Networks and systems

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Lecture – 49
State-variable methods (5)
Example
State equations for discrete-time systems
Solution in time domain and z-transformation domain

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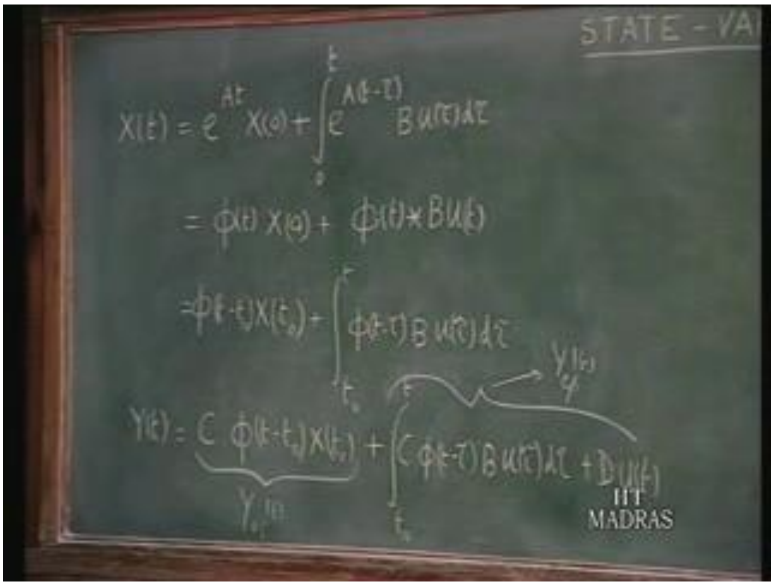

In the previous lecture, we derived the solution for the state and output equations of a continuous time system in time domain. The results as would recall or else put down here the state at any point t is given by e power At times x 0 which is the initial state plus the transformation that is brought about under the influence of the forcing function ut. A more compact notation for this state transition matrix is to write phi t instead of e At.

So, phi t is an alternative notation for e power At. So, we can put this in more compact way as phi t a system transition matrix multiplying the initial state plus the convolution of the state transition matrix with b ut. Now, suppose the state is prescribed on not at t equals 0. But an arbitrary point t, not then we can easily establish that the state cannot given is given by x t not multiply by the state transition matrix phi t minus t not.

But now, the integral here should start from t not because this is once the state is prescribed at t not the further changes that have brought about by ut or given by this integral with a lower limit being t not instead of t. So, then once you have established in an expression for xt yt is after all cx plus du. Therefore, this entire thing multiplied by c matrix plus du ut will be the solution for yt the output vector.

Now, as you would recall this portion is called the 0 inputs solution when u of t ut is 0 this is the solution that is called the 0 inputs solution. And this portion of the solution is called 0 state solutions. So, if the original state is 0 a starting from the x t not equals 0 then this is the entire solution. So, this is the 0 state solution and 0 input solution it has the same connotation and the same meaning as attributed them in in the context of the solution by the Laplace transform domain, which you already know. So, let us now used this ideas and to work out a problem a numerical problem in the to find the solution and time domain.

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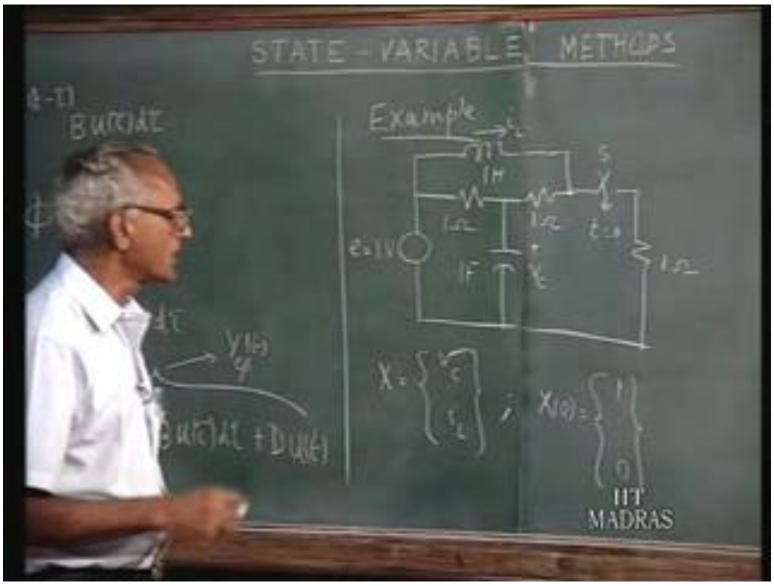

For this purpose we take the same example that we have used while illustrating the Laplace transform technique. So, the example that we take is we have a switch here which is closed at t equals 0. And we have an inductance at this point. Now, the driving source is a 1 volt dc source. This is 1 ohm, this is 1 farad, this is 1 henry, this is 1 ohm and this is 1 ohm. We take the state variable to be il and vc is a form the natural choice.

Now, s is closed at t equals 0 after the steady state has been reached. Therefore once the steady state has been reached the switch open this capacitor charges to 1 volt The inductor current becomes 0. So, if you take the x vector is state vector to be vc and il we know the initial state is 1 and 0. Because, vc is going to be 0 and il is going to vc is going to be 1 and il is going to be 0. So, we have form the state equation for this already in the context to the Laplace transform solution.

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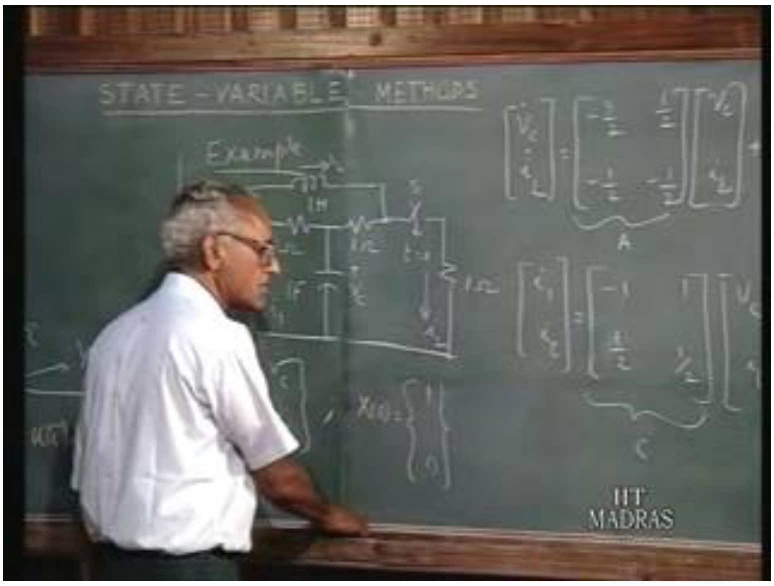

So, i will just put them down vc dot il dot equals minus 3 by 2 half minus half minus half vc il. So, this is a matrix 1 1 e this is the b matrix. Now, let us take the output quantities to be i 1 i 2. When we worked out to this problem in the Laplace transform domain we took this as a output quantity just to for a shake of variety that. So, take both i 1 and i 2 to be the output quantities.

We also have advantage of comparing the solution that we get for i 2 with the 1 which we obtain, when using the Laplace transform. So, let us take i 1 and i 2 as a output quantities. So, this is the y vector this is related to the state vc il plus another matrix multiplying the input quantity input quantity instead of, u i am writing u e because this is after all a voltage source.

