Networks and Systems

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Lecture – 46

State Variable Methods (2)

Formulation of state equations extended form solution by Laplace transform method

In the last lecture, we have acquired ourselves to the concept of the state of a system; what are what is meant by state variables. And the nature of the equations connecting the state variables, with the input and output quantities. We worked out the example where the state equations are formed when, I say state equations I mean; the state equation and the output equation put together. And we used an adhoc approach, in writing down this state equation pertaining to that example.

There are stream line methods of doing this 1 which chooses topological methods, there are theoretical concepts and use a systematic way writing the state and output equations, in matrix form taking the various that the circuit parameters into consideration. We will not deal with that but, we will take up another general method; which works reasonably well in network and some moderate complexity I will first demonstrate this through an example and then; discuss the general state that involved in adopt in this procedure.

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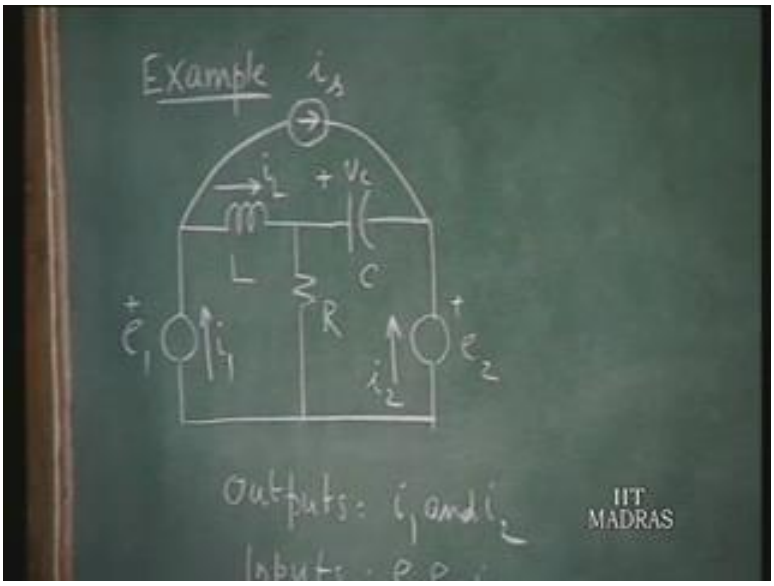

Let us consider: a network like this containing an inductor a resistor and capacitor and there are 3 sources e 1 e 2 voltage sources and a current source i s. So, this is an example how various representative elements 1 resistor, 1 inductor, 1 capacitor 2 voltage sources and current source. So, the input quantities are e 1 e 2 and i s time varying quantities that the sources. And we decide to find the solution for i 1 and i 2. So, i 1 and i 2 are the output quantities of the responses.

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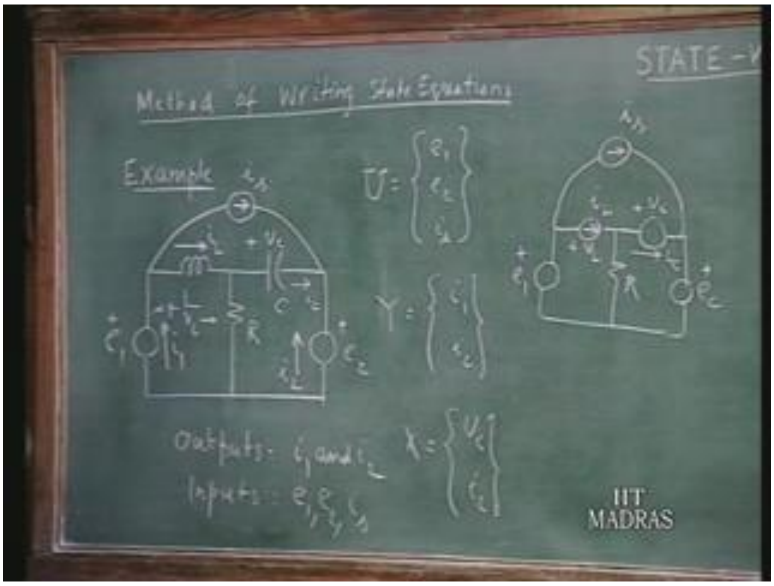

So we straight away see that the u matrix that the input quantities are e 1 e 2 and i s. The output quantities are i 1 and i 2. So, y, I will write capital U and capital Y i 1 and i 2 as I mentioned, in the last lecture its convenient to take the inductor current and capacitor voltage as state variables in the system like this. Therefore the state variables are iL and Vc I will take Vc first and then iL. Therefore, let me put Vc and iL these are the state variables. So that is the natural choice.

