect to tou.

Therefore, e to the power of minus 2t can be brought outside; 2 e to the power of minus 2t. Then, 0 to t e to the power of minus 3 tou and e to the power of minus plus 2 tou. You have got e to the power of minus tou d tou and then, for the second part you have e to the power of minus t that can be pulled outside and the integral from 0 to t e to the power of minus 3 tou and e to the power of plus tou. Therefore, e to the power of minus 2 tou d tou is what you are getting. And if you carry out this work the answer will be finally, minus 3 upon 2 e to the power of minus 3t plus 2 e to the power of minus 2t minus e to the power of minus t upon 2 for t greater than 0. Of course, for t less than 0 it is going to be 0.

Notice that in this particular case, we have found out the complete solution. After all the input now is e to the power of minus 3t. That means; the force part of the response will have only this particular complex frequency which is equal to minus 3. So, this is the first part of the response. This is the response which you would obtain by taking the particular integral solution of the differential equation. And this part is the natural response which contains the response when there is no particular input, no sustained

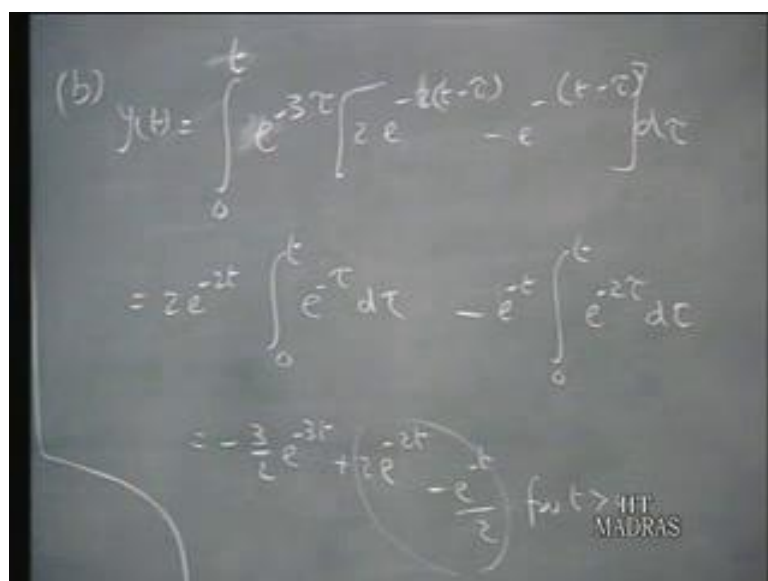
inputs. Actually as a matter of fact you can see, the impulse response what do you mean by impulse response? An impulse appears at t equals 0 and afterwards there is no input.

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That means; whatever follows is kind of natural response of the system. Therefore, the impulse response here, whatever frequencies are present here is the natural frequencies of the system. And the force response for e to the power of minus $3t$ will have also e to the power of minus $3t$.

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So, you can recognize this to be the force response for the particular integral solution and this is the natural response or what comes as the complementary solution of the differential equation.

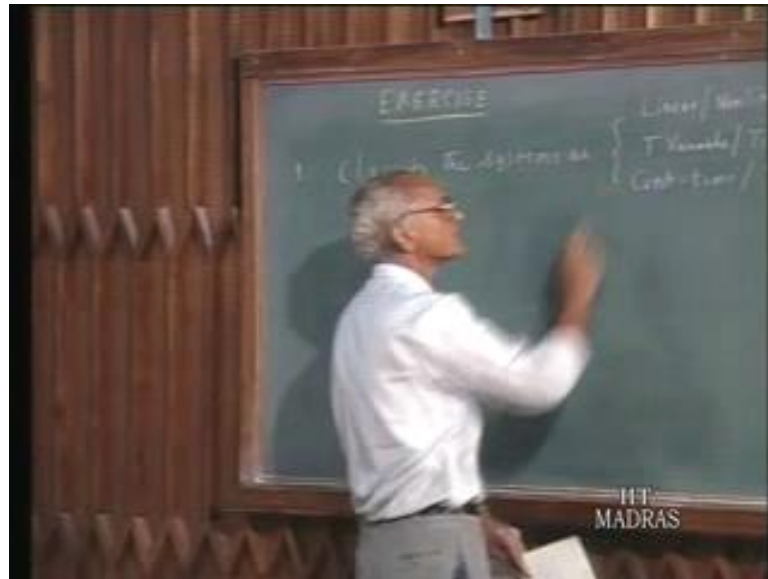
So, at this stage let us we have, let us summarize what we have learnt so far. In the question of introductory system concepts, we have seen how systems can be analyzed particularly transient behavior of the systems. We have looked at various methods; the classical differential equation approach which, entails a lot of involved calculations to find out both the complementary function as well as the particular integral solution and even more so; finding out the initial conditions in a differential equation of a large order. Then we said equivalent information of the differential equation can be obtained, through the system function concept h of s or through the frequency response function h of $j\omega$.

