**Networks and Systems**
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**Lecture – 47**
**State-variable methods (3)**
**Zero-input and 0 state responses**

In the last lecture, we saw how the state equations and solved to get the output function using, the Laplace Transformation technique. To recall what we have done, we started with the state equation.

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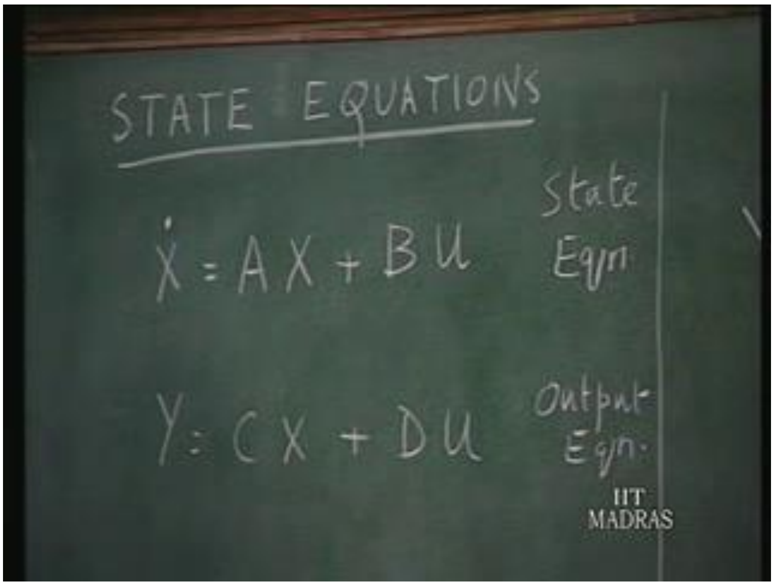


The X dot equals AX plus Bu which called the state equation. Y equals CX plus Du the output equation. These are in time domain. Collectively they will call state equations; even though this particular matrix equation is referred to discrete equation. Now, this is a, these 2 equations are actually matrix equations where, each matrix is have an appropriate dimensions X X dot u and Y are columns or vectors. A B C D are matrices appropriate dimensions.

Now, in the Laplace Transform domain the Laplace Transform each 1 of these matrices; that means, each in the single entry in the matrix is transformed in to the Laplace Transform domain. And using that, we arrived at this final solution for the output.

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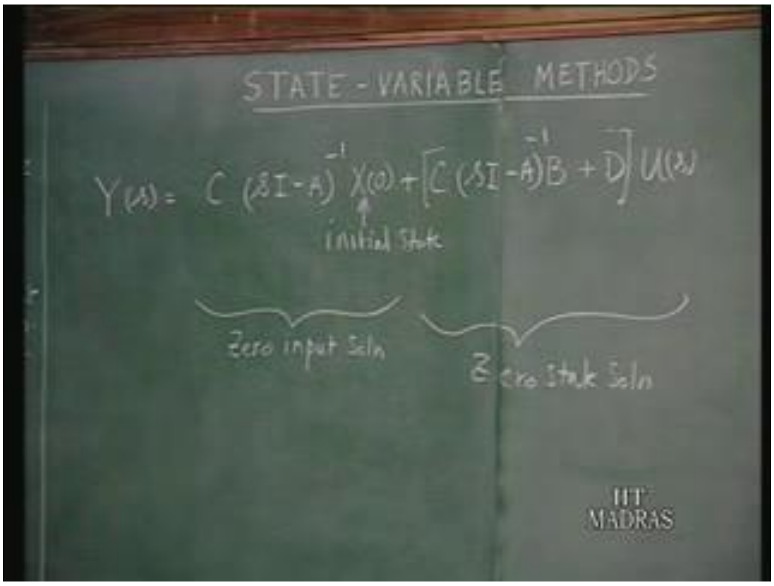


Y of s C times s minus A inverse X 0 plus C times sI minus A inverse B plus D multiplying u of s. So, this is the final solution and the transform domain. And once in find inverse transformation you will get the output in time domain. Now, before we do that we recall, I mentioned that if the initial state was 0; that means, X 0 by the way X 0 is the initial state vector; initial state. It is not X of s which s substituted by 0 it is not that, it is X 0 is X of t when t equal 0. That is the initial state.

If the initial state was 0 this would be the solution. Therefore, this part of the solution is called the 0 state solution. This is the 0 state solution. On other hand, if you had a non 0 initial state that 0 input that is, if the system which some initial energy is allowed to go on its own. This would be the solution and that would therefore, be called the 0 input solution.

Now, to find out the nature of the 0 input solution and 0 state solution, we should look at these terms carefully. What we are having here is sI minus A inverse I is unit matrix of dimension n. A is also a square matrix of dimension n. Therefore these 2 are conformable for addition or subtraction.

So, if I take sI minus A inverse sI minus A.

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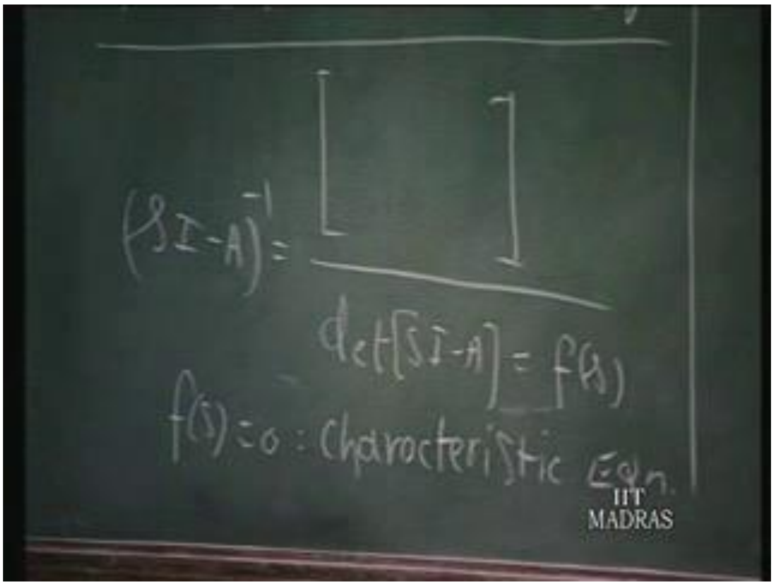


Is a square matrix of order n. If, I take the inverse of that you will have, a matrix which corresponding to a joint of this divided by the determinant of sI minus A. The determinant of sI minus A as you can see is a polynomial in s. Because, A is a matrix with constant coefficient and therefore, there is s only the diagonal terms. So, if you evaluate the determinant you will have a polynomial in s of degree n where, n is the dimension of the state vector.

So, if I call this F of s, the determinant of sI minus A turns out to be a polynomial s of dimension for degree n and this is F of s turns out to be the characteristics polynomial that, we have been talking about in terms of in our system evaluation earlier on. So, F of s equals 0 turns out to be the characteristic equation of the system. So, when you have in the Laplace Transform domain C time sI minus A inverse X 0 and so on. The denominator of this, in this each term in the matrix the denominator we have, F of s as the denominator in each 1 of these terms of the matrices, similarly, here.

Therefore the poles of the term that, come about from here.