And this turns out to be minus 1 1 0.05 or half half and then 1 0. So, this is the c matrix and this is the d matrix. That’s what you are having. So, in order to find the solution in 1 of these forms we have to find this state transition matrix e power At or phi t as it is called. So, the first thing to do would be to find out the Eigen values.

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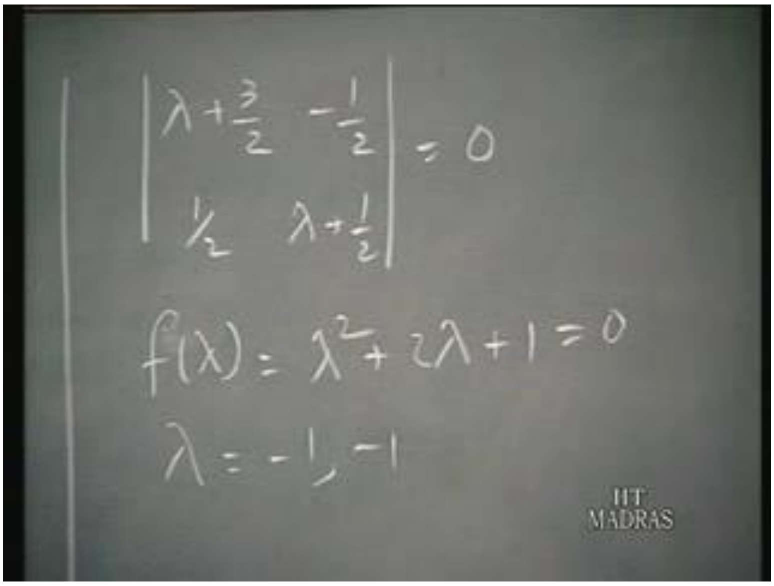

So, lambda the characteristics equation is determinate lambda i minus a. So, lambda plus 3 halves minus half half lambda plus half, this determinant equal to 0 is the characteristics equation. And this turns out to be this is f lambda equals lambda squared plus 2 lambda plus 1 equals 0, this is the characteristic equation. So, the Eigen values are minus 1 minus 1. So, it is a case our repeated routes.

So, that 2 Eigen values, but there is only 1 distinct Eigen value that is the particular Eigen value is repeated twice.

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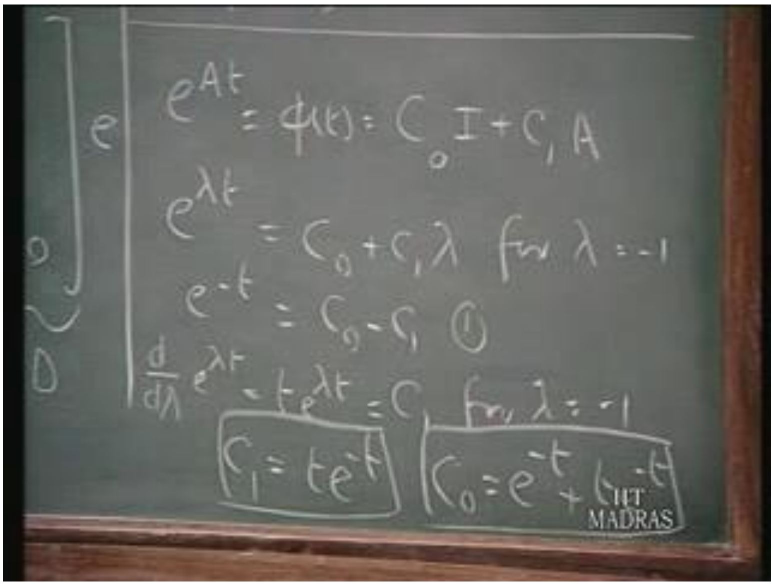

So, now we know therefore, that e power At which we also call phi t is A square matrix of order 2 and then this is given by a polynomial of degree 1 c not plus c 1 a polynomial un matrix polynomial matrix polynomial. So, each 1 of this terms represents a matrix of order 2 C not i plus c 1 a. Now, how do we arrive at c not and c 1; the procedure is already discussed in the last lecture.

So, we take the corresponding scalar polynomial e power lambda t would be c not plus c 1 lambda for values of lambda which corresponds to the Eigen values. Whereas, this is true for all values identically whereas, this equation haler equation is true only for lambda being 1 of the Eigen values. So, in this case this is for lambda equals minus 1. So, that gives to you equation 1 equation.

So; that means, e power minus t equals c not minus c 1 that is 1 equation. To obtain the second equation what we have to do is since this route is repeated twice you have to take the derivative of this with respect to lambda and then that equation will still be valid for lambda equals minus 1. Because lambda minus lambda plus 1 whole squared is a factor here. So, taking the derivative of this d by d lambda of e power lambda t which is equal to t times e power lambda t equals, taking the derivative on the other side which is equal to c 1.

This is also valid for lambda equals minus 1. So that means, c 1 is straight away obtain as substitute in t lambda equals minus 1 t e power minus t. So, we have got an expression for this c 1 and from this you can get an expression for c not. So, c not will be e power minus t plus t e power minus t. So, we have got c 1 and c not and since e power At is c not i where i is the unit matrix of order 2 plus c 1 a is we know now expression for c power c e power at. So, let me put that down here.

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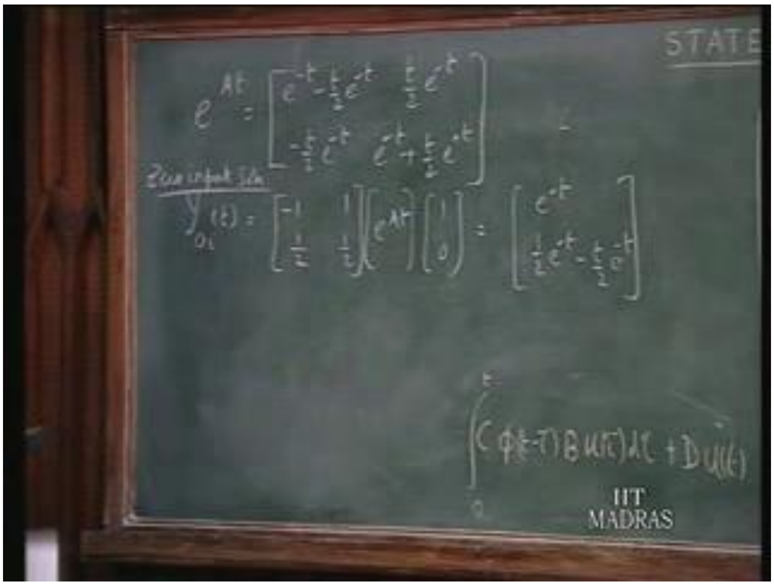

So, you have e power At will turn out to be e power minus t minus t upon 2 e power minus t t by 2 e power minus t minus t by 2 e power minus t e power minus t plus t upon 2 e power minus t. That is e power at. So, we have to calculate yt as c times because we know the expression here the initial state at t equals zero So, we have to take t not equals to 0 and use this expression. So, let us say let us find out y 0 i t; this is the 0 input solution.

So, c times the matrix c is known to us that is minus 1 1 half and half C times e power At which is this 1 times xt 0, which is x 0 in our case happens to be 1 0. Use this matrix here and compute the product of this 3 matrices and you can show that the solution is e power minus t half of e power minus t minus t upon 2 e power minus t. That is the solution for the 0 input component of the output.

That is i 1 equal's e power minus t and i 2 happens to be half e power minus t minus t by 2 e power minus t. To find the 0 state solutions you must compute this quantity. Where t not is now 0. So, we take these to be 0 and this is what we should take. So, we should now like to find out a product of these before we integrate.