We will exploit these 2 particular functions, when we are discussing the Laplace Transformation and Fourier Transformation techniques at a later point of time. We also said that equivalent information can be provided by the impulse response method; impulse response h of t or the step response a of t . We have looked in some detail only the impulse response as far as our previous discussion goes. A similar analysis can be given in terms of the step response as well.

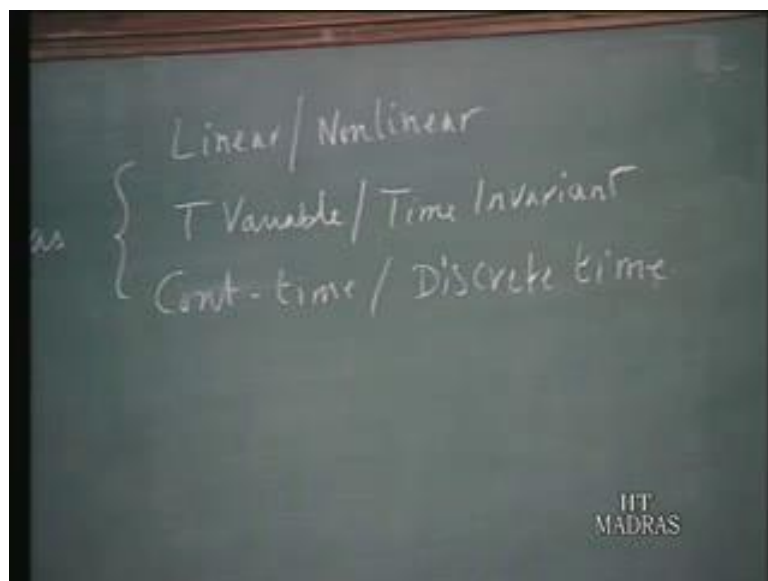
Step response is called a of t , as we have mentioned here, almost identically we can carry out the analysis. We have also some kind of convolution integral involved there also, but we will do this later after we study Laplace Transformation methods. So, impulse response and step response also can be used to characterize the linear system and here again we are calculating all this in time domain only.

So, these are all different ways of describing the input output relations of a linear system. The differential equation approach, system function approach both as a function of $j\omega$ or in time domain using either the impulse response or the step response sometimes called indicial response. The step response is also called indicial response. Some of these techniques we will return to later, when we are talking about Laplace Transformation techniques. And so we will close our discussion of the introductory system concepts at this stage.