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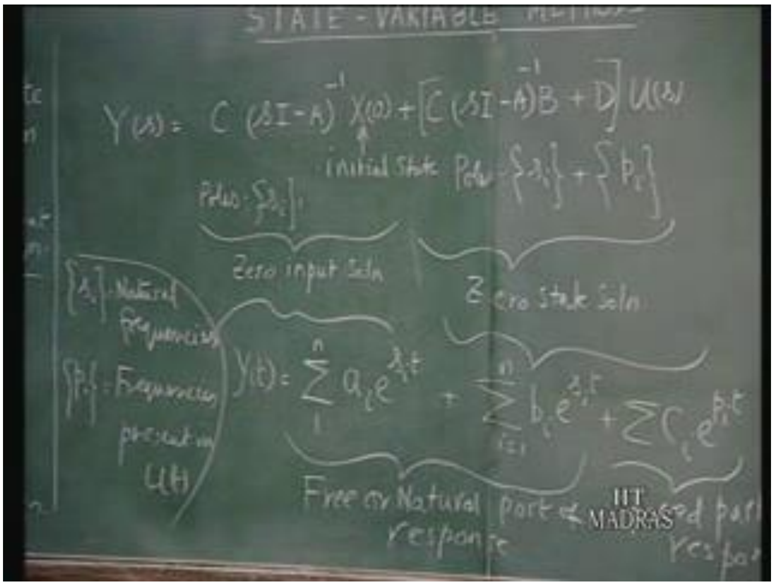


In this or the set of zeros of F of s equals 0. The characteristic equation and the characteristic equation the roots of the characteristic equations are; suppose s i these are the natural frequencies, s i is a set of natural frequencies. So, I will write here: s i of the natural frequencies natural frequencies of the system, which will be the roots of the characteristic equation.

Whereas, here when you look at this, you have the poles come about not only from sI minus A inverse, but also u of s, after all u of s is a forcing function. Therefore you look at the Laplace Transform of u of t then, you have poles arising from that. Therefore, as far as this term is concerned your poles, not only the natural frequencies s i; also p i where p i are the frequencies present in u of s. So, p i will be the complex frequencies present in u of t.

So, corresponding to the complex frequencies present in u of t, you have additional set of poles p i here. So, when you take the Inverse Laplace Transform of this Y of s. So, Y t equals you have for the 0 input solution, we have poles which correspond to natural frequencies only. Therefore, you have a set of terms like a i e to the power of s i t. Assuming that all the natural frequencies are distinct, all the poles are distinct; you have a set of terms like this. And this is of order n you have 1 to n. As far as the second group of terms are concerned, you have poles s i plus p i.

Therefore, you have another set of terms p i e to the power of s i t i from 1 to n plus, the third group of terms c i e to the power of p i t where, p i are the natural frequencies or the complex frequencies present in the u of t. And depending upon the nature of the function, you will have a number of terms we cannot specify that in advance what they are. So, therefore in the final solution for Y t you have this.

Now, this is a single output system, if there is only 1 output; this is a scalar. So, this a i s i t this is, all this will add up to 1 single entity. On the other hand, if you have multiple output solutions, this is a matrix, this will be a column and therefore, this will be appropriate columns a i b i c i will be, these are the whole group of terms will be a column. This will be a column, this will be a column. But for clarity let us, imagine that we have a single output system therefore, these are individual each of them is a scalar function.

Now, therefore, the 0 inputs solution will have, only the natural frequencies present, this is again the 0 input solutions. This is the same division or same appear here also. Therefore the 0 input solution will have only a natural frequencies present. Because, it is a it is not driven by any external agency, the initial energy will allow the system to go on its shown. These are the natural frequencies. But the 0 state solutions, even it thought even if the initial conditions are 0 because, the steady state solution may not match the initial condition that we have 0. Therefore, you have in the forced response in the 0 state solutions not only the natural frequencies, but also the forced response.

So, we observed now if you take only this portion that will be normally what, we refer to as the forced solution of the response, forced part of the solution, which we normally get from the particular integral solution of the differential equations. So, this is the forced part of the response. On the other, hand these 2 groups of terms have natural frequencies present this is for free response or the natural response; free or natural part of response. When you solve normally differential equation of high order you have particular integral solution and the complementary part solution.

The particular integral solution corresponds to this; that is the forced part of the response. The complimentary solution corresponds to both of these. But on the other hand, if the initial conditions are 0 the complimentary part will correspond this. So, that is the distinction between the various components of the response. And now what we can see is; suppose you are interested in superposition the responses due to the individual excitations, you apply, you want to apply the principle of superposition. We like to see what parts of the response; to what parts of the response you can apply superposition. After all you have this equation u of s is the input quantity. Therefore, if u of s can be expressed u 1 of s plus u 2 of s plus u 3 of s plus and so on, you can see certainly this is linear in proportional to u of s.

Therefore, you can apply superposition to this part. But you cannot apply superposition with this present, the total solution because if, you apply if you find the solution for u 1 of s, total solution you will get this, when you apply this find the solution for u 2 of s, you get again this same solution. Therefore, this is taken into account twice. Therefore, superposition principle will be valid only if you take the 0 state of solution. It is also valid if, you take the forced part solution that is, there are particular integral solution which we have already saw.

On the other hand, if you replace the initial conditions, initial state initial condition in network by equivalent sources, this will also be thought of as equivalent source; that means the initial state is assumed to be 0 and the initial conditions are replaced by equivalent sources. Then you can apply superposition to the whole solution because, this portion will be absent. So, we can summarize this; what we said now is.

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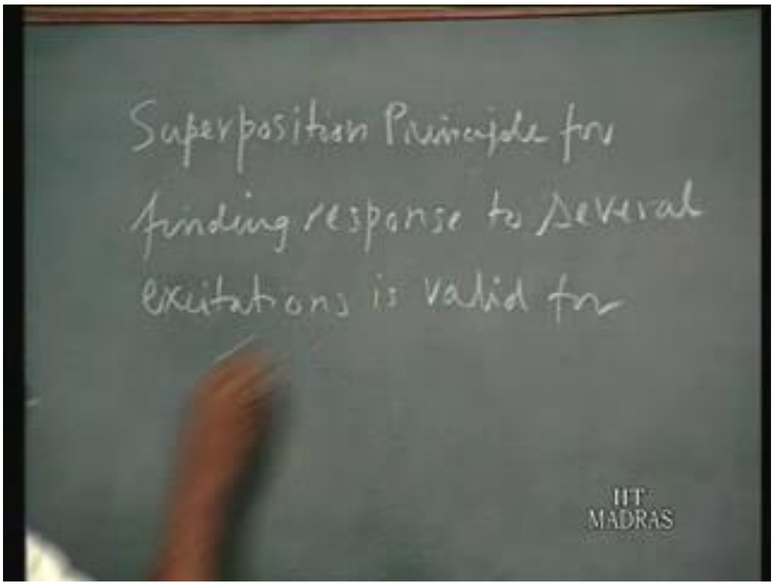