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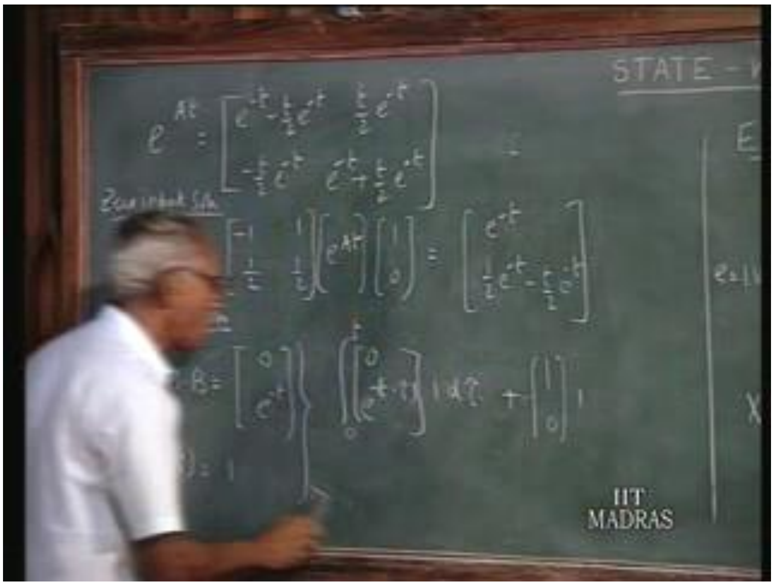

So, 0 state solution we have to find out c first of all c phi t times b. Suppose, you do that then we can after all substitute t minus tow for t later on. So, let us not complicated picture at the stage let us find c phi t time's b. So, we know the matrix c here we know the matrix b and we know e power At which is of course, phi t.

So, if you use 3 matrices and multiply this out it turns out that is 0 e power minus t that is c phi t times b. But what we have to do is take c phi t minus tow b u tow and integrate from 0 to t. We know that ut the input here e that is the same as e we have instead of u we have used the symbol e and that is equal to 1. So, u tow also happens to be 1. So, we using this to do this integration now, what we have to do is therefore. Integrate from 0 to t c phi t minus tow b; that means, 0 e minus t minus tow times u tow u tow ut equals 1 u tow is also equal to 1 times d tow.

That is 1; that is the first part and then in addition you have d ut; d happens to be 1 0 that is this matrix here for column matrix 1 0. So, you have 1 0 times u of t happens to be constant 1. So, this the solution. Now, what we integral of this matrix what is this means is. You have to integrate both this terms separately after all, the product will now be this is equal to.

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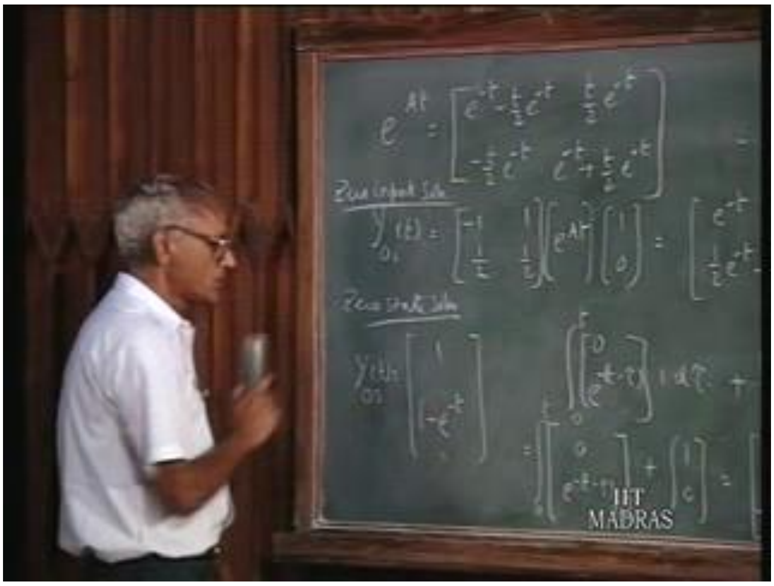

I will write this separately after all 1 d tow that is the multiply multiplying by 1 does not change anything. Therefore; that means, what we have to integrate this term separately and this term separately after all, the result column can write this actually 0 e power minus t minus tow t tow. That is to be 0 to t T tow t tow is already there plus 1 0 that is the second.

So, this means that the first matrix is integral of this which is of course, 0 the second matrix you have to integrate the second term in this matrix you have to integrate with reference to t tow. Therefore, e power minus t is a constant. Therefore, you have e power minus t and then 0 to t e power tow t d tow the balance term here e power tow d tow that is the meaning of this. Integrating of matrix means every 1 of the term must be integrate separately plus 1 0.

And when you carry out this integration the answer to be finally, it will be e power minus t. So, 0 states solution will be 1 1 minus e power minus t 1 1 minus e power minus t. So, that is the 0 input solution, this is the 0 state solutions. The total solution will be for after all y the output matrix output vector is i 1 i 2.

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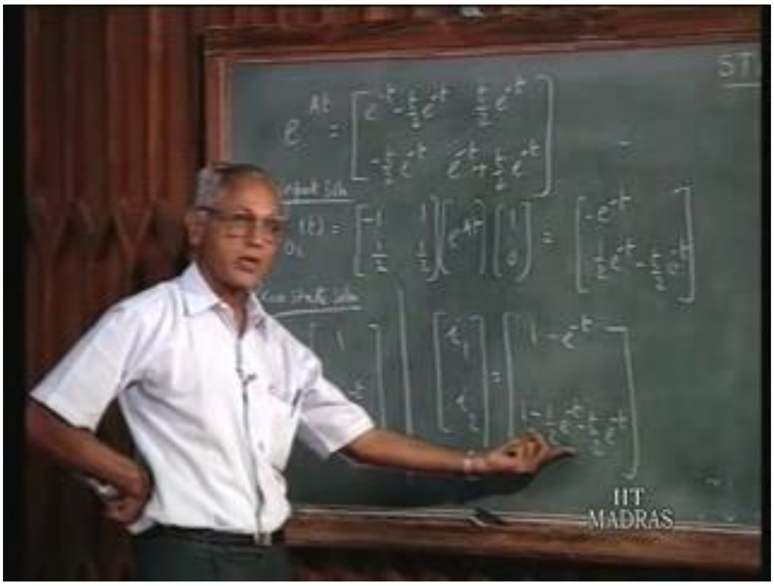

So, i can write now i 1 i 2 is the sum of these this is the 0 input solution this is the 0 state solution, it will be 1 minus by the way 0 input solution it should be a minus here; that is what we should have. So, it 1 minus e power minus t and 1 e power minus e power minus t and half e power minus t. Therefore, minus 1 half e power minus t minus t by 2 e power minus t, that is the complete solution for i 1 and i.

If you compare the result that you have got for the same problem for this output quantity using the laplace transform domain, you would observed this is the same answer that we obtain for i 2. Here, we have got additional output i 1 that will happen to be 1 minus e t. So, this illustrates the time domain solution of the state equations using the trade state transition matrix e power at. So, far we have discussed the solution formation of the state equations for the continuous time system and the solution in time domain and the laplace transform domain.

We would like now to extend these concepts to discrete time system. And as a matter of that the solution of the state equation the formation solution of the state equation in discrete time domain is at live as great importance. Because, after all for computer simulation computer solution or state equation even in continuous time domain we will approximate this by taking small increment of time at a time. So, it is the solution is obtained a discrete time basis.