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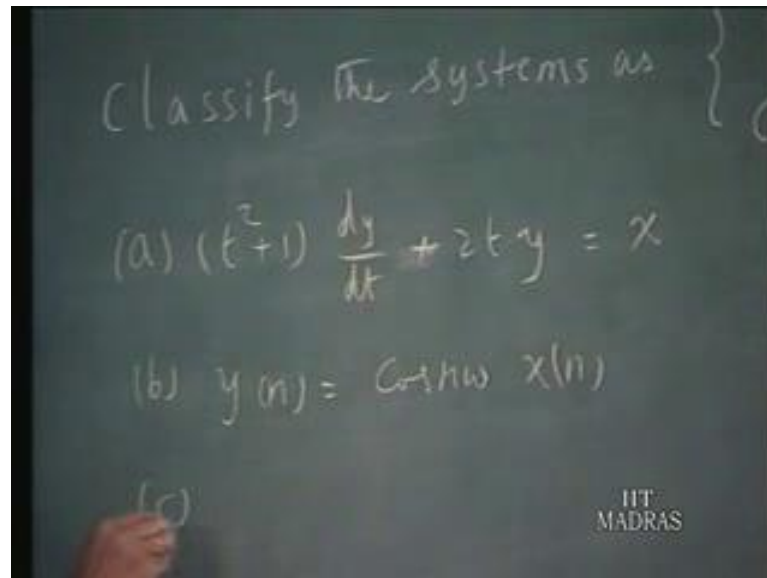


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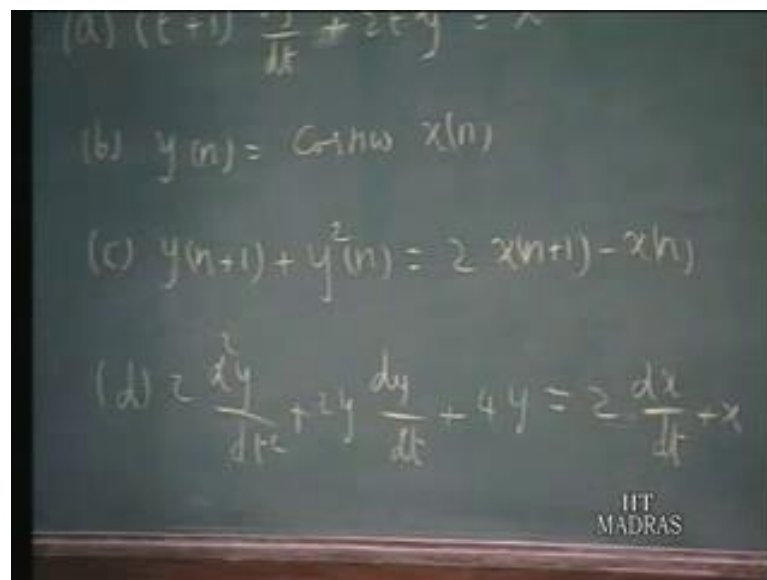
Let me now; give a set of examples for you to work out in the form of an exercise. First question in the exercise is: Classify the systems given by the following equations as linear or non-linear, time variable or time invariant, continuous-time or discrete time.

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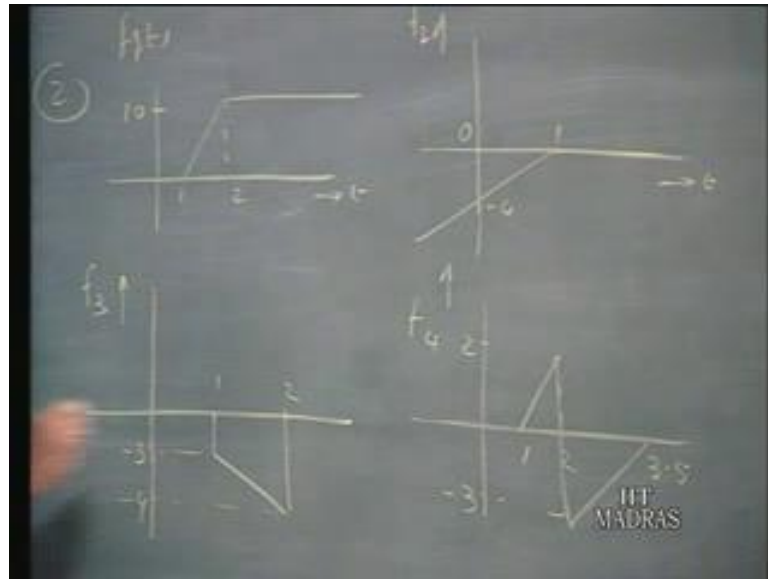
So, the equations are: a; $t^2 + 1 \frac{dy}{dt} + 2ty = x$, b; $y(n) = \cos n\omega x(n)$