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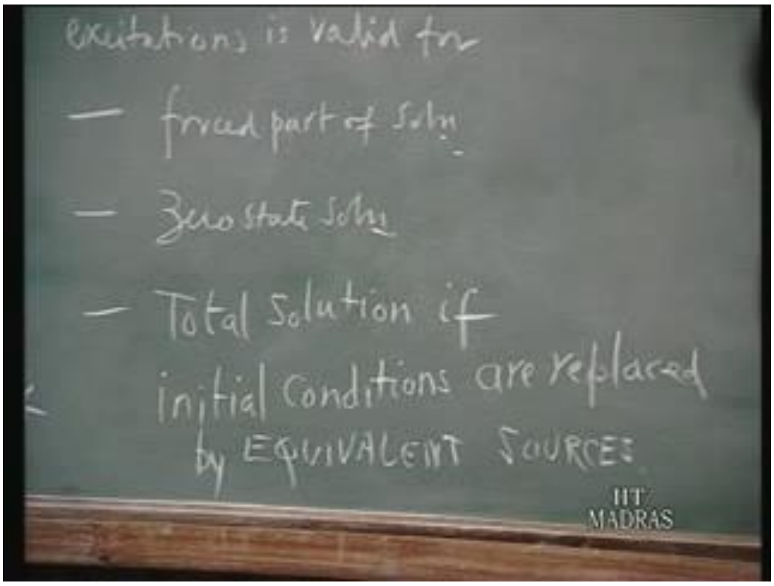


Superposition of responses; superposition principle for finding response to several excitations is valid for: forced part of solution that is, this portion. It is also valid for 0 state solution that is, this entire thing. It is also valid for total solution if, initial conditions are replaced by equivalent sources. This is in the context of networks; when you replaced capacitor voltages, capacitor initial charge in capacitor and using the current inductor by equivalent sources then, you can assume this to be essentially a 0 state solution, 0 state situations. And therefore, you can use still use the principle of superposition because, you take the initial conditions it accounts only ones, not several times.

So, that is the import of these equations. And now, in the last class we have also seen that, if the input if the initial conditions are 0; that means, we have only the total solution only 0 state solution and the 0 state solution is proportional to u of s. Therefore the response in transform domain is, proportional to the input in transform domain and the proportionality factor is given by this quantity.

So, let us look at that little more closely.

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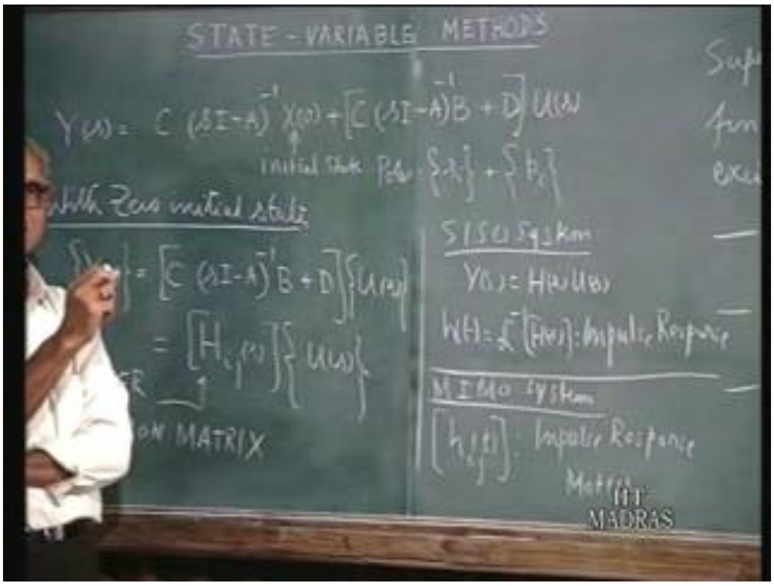


This is the general solution for the output. So, with 0 initial state, we have Y s the output vector is C times SI minus A inverse B plus D a matrix multiplying the input vector u s. So, the output vector is related to the input vector by a matrix which may call H i j s. So, H i j of s is called the transfer function matrix. In the case of a single input single output system this happens to be the familiar transfer function H of s.

So, the, if you have single input single output system, single input single output system. We have simply Y s equals H s times u of s. And the impulse response h t is the Inverse Laplace Transform of H of s. This is the impulse response a concept which, we are familiar with already. So, the impulse response is obtained as the Inverse Laplace Transform of this quantity, if it is a single input single output system.

If, you have multiple inputs multiple outputs systems then Y s is related to u of s by this matrix and if you take the Inverse Laplace Transform of this, That means: Inverse Laplace Transform of each individual term, you have a matrix h i j of t. This is called the impulse response matrix. What is the meaning of the impulse response matrix? A particular term h i j t represents the response at the i'th location, due to an impulse at the j'th the j'th input being a pure impulse all over the input being 0. So, if you expand the matrix equation you will immediately see that, h i j t represents the i'th output due to an impulse at the j'th input with all other input is being equal to 0.

So, that is generalization of the impulse response of that we know for single input single output system. And the impulse response matrix is obtained as Inverse Laplace Transformation of h i j of s matrix which, the dimension of this depends on the number of inputs and the number of outputs. In the case of single input single output system this will be a scalar, the impulse response will be obtain that Inverse Laplace Transformation of this, which form the system function or transfer function C times sI minus A inverse B plus D. That is what we have.

Now, let us clarify these ideas by working out in example, finding out the solution with Laplace Transform approach.

Let us consider this example where we have

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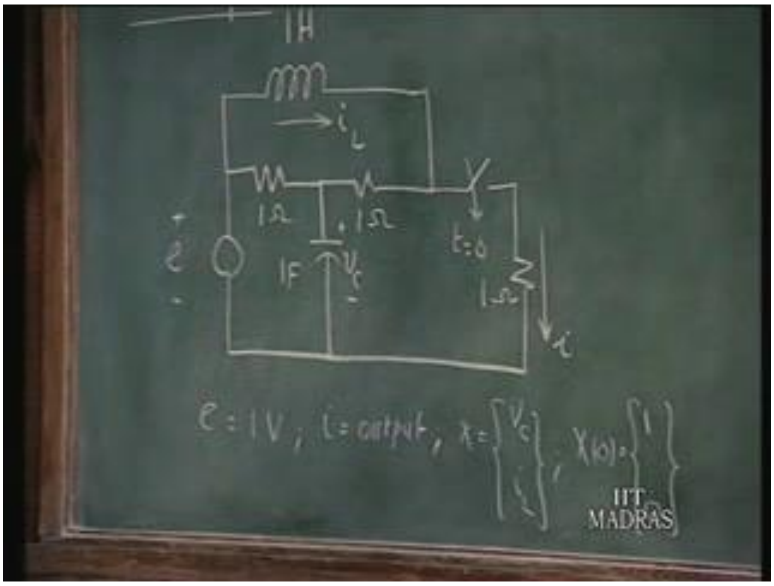


A voltage source e let us take this e to be 1 volt dc. That forms the input quantity single input and i is the output, this is the output. We have 1 inductor and 1 capacitor, we take V c and i L as the state vector v c and i L. So, we have this is correspond to u. This corresponds to Y and this corresponds to this state vector. We close the switch at t equals 0 and we are asked to find the solution for i.