Therefore, the formation of state equations in the discrete time situation is certainly important not only its own write, but also because as an approximation of the equation and solutions in continuous time domain also. So, that is the topic that we will now take up next formation and solution of the state equation per discrete time systems.

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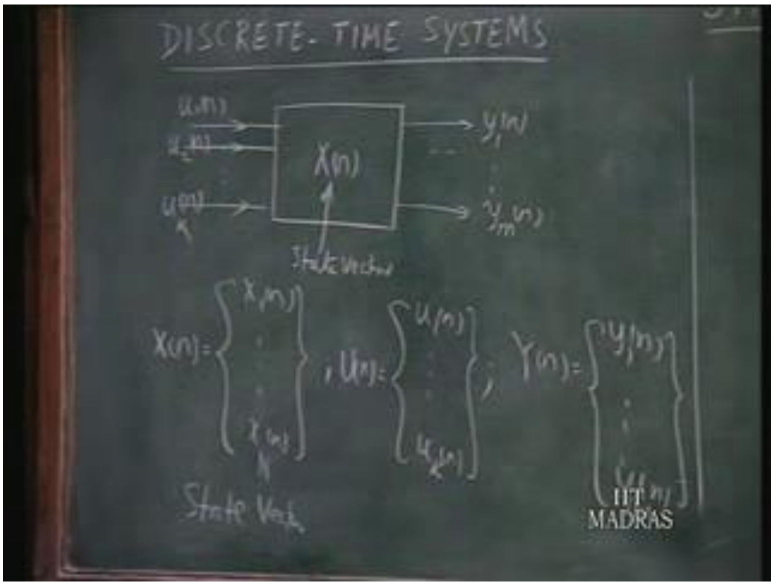

In a discrete time system; in the representation of discrete time system you have several inputs u 1 u 2 let us say uk. These are all the discrete time signal; therefore these are not function of continuous variable, but a discrete variable n. And various outputs are there y 1 n upto let us say ym n. And inside internal variables there is a state vector xn. So, the state vector is coupled to the inputs to the state equation and the output is coupled to the state vector by the output equation.

So, we can say that is state vector xn is now x 1 n these are the state variables let us say there are n such variables xn n. So, these are the various so in order to not to confuse with this discrete time variable n and also the dimension of the state let me use this as capital n. So, that is the departure from the continuous time case because, we are using n for the argument in the case of discrete time systems. We are although use the n for the dimension in the state vector; the continuous time systems.

So, here let us use the N to indicate the size of the dimension of the state vector. So, we have this is the state vector, we have u the input vector u 1 n write up to uk n. And we have the output vector also is a function of n i can write this as u of n also output vector y 1 n that is what we have.

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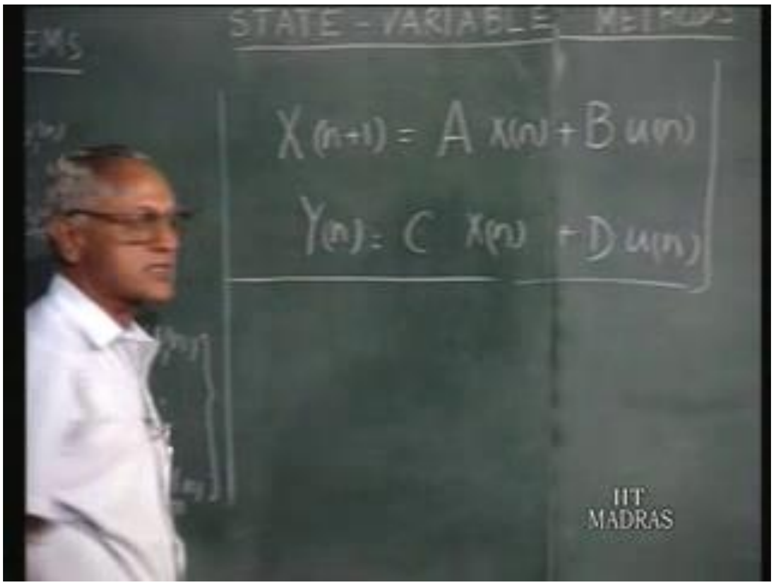

Now, this to form the steady equation we do not have the counter part of the derivative of the state in the continuous time you are talked about x dot equals ax plus bu. For here it is nothing like a derivative because we are talking about signals and discrete points of time. Therefore, the state equations here we have the form x n plus 1 the value of the state at the n plus oneth sampling instant equals a times the state at point n plus bu.

That is the form of state equation. This again the matrix equation where this is a vector of dimension N this is a square matrix of n by N by N this is the vector this is again it has got N rows and k columns corresponding to the dimension of the input vector un. The output yn is coupled to the state by a c matrix c xn plus d un.

So, the form of the state and output equations are similar to what we have got in the continuous time situation except that the arguments now, instead of continuous variable t we have a discrete variable n. And further instead of having the derivative x dot t here you have x n plus 1 that means, this discrete time variable is incremented by 1 step to form x n plus 1. So, this is the form of the state and output equations. A b c d are matrices our appropriate dimensions.

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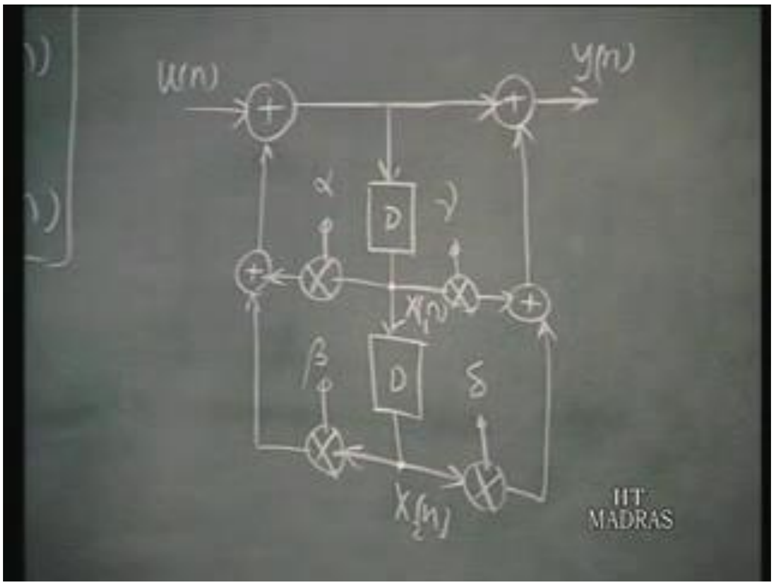

Now, to illustrate how the state equation can be formed from a discrete time model let us take an example formation of state equation, formation of equations. Let us take the model of discrete time system which is given by this figure; there is 2 delay units you have coefficient multiplier alpha. Let us say this coefficient multiplier as a beta. And these 2 are summed up this is given here this is the input un and this goes to an adder that will be yn. And this signal here goes to a coefficient multiplier let us say coefficient multiplier the multiplying factor is gamma and this is an adder here.

Let us say there is another coefficient multiplier here the multiplication factor delta. So, these this signal comes here this goes here. So, this is then the model of a discrete time system of second order. Now, to form the state equation for that it will be convenient to take the outputs of the delay units to be the state variables. Just like, we have taken the capacitor voltages and inductor currents of the state variables in the continuous time case.

In the discrete time case when you have modeled like this the output is delayed it is convenient to take the outputs the delay units to be the appropriate state variables here. By no doubt this is only 1 possible choice and it is convenient choice. So, i will call this x 1 n here and this point the signal x 2 n. So, the state variables are x 1 n and x 2 n.