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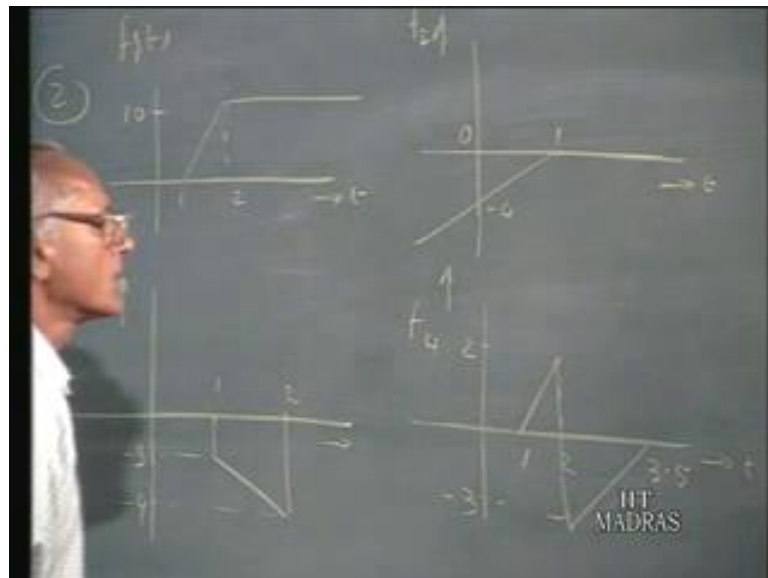
c; $y(n+1) + y^2(n) = 2x(n+1) - x(n)$, d; $2 \frac{d^2y}{dt^2} + 2y \frac{dy}{dt} + 4y = 2 \frac{dx}{dt} + x$ where the usual notation x represents the input and y the output.

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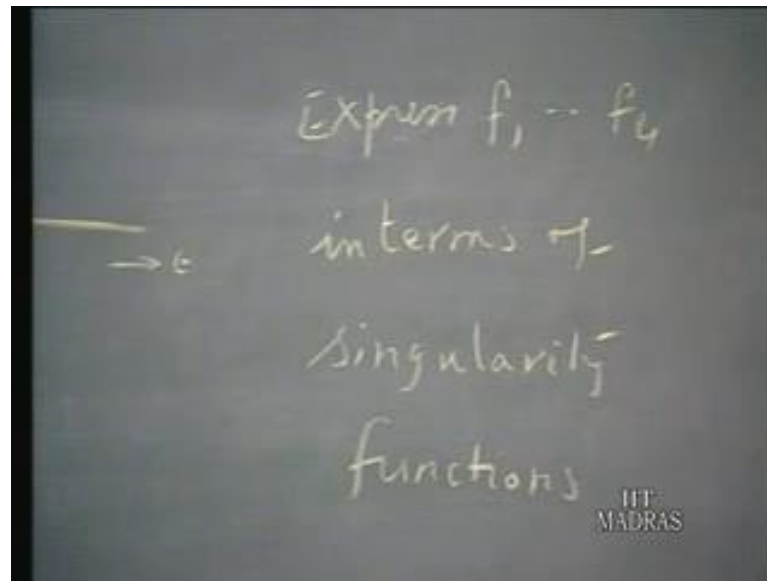


So, this is the first question. Second question; I give you a series of functions of time 1 2 this is 10. This is $f_1(t)$. x axis is always is always time. f_2 ; 1 0 minus 4 and this is 0 from this is f_2 . f_3 ; 1 2 like this, this minus 3 and here is minus 4. Everywhere else is 0. This is f_3 . f_4 ; 1 this is 2 comes down, this is 3.5 and value here is 2 value is minus 3.