Now, to form the solution we need to know something about the initial conditions, we will assume that is circuit is steady state before the switch is closed. And therefore, the initial vector X 0 will be composed of the initial value on the voltage at the capacitor and the initial value of current to the inductor. When this switch is open and kept open for a long time ultimately, this capacitor will charge to 1 volt which is the supply voltage. And at that time there will be no current in the inductor because, the both these 2 are same potential and so, we say X 0 equals 1 volt and i L 0 equals 0. So, this is the initial state.

So, the initial state is 1 0. Now, using our method that we have already discussed, for framing the state and output equations. It can be shown that the equations corresponding.

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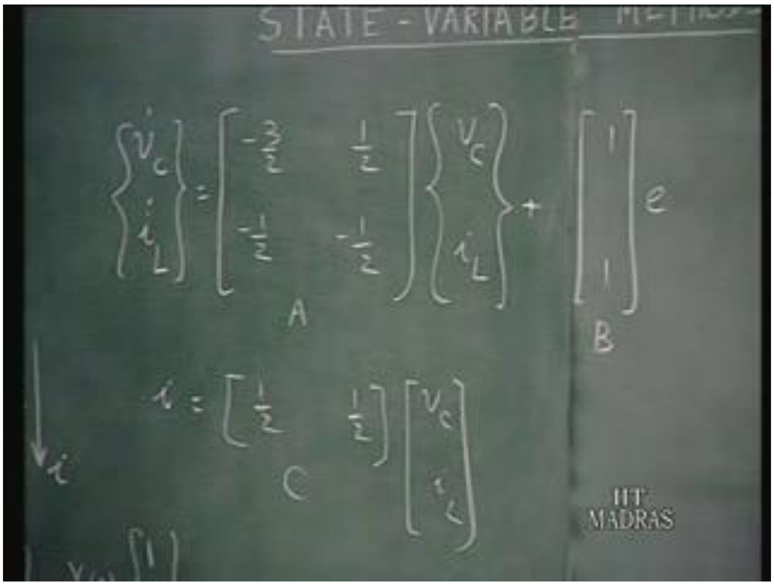


To the state equation corresponding to this particular network can be written down. I will not go to the details; I will just give the final result. This is e minus 3 upon 2 half minus half minus half and 1 1 that is the state equation. Corresponding the output quantity i this is Y this will be half half V c i L and the matrix D is absent that is going to be 0. So, this is the first, the state equation. The second is the output equation. Therefore this is the A matrix. This is the B matrix. And this is the C matrix. D matrix is being equal to 0.

So, in order to find the solution, we must first find out s I minus A inverse that is most important part of the solution.

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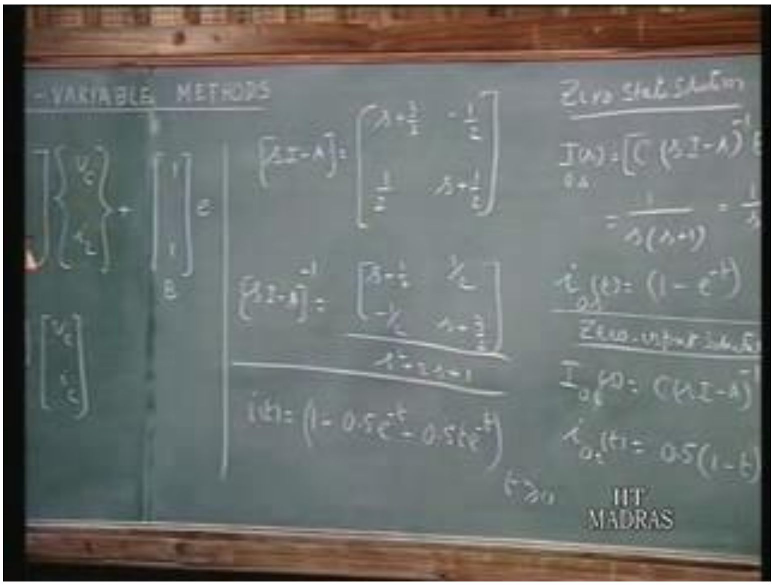


So, sI minus A this is A matrix. So, sI minus A turns out to be s plus 3 upon 2 minus half half and s plus half that is sI minus A. We have to find out its inverse; sI minus A inverse, by the usual rules can be shown to be equal to s plus half half minus half s plus 3 upon 2 divided by s squared plus 2s plus 1. That means: s squared plus 2s plus 1 is a common denominator for each 1 of these entries. So, to simplify our writing I put this as common denominator; that means, each term is in the matrix has got the common denominator. That is sI minus A inverse.

So, in order to find out the output I of s, we will let us find out the 0 input solution and the 0 state solutions separately. So, 0 state solution I of s equals C times sI minus A inverse B plus D times u of s which happens to be in our case, E of s which happens to be 1 by s because, the input source is 1 volt dc. Therefore, its Laplace Transform is 1 by s. And we also have that D is 0 in our case.

So, we have the matrix C and D, we know this is C and this is D and you multiply that by sI minus A inverse, the final solution will turn out to be 1 over s times s plus 1 1 over s times s plus 1. This will be 1 by s minus 1 by s plus 1. So; that means, the 0 state solution for the output 0 state will be 1 minus e to the power of minus t valid for t greater than 0. That is the 0 state solution.

The 0 output solution, some 0 input solution will be I will say 0 state 0 input solution 0 oi of s equals C times sI minus A inverse times X 0; X 0 is being the initial state vector 1

0 and using the appropriate numerical values, you can show this to be 0.5s over s plus 1 whole squared. And taking the Inverse Laplace Transformation of this, the 0 input solution will be 0.5 times 1 minus t e to the power of minus t valid for t greater than 0. So, these are the 2 components of the solution. And the final solution i t the sum of these 2, which is the combination of these 2 terms will be 1 minus 0.5 t e to the power of minus t minus 0.5 t e to the power of minus t amperes, valid for t is greater than or equal to 0. That is what you are having.

So, the steps are straight forward; perhaps the most complicated part is only to find out the inverse of this. And then of course, you have to take the Inverse Laplace Transform in various quantities. So, you observed that the 0 state solution contains not only the natural frequency, which is coming minus 1, but also the forced part of the solution. Because, the forced the input is dc the output is also dc.

Whereas, the natural part of the solution which is the 0 input solution contains 3 frequencies minus 1 actually, if you take the characteristics equation s squared plus 2 s plus 1 equals 0 which means: there are 2 roots of the characteristic equation at minus 1 repeated routes. And because of that, you have got a t term e to the power of minus t. They are not distinct routes that the characteristic values are repeated to routes that is why, you get t e to the power of minus t as well.

Now, if you are asked to find out the transfer function of the system related in to the input and the output, this is also straight forward.

The H of s in this case.