The advantage of such a choice is if this is x 1 n the this signal at this point will be after all this is delayed by 1 unit to form x 1 n. Therefore, this signal must be a x 1 of n plus 1.

So, we can immediately write down the value of the signal in terms of the other state variables and input quantity is quite conveniently. Similarly, if this is x 2 n the input to the delay unit must be x 2 of n plus 1. So, you straight away you have 2 quantities corresponding to x 1 of n plus 1 and x 2 of n plus 1. And this is the very convenient way of writing down the state equation.

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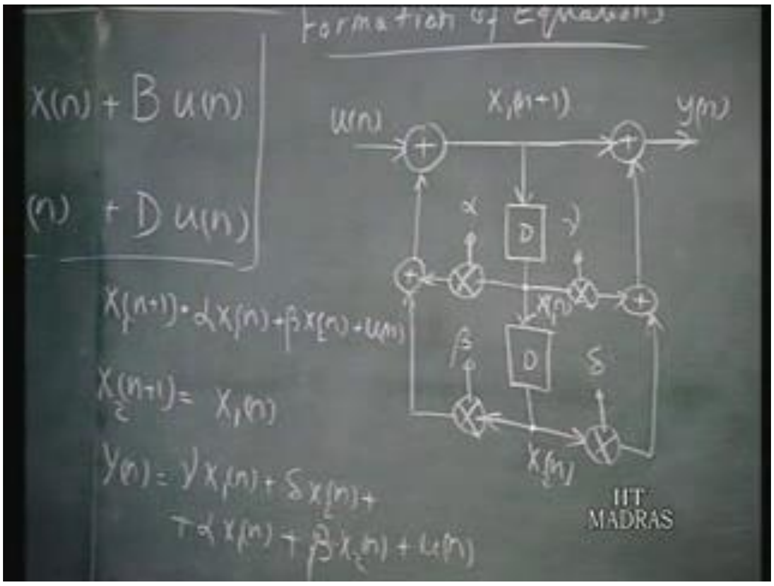

So, let us do that, we have here x 1 of n plus 1 that is the signal at this point is after all, you have this is the signal here is alpha x 1 n. The signal here is beta x 2 n. Therefore, alpha x 1 n and beta x 2 n are fed to the adder and the signal that is fed to this adder now is alpha x 1 n plus beta x 2 n. That combine with un produces this signal which is x 1 of n plus 1. Therefore, x 1 of n plus 1 will be alpha x 1 n plus beta x 2 n plus un.

So, that is x 1 of n n plus 1 X 2 of n plus 1 that is this signal is after all x 1 n. So, there is no difficulty in identifying that with the state variable. So, x 2 n plus 1 is after all x 1 n in this case it turns out to be quite a simple expression for x 2 of n plus 1. The output yn there is only 1 output here is x 1 of n plus 1 for which we already have an expression plus the signal here.

The signal here is gamma first of all let us put gamma x 1 of n plus delta x 2 of n plus that is, what we have written here its sum of these 2 signals plus x 1 of n plus 1 x 1 of n plus 1 is already known to as alpha x 1 of n plus beta x 2 of n plus un. That is the input quantity so we have got a 2 equations corresponding to the state and the output equations.

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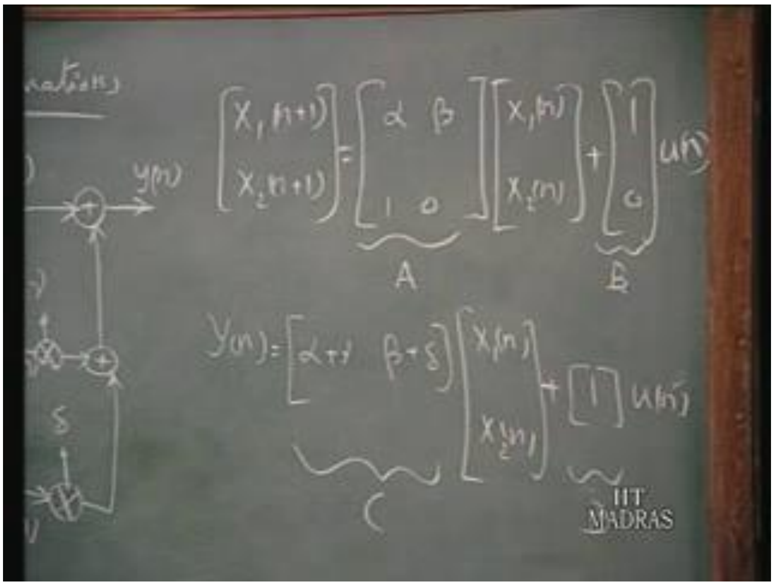

So, the whole thing can be out on now in matrix form. X 1 n plus 1 x 2 n plus 1 equals alpha beta 1 0 of x 1 n x 2 n plus 1 0 un. This is the a matrix this is the b matrix. And yn there is a scalar this is only 1 input alpha plus gamma beta plus delta x 1 n x 2 n plus 1 the matrix d turns out to be a pure scalar. So, this is the c matrix this is the d matrix.

So, this is single input single output system. Therefore d matrix turns out to be matrix of order 1 by 1 that is the scalar and so this is 1 confirm the state and output equation for discrete time systems given by a model of this type. The point note here is that the outputs of the delay unit turn out to be convenient choices for the state variables.

Just like the continuous time case there is no fixed there is no we have a choice we state variables do not have to be necessary this. You can have other choice also, but this is the convenient choice. Now, we have to look at the solution of this equations we will do this again in 2 parts: will try to find the solution in time domain and also in the transform domain. But the transform domain that we have to use in this case is the z transform domain.

Therefore, we first try to find out the solution for this state equation and time domain that is the discrete time domain. And later on we will find out the solution in the z transform domain. So, we take up the solution of the state equations in time domain.

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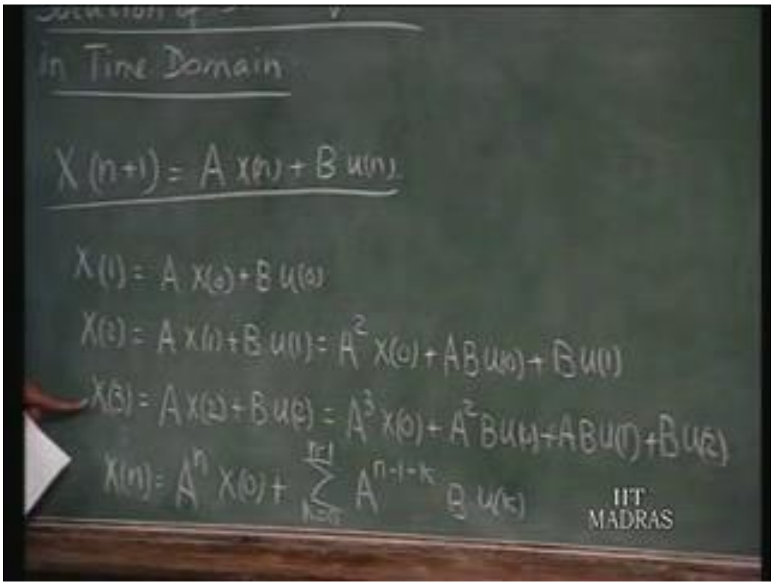

So, this is the equation which we have to solve for x n plus 1 is a xn plus b un. We can find out the solution very convenient way by looking at step by steps solution of this matrix equation. Suppose, i want to find out the value of the state when n equals 0 for example. So, x 1 that is x 1 equals a times x 0 plus b time’s u 0, that is n equals 1.