Now we would like to express X dot interms of X and U, Bbecause after all you have to write X dot equals AX plus Bu. And what is X dot X dot is Vc dot and iL dot. So the current in the capacitor is Cv C dot. So, we have to express the current in the capacitor in terms of the state variables and the input quantities. Similarly, iL dot the derivative of the current in the inductor is related to the voltage across the inductor VL is L ViL by dt. Therefore, we have to express the voltage across the inductor and the current to the capacitor interms of iL Vc e 1 e 2 and i s.

So, to do this it will be convenient for us to write down the equation, we can replace Vc the capacitor by a voltage source is equal to Vc and the inductor by a current source of value iL. Let us see do that. So, I have i s here and the inductor is replaced by a current source iL and the capacitor is replaced by a voltage source Vc and then; you have resistance R and you have the other sources e 1 and e 2. And we have to find out the voltage across the inductor that is VL. So, we call this VL. And the current through the capacitor I would like to call that ic.

So, that the current here ic and the voltage across the inductor is Vi. So, this is what we are interesting finding out we it is only a convenient representation type putting representing inductor by a current source and a capacitor by a voltage source. You do not have to do it if you can write down the equations for VL and ic in terms of iL Vc e 1 e 2 and ic inversion method its possible. But, what we want to do emphasize that we have to express this voltage and this current in terms of various sources and the non dynamic element that are involved there resistance that is what; we are interested.

Therefore, it is convenient for us to replace this by a sources, because it is cleaning to focus that we have to express the response quantities in the circuit due to the various sources. And also in terms of iL and Vc. So, that you have to express the in terms of iL and Vc so iL and Vc are node sources, it becomes very convenient for us to be solved. So, in this network we have to find VL and interms of i s and iL Vc e 1 and e 2. So, we have several sources now really there are 5 sources. And we have to find out to response quantity VL and ic you can solve this network, in any manner you like you can use the principle of super position take 1 source at a time and find out the contribution of that e source to VL to ic and ad them up.

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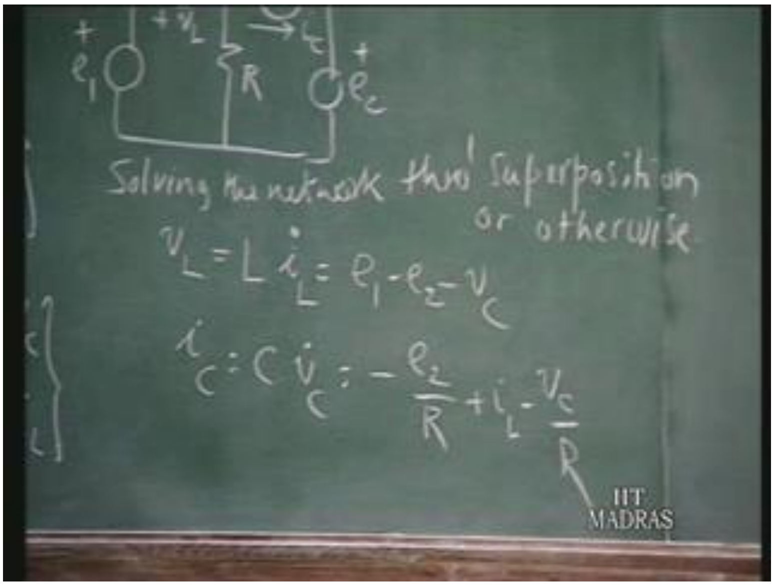

Otherwise, if you any other method you can find out the solution in 1 shot; whatever, might be you can show that VL equals solving this network through super position or otherwise, I will write that there is no compulsion onwards part that you should be only in superposition it can be any other method VL turns out to be VL of course, is know L iL dt that we know. This turns out to be e 1 minus e 2 minus Vc that you can see from this loop VL equals e 1 minus e 2 minus Vc.

So, in according equation for iL dot. Similarly, ic the current which is C Vc dot the current here C times Vc dot can be shown to be minus e 2 by R plus iL minus Vc by R. So, from these 2 equations we can get iL dot and Vc dot by dividing right through by L in the equation and see by the other equation. So, we have 2 expressions for iL dot and Vc dot. The derivative of the state vector interms of e 1 e 2 Vc and iL e 1 e 2 are the state are the input quantities and Vc and iL of course, a state variables.