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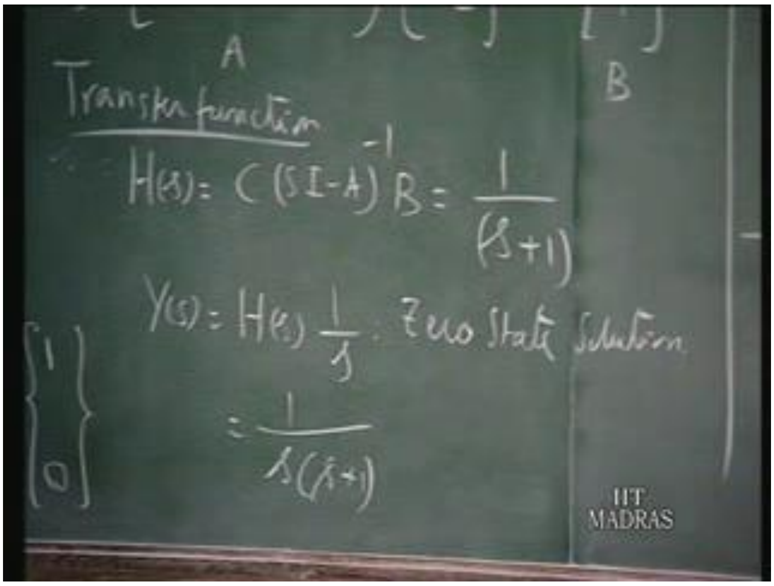


The single input single output system C times sI minus A inverse B because, D is absent and that terms out to be to calculate that that terms out to be 1 over s plus 1. That is the transfer function. We know, that the transfer function relates to the Laplace Transform of the output, the Laplace Transform of input with 0 initial conditions. Therefore, if you take Y s equals H of s times the input 1 over s. This must correspond to the 0 input solution, 0 state solution. The initial condition is being 0 and that is indeed equal to 1 over s times s plus 1 which, we have already found out to be the case. This is 1 over s times s plus 1.

So, the transfer function times the input will give you the response which is; the 0 state solution. Because the transfer function is defined as Laplace Transform as output in Laplace Transform of the input in 0 initial condition which means; 0 state conditions. So, this example illustrate: what we have discussed, by way of the Laplace Transform method of solution of the state and output equations.

Let us now discuss how to solve the state and output equations in time domain. After having discussed the Laplace Transform method of solution of the state and output equations. Let us now discuss the methods of solution of these equations straight away in time domain, without having to go to the transform approach.

To do this.

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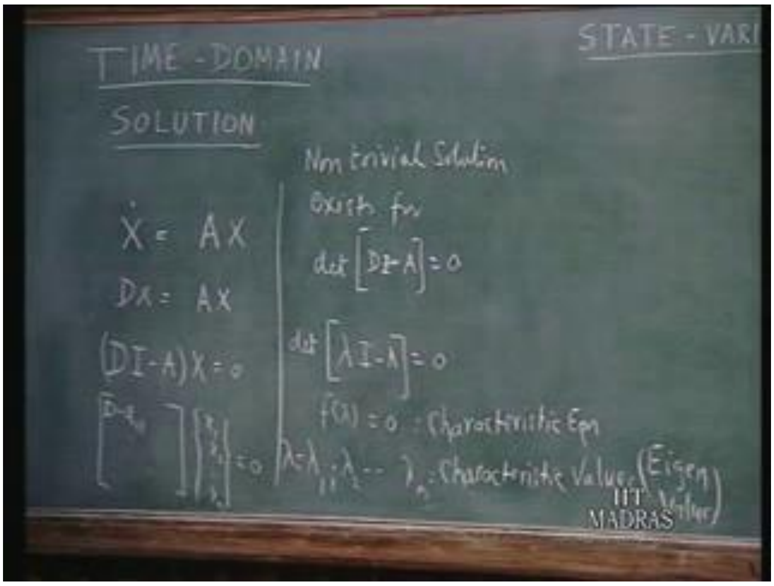


Let us first discuss the solution of the homogeneously equation where, the forcing function is 0. So, we have a couple of, we have coupled first order differential equation X dot equals A X. Now, X dot equals X 1 dot X 2 dot and so on this vector. Using the differential operators symbol, instead of X dot I can write this as D X equals A X where, D is the differential operator; that means, D of X 1 D of X 2 like that it present vector. We can combine these 2 terms into 1 write D I minus A X equals 0 whereas, this is the matrix; that means, you have got here matrix operating on x 1 x 2 x n equals 0 this is the matrix equation, where the first entry here will be D where D is the differential operator minus A 1 1 where, A 1 1 is the first element in the matrix like that it goes on.

So, this is a matrix with a with differential operator D. Now, in order to find the solution for this, this is homogeneous equation. So, no trivial solutions exist for a set of a matrix equation like this, when the determinant of this is going to be 0. Otherwise x 1 equals 0 x 2 equals 0 x n equals 0, there is a trivial solution. A non trivial solution will occur, when the matrix of this, when the determinant of this matrix is equal to 0. So, we will say non zero solution non trivial solution exists for: determinant of D minus A DI minus A equals 0.

So, when you have that, this means; this is the auxiliary equation that we have for in order differential equation approach. That means: the determinant of instead of D the auxiliary equations and differential equation of approach we write M. But here let us write lambda; lambda I minus A determinant of that equals 0. So, when you have for values of lambda for which, this determinant is 0 you have non 0 solutions. And this turns out to be a polynomial in lambda, which is the characteristic equation, this is equal to 0. This terms out to be characteristic equation.

In the earlier analysis, we use the symbol s in the Laplace Transform approach. It is the same equation, but in this in conventional to use the term lambda in the differential equation approach that is why, we use this lambda. So, f lambda equals 0 it is a characteristic equation just like F of s equals 0 it is a characteristic equation, when we talked in the Laplace Transform domain. When talked about this Laplace Transform domain; both are 1 at the same.

So, this particular characteristic equation has got routes lambda 1 say, lambda equals lambda 1 lambda 2 up to lambda n these are called characteristic values or risen values. So, in terms of this characteristic equation, this characteristic values or also called Eigen values. So, whenever lambda equals lambda 1 lambda 2 and so on then, this particular characteristic equation is satisfied and these are characteristic values. And once you have for such values of lambda 1, you have a non trivial solution. That means: this particular equation has a non trivial solution whenever, D equals lambda 1 lambda 2 or lambda n; that means, the non trivial will be.

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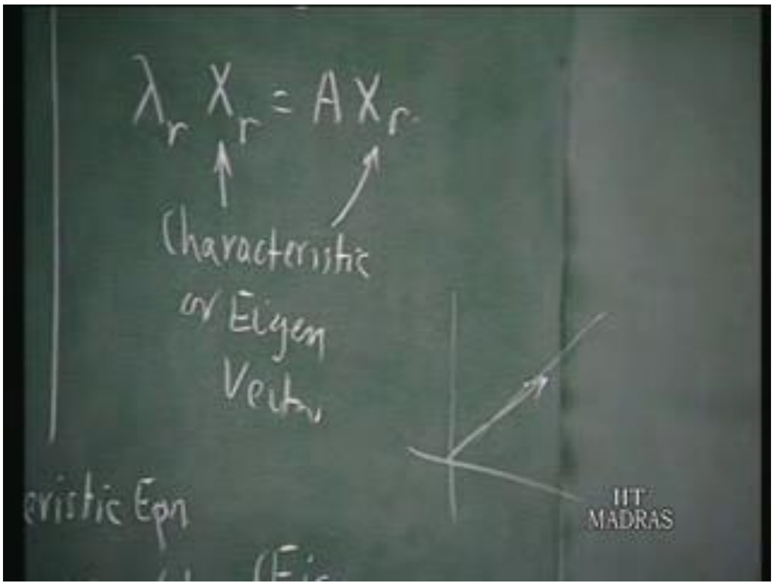

For each lambda, for each Eigen value lambda r you have a solution lambda r times X r equals A times X r. So, look at this X r is the characteristic vector or characteristic this called characteristic or Eigen vector. So, for given matrix A we have a special square matrix A, we have a special property that, there are certain characteristic vectors a column X r such that, when you multiply this X r, pre multiply this by a square matrix A its equivalent to multiplying in this by a scalar constant lambda r. That means: if you have n dimensional space if, you consider think of this X r as a vector n dimensional space. Then when you multiply this matrix A with all what it does is; enlarge that vector or decrease that vector by scale factor lambda r. That means: the direction of the vector does not change.