So, if you know the initial state and of course, the forcing function from n equals 0 onwards you can find out the state at x n equals 1. Now, let us use this information to calculate x 2 when n equals 2 x 2 is after all a times x 1 plus bu when n equals 1. So, you know the solution for x 1. So, you substitute over here so you get a multiplied by a x 0 that is z squared x 0 plus a times b u 0. So, ab u 0 plus b times u 1.

So, that is the solution for this state at the sampling instance at n equals 2. Let us continue suppose n equals 3; that means, you increment the value of n by 1 unit. So, x 3 will be a times x 2 plus b times u 2. So, a times x 2 means i must multiply this by a; that means, a x 2 plus b times u 2, where these are all matrices. So, when you multiply this by a pre multiply this by a you get a cubed x 0 plus a squared b u 0 plus ab u 1 plus b times u 2.

At this stage you find a certain of them in this expressions. So, you continue this you would expect u 1 to be a power n multiplied by x 0. That is as far the rest of terms are concerned you find a squared b u 0 ab u 1 b u 2. So, if you think of this a general form akb let us say that k plus b the index the power of a plus the argument of u add up to 2 two 0 2 1 plus 1 2 a 0 of course, 0 plus 2 2.

So, you now can see that a general expression for xn would be xn would be a power n times x 0 plus summation of terms like this, a n minus 1 k minus k b ukN minus 1 a n minus 1 minus k b uk k ranging from 0 to n minus 1. So, k equals will be a n minus 1 b u 0 that is corresponding to this for n equals 3 and k equals n minus 1 a 0; that means, unique matrix times bu 3 that b u 2 that corresponds to this for case n equals 3.

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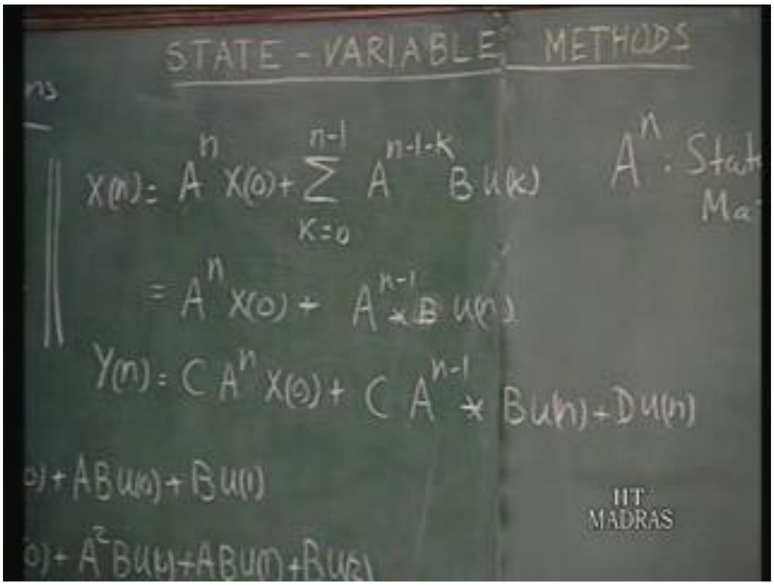

So, the final solution therefore, will be I will put this here x n is a power n x 0 plus summation a power n minus 1 minus k b uk k ranging from 0 to n. So, that is the solution that is the solution for the state vector xn. 2 things you would note 1 is a power n is like a system transition matrix that we have in the continuous time equations. This is the state vector at the value at discrete variable being equal to n.

So, x 0 has been transformed into xn by multiply in this by an. So, this corresponds to the state transition matrix so, we will note that straight away. An has the same role as the state transition matrix this is also called state transition matrix in the discrete time situation. So, instead of e power At an turns out to be this state transition matrix in the discrete time situation. Second point you would notice that, this summation is in the nature of a convolution summation after all u n minus 1 minus k.

So, as increment k the sum of these 2 happens to be the same just like you have some xt minus tow y tow. So, the sum is always equal to t similarly the sum of these 2 is equal to sum of the 2 indices happens to be n minus 1. Therefore you can put this in more compact fashion an x 0 plus the convolution now an minus 1 un b un, i must put in b un here. So, the convolution of a n minus 1 b un will be the summation with at the running index k here a n minus 1 minus k times b uk.

So, it is a convolution of a n minus 1 and b un. So, that is xn so yn would be c times an x 0 plus c times a n minus 1 convolved with b un plus of course, d un which is a direct coupling with the input signal un. So, this is the output. Now, i will rub this off.

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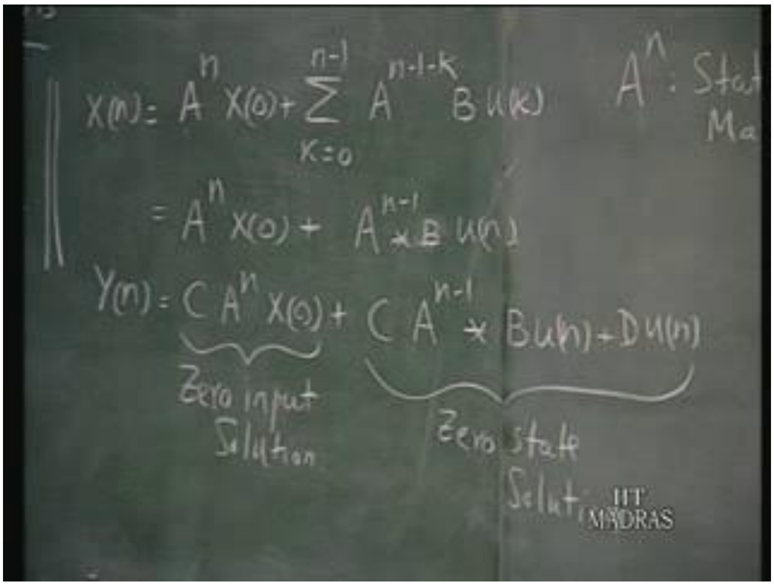

You observe now, that thus the continuous time situation. We can call this the 0 input solution. This is how this state has been transformed to the 0 state a particular state and coupling with this state as far the output is concerned it is to the matrix c c an x 0 is a 0 input solution. And this portion will be the 0 state solution that is the initial state is 0 the solution that would be obtain is this.

And the convolution here does not involving integration involves only summation that is what you are having.

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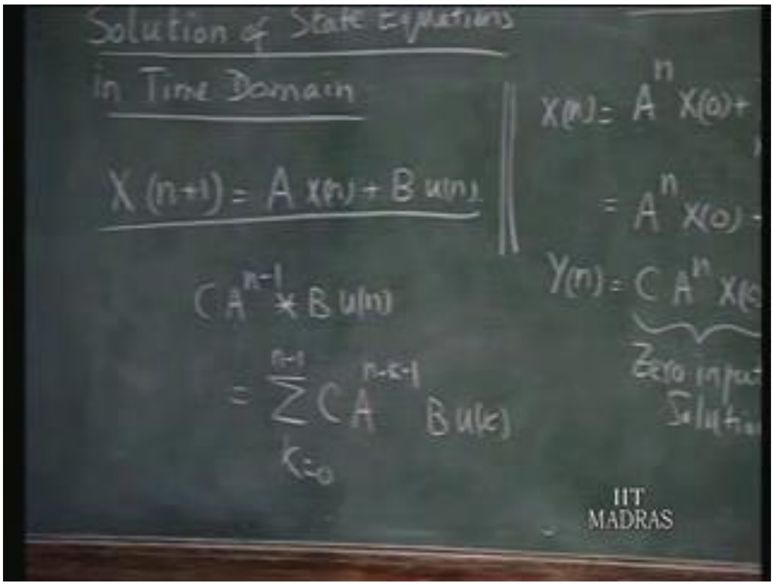

So, to specify clearly what c a n minus 1 convolved with b un means this it is summation of the matrix c multiplying a n minus k minus 1 b uk and the range of summation this k from 0 to n minus 1. That is what we have seen. So, this convolution means this c a n minus 1 k b uk.