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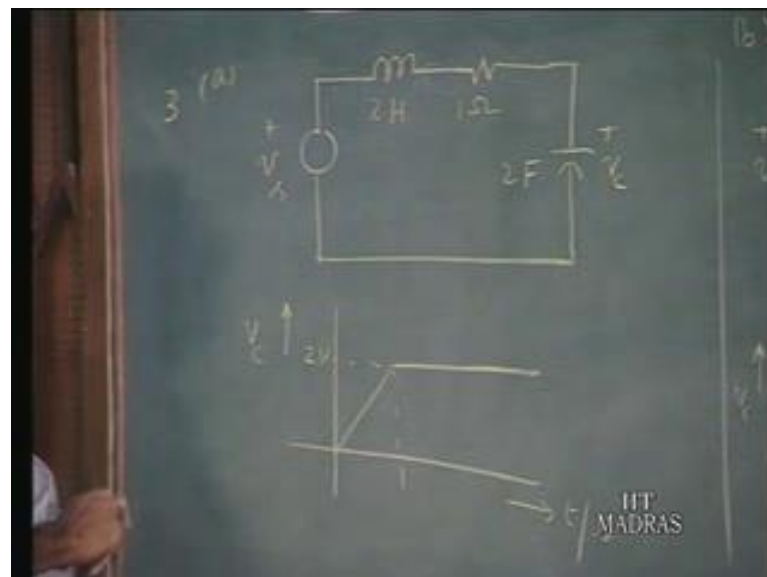


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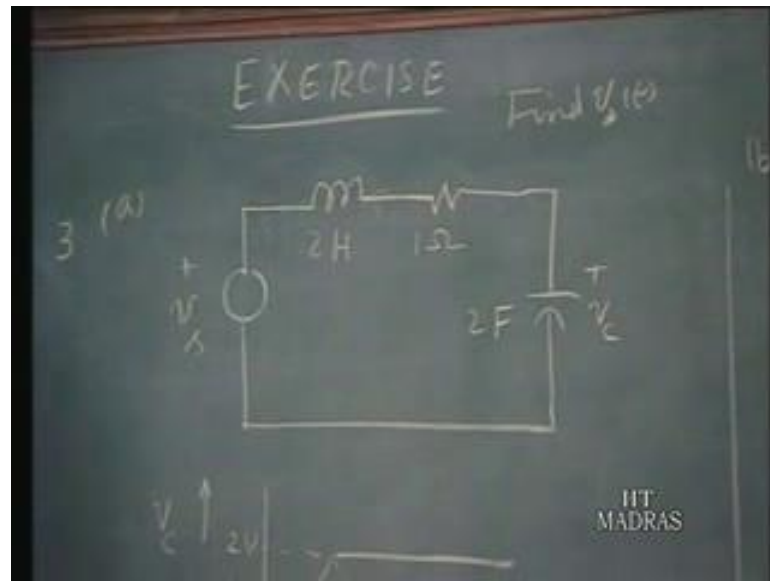
So, 4 functions of time are given to you. The x axis always in seconds; function of time and f_1 to f_4 . Express f_1 to f_4 in terms of singularity functions. So, in terms of step functions, ramp functions etcetera you express this find out expressions for this in terms of step functions and ramp functions.

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that is, the singularity functions. That is the second problem.

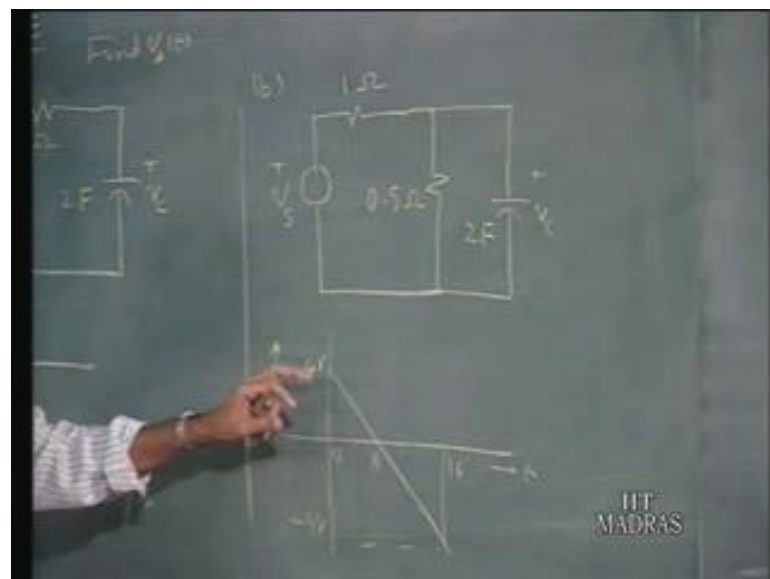
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As a third example, let us take this circuit where source v_s of unknown wave form is driving a circuit containing a inductor, resistance and a capacitor, values are given. v_c has got this particular variation. The wave form of v_c is given. Find v_s of t and express this in terms of singularity functions and sketch its wave form v_s of t in this given this circuit and this wave form of v_c t ; this is part a.