So, if you think of its 2 dimensional situation if, I have a particular vector like this x and y if you multiply this by A 2 by 2 matrix that, this matrix by A 2 by 2 matrix this, this vector the direction of the vector will not change because, all it does is when you pre multiply this by matrix A this vector will be multiplied by a constant. That means: the same direction will be maintained; it will either increase or decrease. That is the meaning of these characteristics vectors.

So, for each matrix A we have a set of characteristic vectors and the characteristic vectors, each characteristics vector is related to or associated with a characteristic value which is called Eigen value. And further more this Eigen values are obtained or characteristic values are obtained by the equation determinant lambda I minus A equals 0 which the same as f lambda equals 0 this is the characteristic equation.

So, this is the background which we would like to have. Furthermore, we also would like to have this all by a way of background. We would also like to have an introduction to the concept of a polynomial of matrices; just like you have a polynomial.

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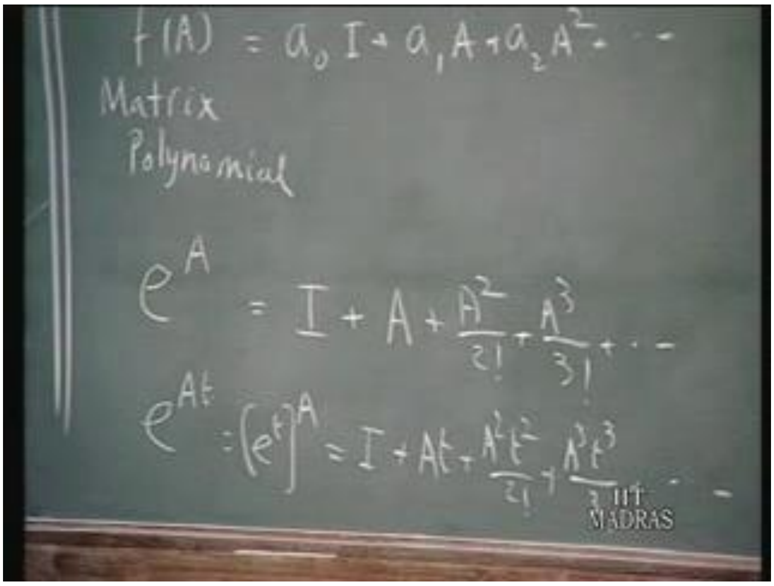


f of x suppose you got a not plus a 1 x and so on and so forth. We can also thing of a polynomial of matrices square matrices. Suppose, f of A this is the matrix polynomial. So, you have just like a matrix a polynomial x a constant quantity a variable x single a scalar x. Here also you have a not I because, these all terms are matrices a not I plus a 1 A plus a 2 A squared and so on and so forth.

So, this is the matrix polynomial with a various coefficient a not a 1 a 2. Each 1 of these terms is again A matrix. If, the matrix A is n'th order matrix that is the square matrix for order n.I is the unit matrix of order n A is of course, a square matrix of order n A squared also is a square matrix of order n because, here multiplied by A the number of columns and rows will still remain as n. Therefore each 1 of these terms represents a square matrix of order n and therefore, the compactable for edge.

So, we can think of this matrix polynomial only for square matrices and once you have a square matrix, we can define matrix polynomial like this. In particular; we would like to know what is meant by e to the power of A. So, we know e to the power of x equals 1 plus x plus x squared by 2 factorial x cubed by 3 factorial and so on and so forth. Similarly, we write here as: the first term is in the scalar case is 1. But in this case, we must thing of matrix I unit matrix of dimension n.

So, A plus A plus A squared by 2 factorial plus A cubed by 3 factorial and so on. So, that is what is meant by matrix exponential. That is e to the power of A where, A is a matrix is written in this form. Suppose, I have e to the power of At where t is a scalar. This can be this thought of e to the power t raised to exponent matrix exponential like this e to the power of At. You can still like this; you will have this will be write this as 1 plus At plus A squared t squared by 2 factorial. After all any matrix multiplier by a scalar is another matrix square matrix At. So, you go on multiplying this plus A cubed t cubed by 3 factorial etcetera. So, that is meaning of e to the power of At.

So, with this background let us now, look at the solution of this matrix equation X dot equals A X.

Before trying to solve the matrix equation let us look, at the scalar counter part

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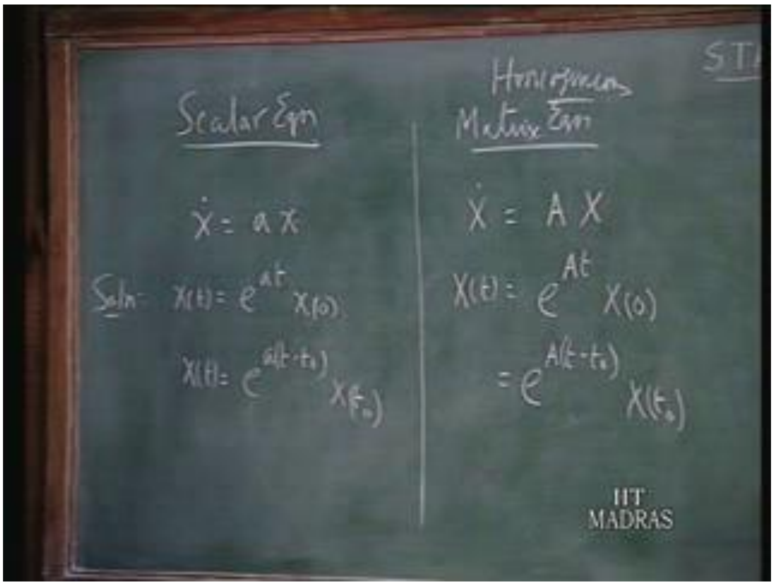


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Suppose, I have X dot equals a x that is dx by dt equals a times x where a is the constant. In the scalar equation what we do is; when you got dx by dt is equal to a x. Then, the solution for that we know is X equals e to the power of a t times a constant which we write when t equals 0 X of t when t equals 0 this becomes 1 X 0 that is, the particular initial condition. So, we can the solution for that is, solution for this equation is this 1.

If, instead of t equals 0 you are given the initial value of X of t at another point t equals t not then, you can write this as X of t equals e times a to the power of t minus t not times X of t not. So, when t equals t not this becomes 1 and X of t not equals coincide to this.

So, these are the alternative forms solution of the scalar equation where, the initial conditions are prescribed at t equals 0 or arbitrary some other point, you can make yourselves. So, the arbitrary constant that is involved in the solution of this equation, can be expressed in terms of X 0 or X t not whatever here you are having. This is the scalar point.