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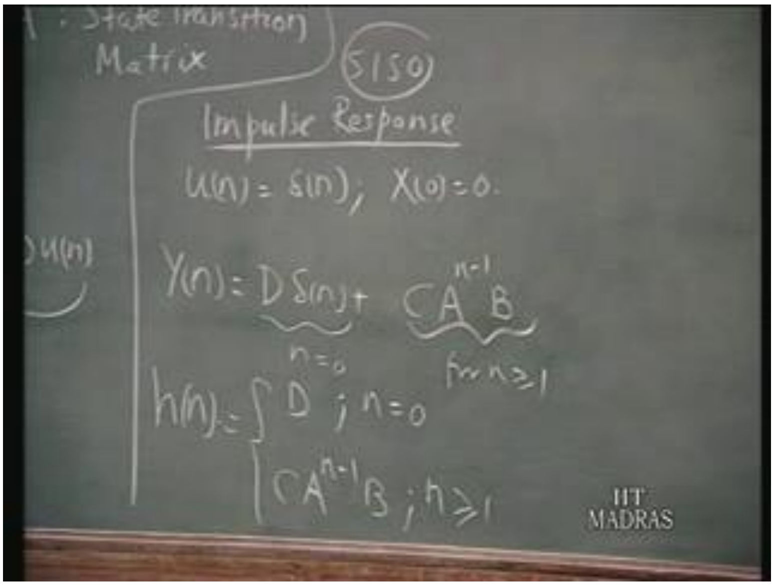

Now, let us see using this how do we find out impulse response of the system. Impulse response means, the input un is delta n and the initial state is 0. Because whenever you talk about impulse response we assume that initial conditions are 0. So, x 0 is 0 and un s delta n. So, all we have now we have of course, x 0 is 0 therefore, this termed occur and as far as yn is concerned you have this component and this component.

This component is straight forward so, as far as yn is concerned d times un un happens to be delta n therefore, d times delta n that is 1 component. As far as this component is concerned you have the k only when k equals 0 you have a value because that is an impulse. And when k equals 0 u 0 happens to be 1 because that is delta n. And therefore, this is c a n minus 1 times b C a n minus 1 times b and that will have values after all, this is valid only for n equals 1 onwards.

When n equals 0 this becomes n k equals 0 summation is from k equals to 0 k equals n minus 1 when n equals 0 is no meaning. Because k equals to 0 minus 1 it will become. So, this summation will have meaning only from n equals 1 onwards not when n equals 0 therefore, this component arrives at from n equals 1 only. That means, this is where this arises for n greater than or equal to 1.

And this is of course, is has a value for at n equals 0. So, we can say that yn equals d per n equals 0 and this is equal to c a n minus 1 b for n greater than or equal to 1. I could have written here as u n minus 1 as a delayed step function, but we have used u in other contest. Therefore, i do not want to confuse the picture by rating u n minus 1 in the meaning of a step starting at n equals 1 because, u n has been used n another context.

So, i am writing them separately d n equals 0 c a n minus 1 b n greater than or equal to 1. So, that is the impulse response so, this we can write this as h of n. For c this is i assuming that this is single input single output system otherwise this will be again a matrix. Suppose we will to compute to make it simple let us assume that single input single output system. Therefore these are all scalars, these are all scalars.

Now how do we evaluate a power n that is the question that we will like to ask? The question that we now ask is how do we evaluate the state transition matrix a power n. Recall that a i a square matrix of order capital n. That is the dimension of the state where n is the running index in the discrete time system These are the 2 different.

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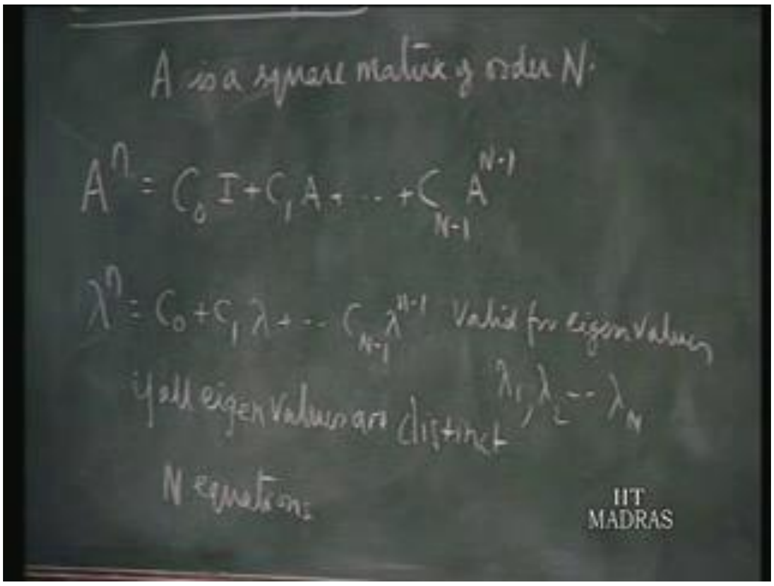

So, as before as in the case of a continuous time system we write an as matrix polynomial of degree N minus 1. So, we write this as c not i plus c 1 a write up to c n minus 1 a n minus 1 that's what we are having.

Now, just as we had in the continuous time case the corresponding scalar equation would be lambda n. Here also, we have that for the same a matrix you have Eigen values we use the same Eigen values same concept lambda n c not plus c 1 lambda c n minus 1 lambda n minus 1. And this equation is valid for Eigen values lambda 1 lambda 2 up to lambda n. If for distinct this is the case for distinct Eigen values. If Eigen if all Eigen values are distinct.

If all Eigen values are distinct we get n equations we get n equations and we used those n equations and evaluate the constant c not to c n minus 1 n constant.

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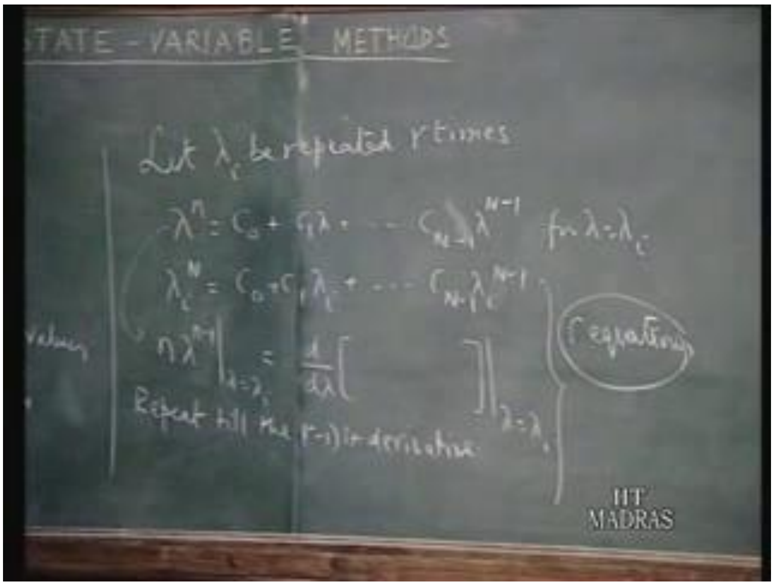

Now, if suppose let a particular lambda i be repeated say r times. We go to the same procedure as we had a followed in the continuous time situation. We have lambda n equals c not plus c 1 lambda plus c n c n minus 1 lambda n minus 1 for lambda equals lambda i. Then; that means, we have lambda i power n equals c not plus c 1 lambda i c n minus 1 lambda i n minus 1. And we take the derivative of this with reference to lambda and then substitute lambda equals lambda i.