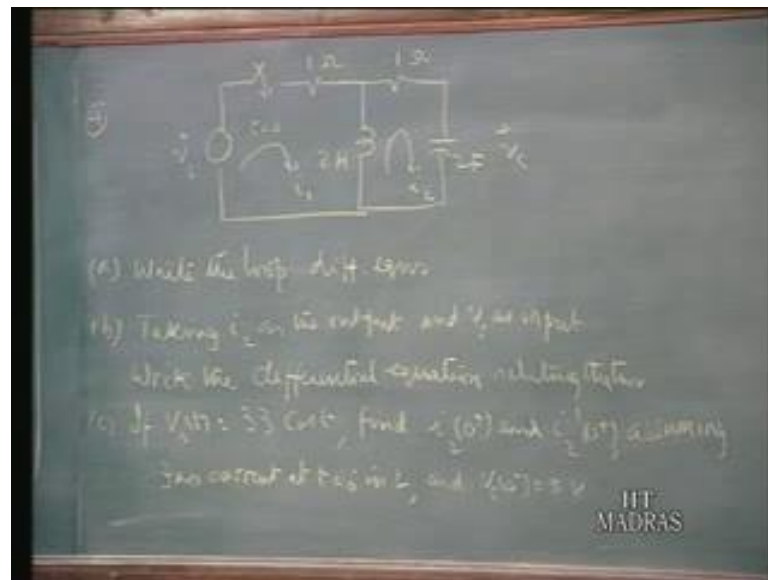
Do similar analysis for this circuit in which, again the value of the voltage v_c is given by means of this particular variation; 4 volts to minus 4 volts.

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It decreases from 0 to 16 seconds and remains 0 for other values of time and if, this is the value and if this is the value of V_c . Find out the expression for V_s of t in terms of singularity functions once again and sketch the form of V_s in terms of using impulses, ramp functions and step functions.

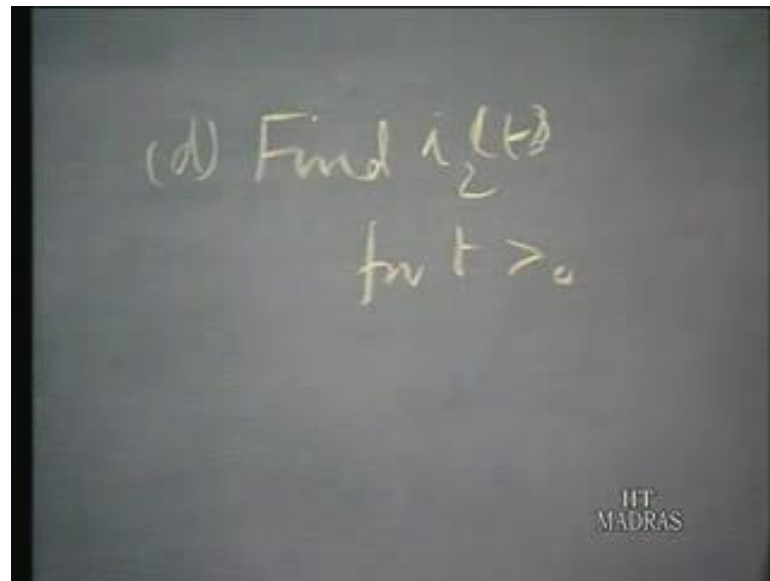
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So, this is 1 problem which is exercised on using the singularity functions. So, these are 2 circuits which you can analyze using, the concept of singularity function. The fourth problem we have, let us take a circuit in which a switch is closed t equals 0 1 Ohm 2 Henrys 1 Ohm and 2 Faraday. We call this i_1 loop current call this loop current i_2 . So, this is a exercise in differential equations.