Now, the matrix equation in a, the homogeneous matrix equation; that means, there is no forcing function without X dot equals A X where, A is the matrix X is a vector and X dot is a vector. So, here also it turns out, we will demonstrate in a moment that, X of t that is the solution for that we will turn out, to be e to the power of At times X 0 just like here, instead of a scalar here you have a matrix. Otherwise, the form will be in the same or alternately you can write this e to the power of A t minus t not times X evaluated t not. So, this is the solution that we have got in the matrix. This is the counter part of this.

So, if you remember this scalar solution, you get the matrix solution identical form except that, instead of A you have got e to the A t minus t not.

Now, how does this come back out, just simplification for that

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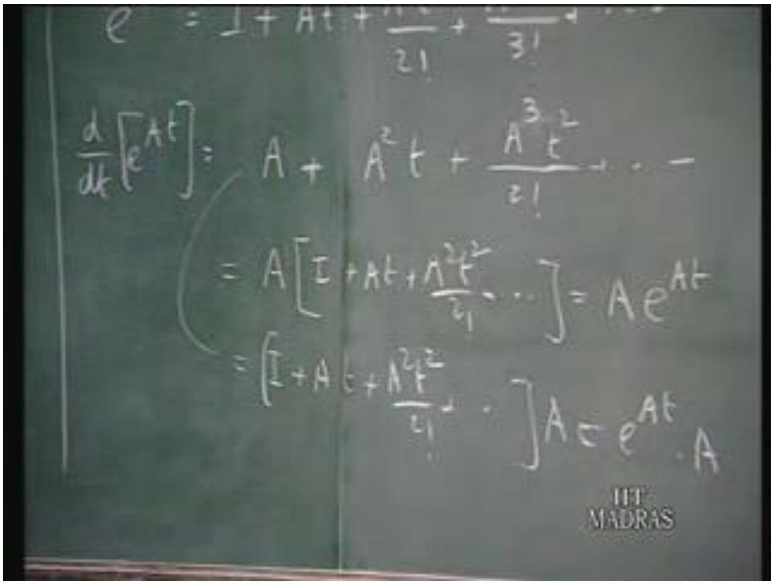


e to the power of At we know, is a unit matrix times At plus A squared t squared by 2 factorial plus A cubed t cubed by 3 factorial and so on and so forth. Now, each is a matrix and each is a function of time. So, suppose I take d by dt of e to the power of At.

That means: you take the differential derivative of each 1 of this terms, in each 1 of this matrices. So, you have this is the constant therefore, that becomes 0. Therefore, here every entry in the matrix A is multiplied by t, this is a constant matrix. Therefore, when you multiply every entry by t, when you take the derivative all you get easier.

Similarly, here this is A squared times t squared. So, every entries multiplied by t squared. Therefore, you take the derivative this becomes 2t. Therefore this is 2 factorial is also a common term. Therefore, you have got A squared t. Then, like wise you get here this is 3 t squared. So, you get A squared by 2 factorial t squared A cubed into t squared by 2. So, this can be a like that it, go it will go on. So, this can be put as; you can take A as a common factor out, you have got unit matrix and At plus A squared t squared by 2 factorial and so on and so forth. And this could be written as A times after all this entire thing is e to the power of At.

So, A times e to the power of At. All this terms are involved powers of A squared. Therefore, you can also write this entire this equation as, I plus At plus A squared t squared by 2 factorial etcetera. And post multiply that by A then also, you get the same equation. So, you can write this also as e to the power of At times A. So, what we find is d by dt of e to the power of At is the matrix A e to the power of At either pre multiplied by A or post multiplied by A.

Furthermore, suppose I have.

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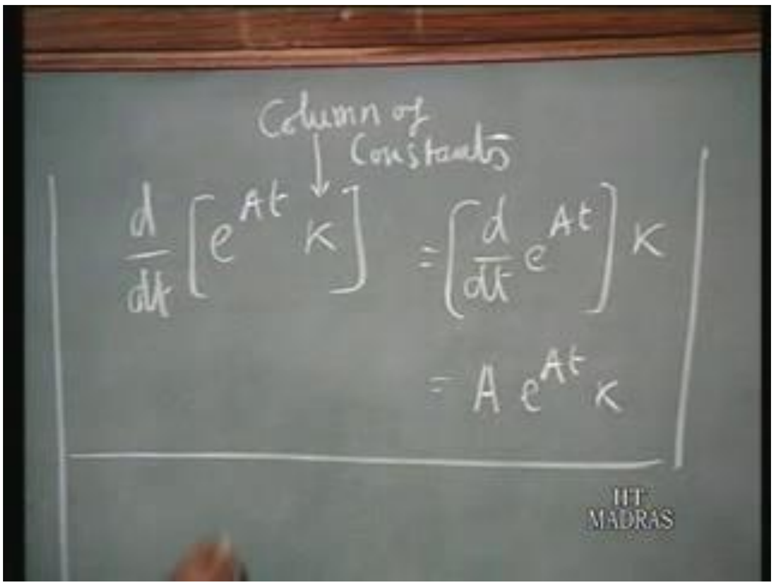

e to the power of At multiplying a column K; column of constants. So, we would like to find out its derivative you can write this as d by dt of e to the power of At multiplied by a constant column K plus e to the power of At multiplied by d by dt of K which is of course, 0. So, that is quite straight forward you will leave it as. And this we know just now, we found out this is A times e to the power of At times K d by dt of e to the power of At equals A times e to the power of At. So, we have d by dt of e to the power of At K is A times e to the power of At plus K.

So, we are looking for the solution for X dot.

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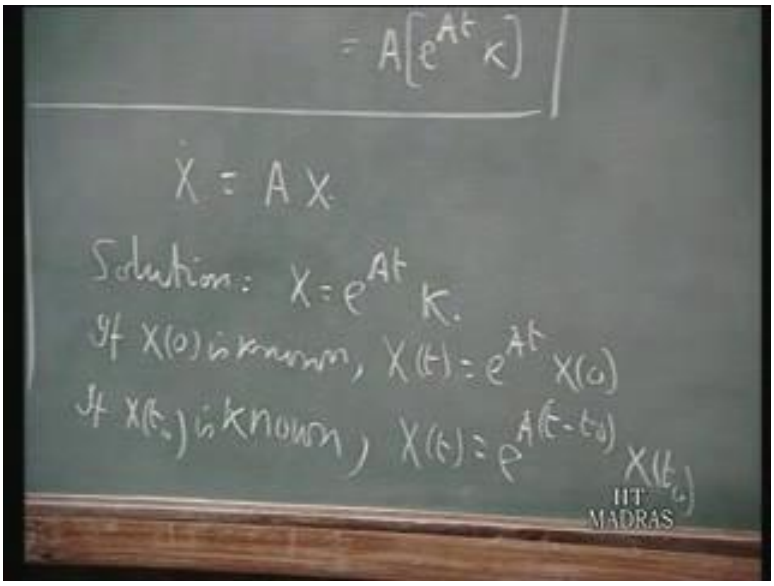


equals A X, we are looking for solution of this matrix equation. You look at this equation here; if X happens to be e to the power of At times K then, the derivative of that is A times e to the power of At K. So, this particular equation as a readymade solution here. After all e to the power of X equals e to the power of At K satisfy this equation because, taking the derivative of that will be A times e to the power of At K. So, solution for this is X equals e to the power of At times a column K. So, that is what we are looking for. That is exactly what we are having here.