That means, we have take the derivative of this n times lambda n minus 1 lambda equals lambda i equals a derivative of this entire expression here that we are having at lambda equals lambda i. And so, you get second equation repeat till the r minus first derivative. We follow exactly the same procedure as we had in the continuous time situation. So, initial value first derivative second derivative substitute at each step is substitute lambda equals lambda i and get r equation although you get r equations.

So, when you a particular Eigen values are repeated r times we are not at a last for the number of equations to solve for the unknowns we get corresponding r equations. So, we have to put all to if the matrix a is of order n, we need to evaluate N coefficients here. And we have n such equations no matter whether some Eigen values are repeated or not.

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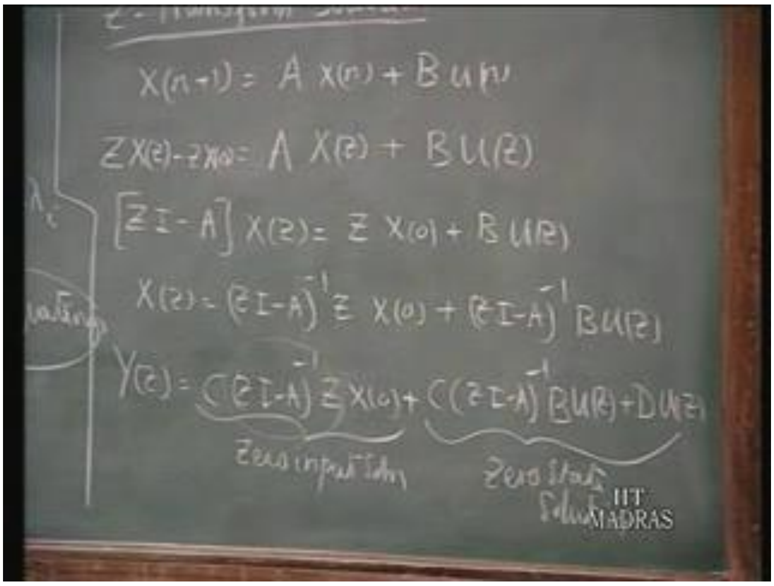

Now, let us f ind out the solution in the z transform domain we have x n plus 1 in time domain as a xn plus bu. So, in the z transform domain suppose x n has the z transform x z. That means, each 1 of these state variables is x 1 z x 2 z and so on. So, you have the entire vector is transformed in the z transform domain xz, this will be a this will be bu z. Here you are advancing the discrete time signal by 1 step n the forward direction.

Therefore, the z transform of that will be z times x of z minus z x 0. So, that is what you have. Consequently, you can find the solution for x of z because after all this is a scalar z multiply x of z you have to combine this and with this. Therefore, in the method that we have done earlier in the continuous time case you put a unit matrix here zi minus a multiplying x of z equals z x 0 plus b uz.

Where x 0 is the initial state this is the time domain evaluated at t equals n equals 0. It should not be confused with x of z where substitution z equals 0, this is the initial state. So, we now have after all this is squared matrix of order capital n. So, xz turns out to be z i minus a inverse times z x 0 plus z i minus a inverse b uz. So, that is the solution for x of z the state vector in the transform domain. And from that you can get yz which will be c times x of z c z i minus a inverse z x 0 plus c times z i minus a inverse b uz plus of course, we have d uz.

So, that is the solution for the output vector in the transform domain and this portion is the 0 input solution and this portion is the 0 state solution. So, when you take the inverse z transform of that you will get y of n. You would notice once straight away that in the time domain expression yn we know is equal to c times a power n times x 0.

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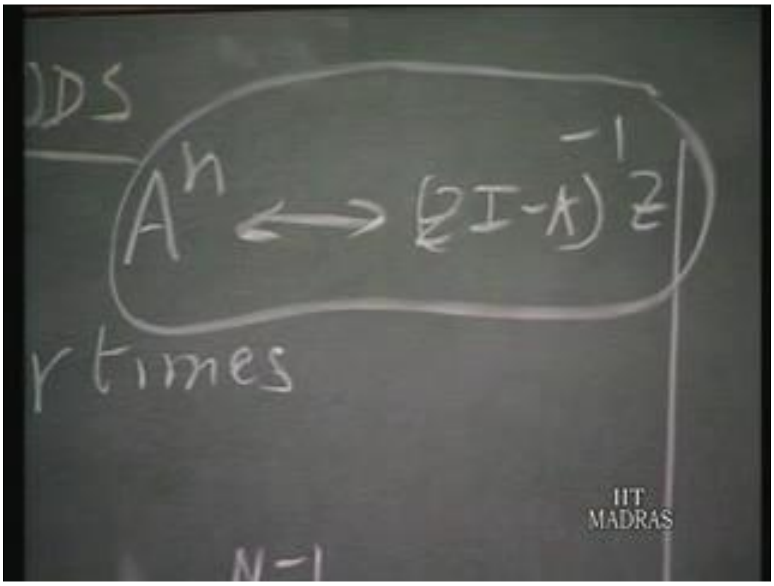

That means this quantity is the transform of an; that means, a power n; the state transition matrix a n in the z transform domain here it turns out to be z i minus a inverse z. So, we will compare the 2 solutions and get a detail in the next lecture and then work out an example showing the complete solution of the state equations in the discrete time domain. So, we now see that yz the transform domain the solution will have this form.

So, in this lecture we started with an example to illustrate the time domain solution of the state equations of the continuous time assume. Then, we saw how state equations can be state variable techniques can be applied to discrete time domain. We have essentially the equations will now be of the form x n plus 1 the state vector is plus oneth instant, is related to the state at xn by multiplication matrix a A xn plus b un. And the output will be yn equals c xn plus d un.

There is no derivative as term possible in the discrete time domain. Therefore, you have xn plus 1 instead of x dot in the continuous time situation. The solution for the state equations in the discrete time domain follows very closely the technique that we followed in the continuous time case. We also here also we have the state transition matrix; this is now, a simpler form a power n instead of e power at.

And the evaluation of a power n can be done in time domain purely using the same procedure that, we had in the continuous time situation using the cayley hamilton theorem. By writing associating a power n with a reduce polynomial of order N minus 1. And evaluating the various constant involved by using the Eigen values, substituting the various Eigen values. Even if take some of the Eigen values are repeated that will not pose any problem.

We have enough equations to evaluate the various unknowns. Another alternative way of evaluate the state transition matrix is to associate this with the z transform the find the z transform domain z i minus a inverse times z corresponds to the state transition matrix. Therefore, if you have found out the z transform the inverse z transform of this is the state transition matrix a power n.

We have also saw, how the solution is obtained in the z transform domain this is what we have got the final step. And we will compare this with the time domain solution and then we will work out an example illustrating the solution for a complete discrete time network in the next lecture.