So, write the loop equations, write the loop differential equations in terms of: i_1 and i_2 and in terms of differential operator d . b taking i_2 as the output and $d s$ as the input, write the differential equation relating the 2. So, you have a second order differential equation relating i_2 as the output quantity y and V_s as the input quantity x . c if, V_s of t equals $33 \cos t$ find $i_2(0^+)$ and $i_2'(0^+)$, the initial value of the second current and its derivative, assuming 0 initial conditions at t equals 0 minus in the inductors and capacitance. Assuming that the inductors and capacitance are initially uncharged or

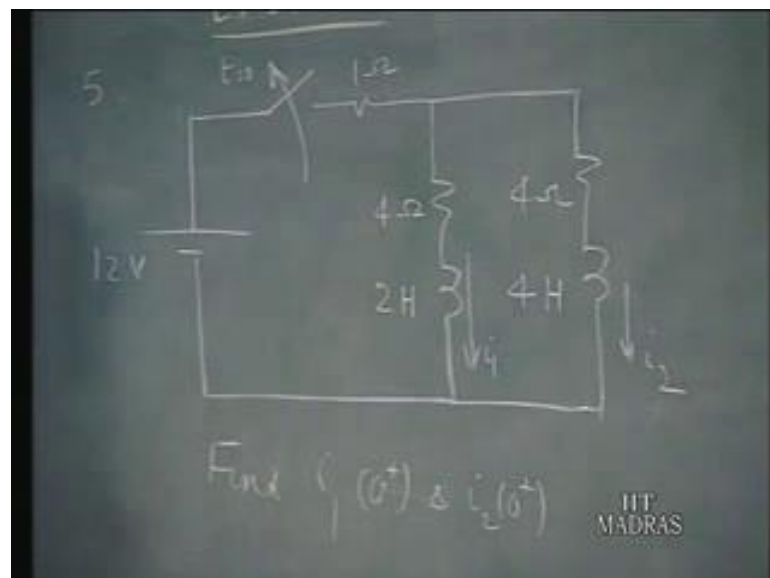
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you know initial current.

We will put this, assuming that assuming 0 current in 0 current at t equals 0 minus in the inductor and the capacitor voltage V_c as 0 minus is 5 volts. And using this information, initial conditions.

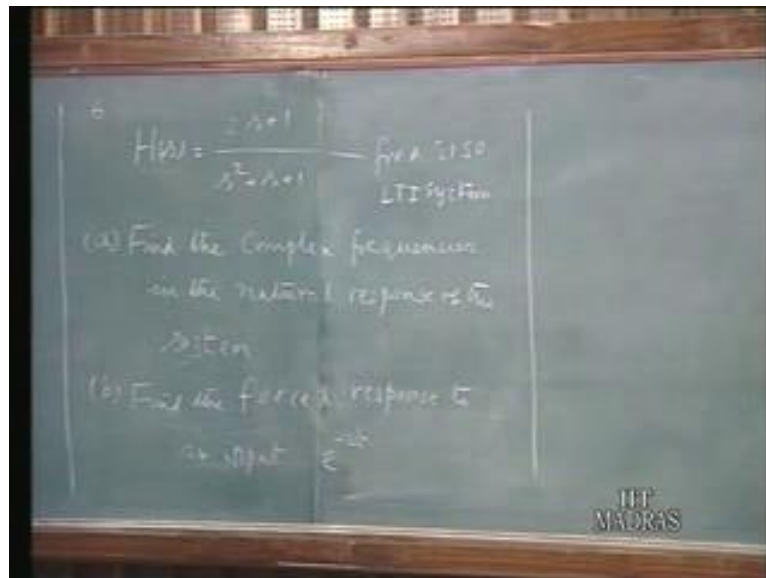
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d find $i_2(t)$ for t greater than 0. So; that means, this exercise in the use of differential equation approach, you have to find the initial conditions and finally, find the total solution for i_2 of t .