Now, K is an arbitrary constant K is an arbitrary constant. So, if you want to find out the arbitrary constant, you must know the value of this vector X at some particular point of time. If X 0 is given, if X 0 is known then, we can write X t is e to the power of At times X 0. On the other hand, if X at a particular point t not is known then, we can write X t not equals e to the power of At times K and from that, you can evaluate K and that can be shown: that X t equals e to the power of A t minus t not times X t X t not. So, that is what you are having. That goes are exactly the solution of that, we have put down here. These are the exactly the solution that we have put down here. So, this is the justification for that.

Now, this e to the power of At is therefore, a very important factor in the solution of the matrix equations.

So, e to the power of At is

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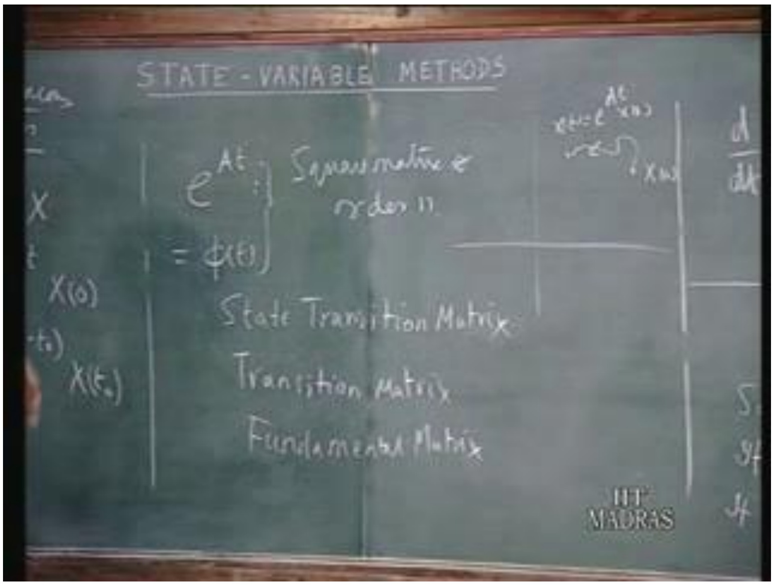


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What is the nature of that? It is a square matrix of order n where; n is the order of factors A. We also can write this as phi t is another way of looking at this, another way of denoting this. And this e to the power of At is referred to as e state transition matrix. I will explain the meaning of this in a moment. It is also called transition matrix. Sometimes, called the fundamental matrix in this context.

So, what this means is X t not multiplied by e to the power of At not means to X t. So, if the state is represented in n dimensional space, suppose you have a n dimensional state, this is the position of the state at time t equals 0. You want to find out the position of the state at time t; you must multiply X 0 by e to the power of At. So, if you multiply that e to the power of At it brings you some another value X t which is equal to e to the power of at times X 0.

So, for different this is the trajectory; that means, the state will move along in n dimensional plane and this is the initial position of the state. If, you want to find out the position of the state at a time t, you have to multiply X 0 by e to the power of at that is why, this is called state transition matrix. Multiplying by the initial vector by that e to the power of at will, make the transition from the initial state 1 to another state; it gives to you the how it transits from 1 state to another.

So, this is the trajectory of the state at every point you have to multiply x 0 by the appropriate value of e to the power of At. So, that is the meaning of the state transition matrix. And this as very useful role, in a system studies involving state space methods.

We have seen some properties of e to the power of At. I will just indicate the important properties of the state matrix.

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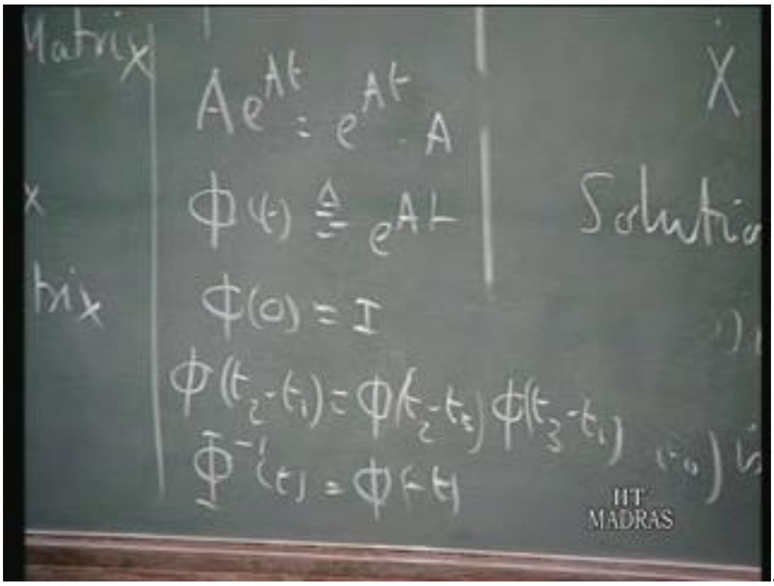

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We have seen that, A e to the power of At equals e to the power of At times A, we have seen that. And this is also phi of t this is also written as, phi t is defined as e to the power of At. We have seen that suppose phi 0 equals identity matrix 1 t equals 0 that is unit matrix. We can also show that phi t 2 minus t 1 equals phi t 2 minus t 3 multiplied by t of t 3 minus t 1. You can show that the product of these 2, will be equal to phi t 2 minus t 1 and finally, you can show that phi inverse of t equals phi of minus t. That means; if you take the inverse of a particular matrix that is, equivalent to making the argument, negating the argument phi inverse of t equals pie of minus t. These are the various properties state transition matrix or the transition matrix as it is called.

Now, to complete this solution then, we must know how to evaluate e to the power of At and that is what we will take up in the next lecture. So, in this lecture, we started with illustrating the solution of the state and output equation for the Laplace Transformation method. We will look at the different components of the response, the 0 state response and the 0 input response. We saw when the principle of superposition can be applied to find the total solution, total response due to individual excitations.

We worked out the example to illustrate this technique and then we started discussion of the time domain solution of the state and output equations. In that we took the homogeneous equation to start with assuming that, the forcing function at the input is 0 that is X dot equals A X. And we said the solution for this equation X dot equals A X will be running along parallel lines to the scalar equation solution which we have already known. Instead of e to the power of at, we have e to the power of at which is a matrix. Therefore, this matrix multiplying pre multiplying the initial state, will give us the value of the state at any other time.

So, if the X 0 is the initial state; to find out the state at any point t, you have to multiply X 0 by e to the power of At that is why, this is called e to the power of At is called the transition matrix of the system transition matrix. So, if you evaluate e to the power of At is different values of t t 1 t 2 and t 3 and mark them out in the n dimensional space that will be trajectory, how the state moves from 1 instant of time to another.

In the next lecture, we will discuss phase of evaluating the e to the power of At and used that, those techniques for the solution of the problem on hand.