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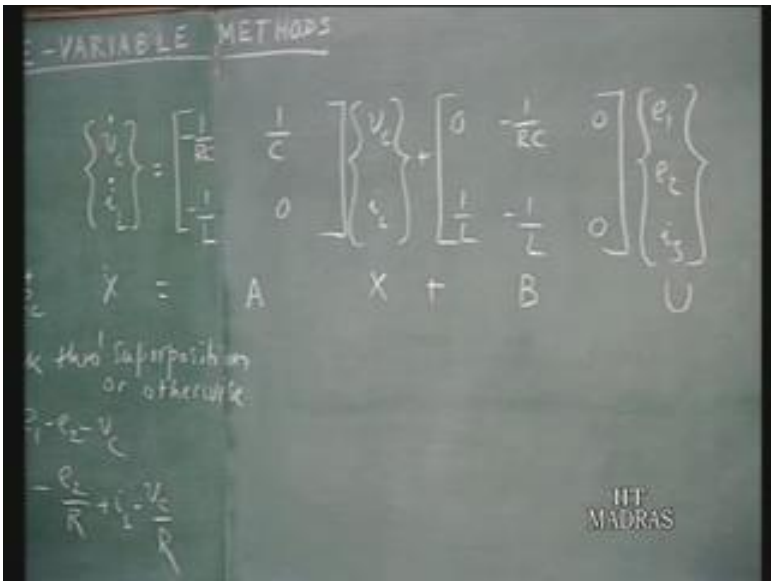

So, we have final result therefore, using this you can write from this you can write Vc dot iL dot this is X dot really is a matrix minus 1 over RC 1 over C minus 1 over L 0 multiplied Vc and iL. This is the matrix A this is matrix X plus Bu u is the input vector which have 2 quantities 3 quantities e 1 e 2 and i s. So, we have to write e 1 e 2 and i s. And we have 3 columns here therefore, and that would turn to be 0 minus 1 by RC 0 1 by L minus 1 by L 0; that is plus B U.

So, X dot equals AX plus Bu that equation has been established. Now, we have to find out the output quantities output quantities are i 1 and i 2 we have to express that in terms of the state variables and the input quantities. We note that I 1 equals I 1 equals iL plus I s. So that is straight forward. And similarly, we can express I 2 interms of e 2 e 1 I s and iL.

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So, I will try this equation first I 1 equals I s plus iL and I 2 equals it can be shown from the circuit diagram e 2 by R minus I s minus iL plus Vc by R. So, you can put this in matrix form as I 1 I 2 equals interms of state variables Vc and iL that will be 0 1 1 by R minus 1 plus e 1 e 2 I s and the 3 corresponding entries here 0 0 1 you see I 1 equals I s plus iL I 1 equals iL plus I s. So other entries will be 0. And I 2 equals e 2 by R. Therefore, I will get 1 by R here e 2 by R minus I s therefore, I have minus 1 sign here minus iL minus iL plus Vc by R that is already be completed.

So, the second equation represents Y equals CX plus DU. So, that is the standard form equation Y equals CX plus DU. So, this is what we do to establish the state and output equations starting from the given configuration.

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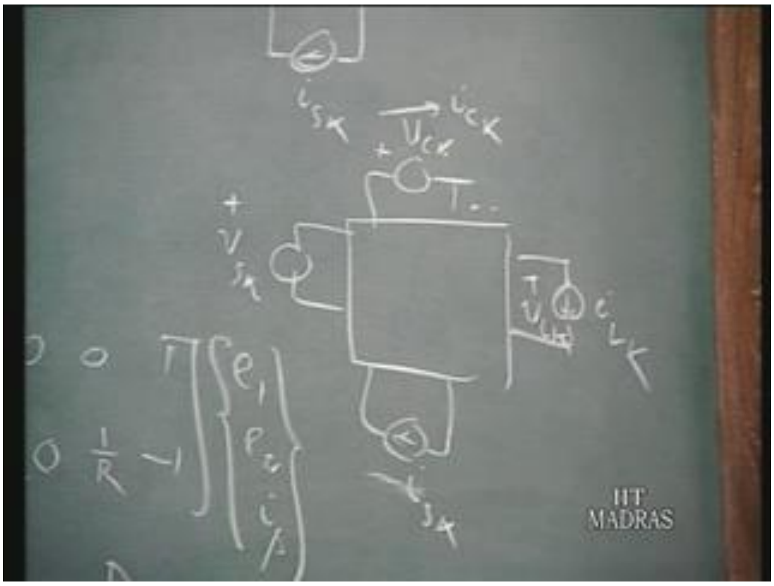