Fifth problem: 12 volts, we split this open at t equals 0 after a long time and you have a circuit in which, you have 2 inductors. This is 2 Henrys 4 Ohms. This is 4 Henrys 4 Ohms. This is 1 ohm i_1 and i_2 . The switch is kept closed for a long time. So, inductors have some currents established in them and once the switch is open find $i_1(0^+)$ and $i_2(0^+)$.

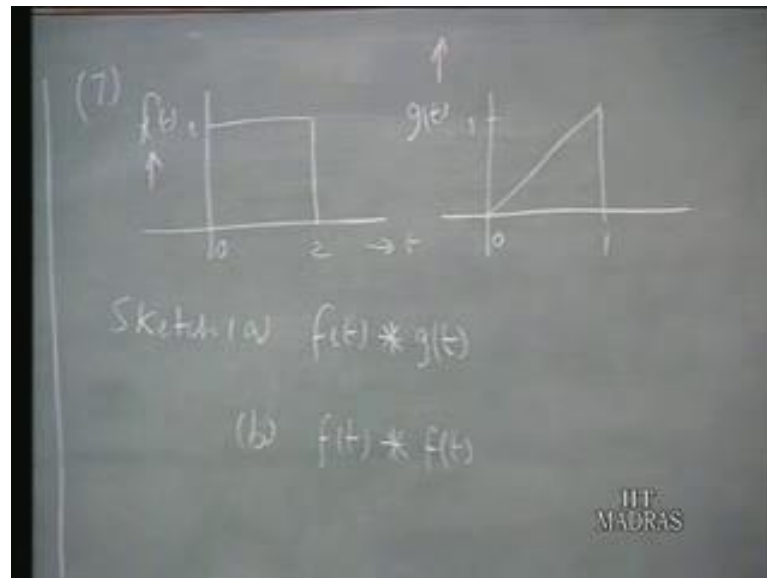
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This is an illustration of the situations where, inductor currents can be discontinuous. So, using the principle that we are talking about, find out $i_1(0^+)$ and $i_2(0^+)$. We assume that the initial conditions have been reached prior to the closure of the, when the switch is closed, steady state conditions are being reached and therefore, the inductors carry some current. Once it is opened out find out the new currents.

Sixth: the system function h of s is given by $2s + 1$ divided by $s^2 + s + 1$ for a single input single output linear time invariant system. A linear time invariant system; single input single output system is this. Find the complex frequencies in the natural responses of the system. Find the complex frequencies in the natural response of the system. b; find the forced response to an input e^{-2t} . If the input is e^{-2t} , what is the forced response of the system?

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That is 6.

Seventh problem: we have 2 functions of time $f(t)$, 0 to 2 seconds which is a value of 1 and $g(t)$ which is a triangular pulse 0 to 1. This is 1 that is, $g(t)$. These are the 2 functions $f(t)$ and $g(t)$ that are given to you. Sketch a $f(t)$ convolved with $g(t)$. The convolution $f(t)$ and $f(t)$. $f(t)$ convolve with itself. So, it would advisable for you to work this out graphically. You can verify them by through analytical working, but a graphical procedure is more illustrative.

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EXERCISE

(8) The impulse response of an initially relaxed linear time-parameter network is $4e^{-2t}u(t)$. Find the response of the same network to (a) an input $u(t)$ (b) an input $e^{-t}u(t)$

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The last problem: the impulse response of an initially relaxed that means; there is no initial stored energy linear constant parameter network is $4e^{-2t}u(t)$ that means, this is $h(t)$. Find the response of the network of the same network, find the response of the same network to a an input $u(t)$. That means find the step response. b an input $e^{-t}u(t)$. That is taking this is $h(t)$, use the convolution principle. Find the response to an input $u(t)$ that is the step response and find the response to a general input $e^{-2t}u(t)$, assuming once again that the network is initially relaxed.