So, let me just review this step that are involved what we do is you have suppose you have a general network in which a various voltage sources. So, suppose say Vsk so, a number of voltage sources and a number of current sources isk. And you have various capacitors suppose: it is Ck and then the Vck and the voltage across that and you have various inductances Lk the current through that iLk. So, you bring the reactive elements and the sources separately well exhibit and whatever, is left inside is non dynamic portion non dynamic part.

It may be purely resistive network or it may also have dependent sources which are non dynamic in character. So, what we are trying to do is you replace this iC Vc the capacitors and the inductors by voltage sources current sources. So, that means; what you are doing now is you are replacing this by a voltage source Vck. And the current and the inductor is replaced by current source iLk. And the others are as before. This is Vsk this is iSk. At this stage we have a non dynamic network packed upon by various sources.

So, you find out the current iCk in this and the voltage VLk in this. So, except Vck and iLK for various case that is various capacitors and inductors interms of Vsk iSK iLk and Vck ILk and Vck that is set constitute the state variables iSk this iSk and then Vsk constitute the input quantities. So, you find iCk and VLk which are interned after all iCk is Ck times Vck dot and VLk is Lk times iLk dot. So, in other words you have found out

Vck dot and iLk dot for various values of K interms of Vck iLk; which constitute the state variables and Vsk and iSk; which constitute the input quantities.

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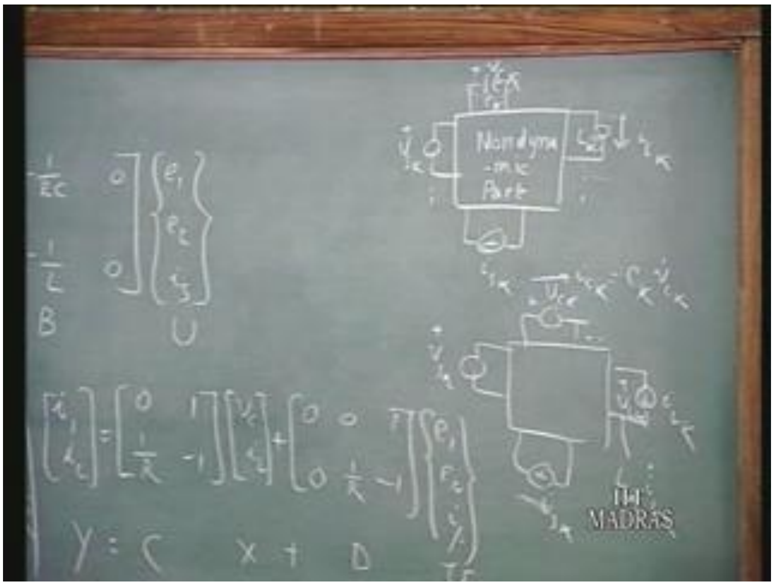

So, first step means; to replace this by sources second step is to find out the currents, in the replacement voltage sources and the voltages across the replacement current sources and identify them with a proportional to the derivative of the state variables. So, you have got n the derivatives of the state variables expressed as a function of the state variables themselves and the input function plus the network parameters which are R L and C. In the same fashion in the same network here you can express the output quantity that you are interested in interms of Vsk iSk Vck and iLk which are the state variables.

So, in analyzing this you already can find out the equations for the output quantities interms of the state variables in the input quantities. So, we have on 1 hand the state equation and then also the output equation. So, this is the general procedure it is a straight forward extension; what you have done with best example: illustrate actually the step that are involved what you have done is we put this in more general same work; where k can be running inductor depending upon the number of inductors capacitors and voltage sources and current sources, what we have discussed so far is the normal form on state and output equations.

X dot equals AX plus Bu and Y equals CX plus Du. It turns out that in special situations we have to deviate from the normal form of the state and output equations such cases arise where, you have loops constitute by a capacitors and voltage sources and cutsets constituted by inductors. And current sources where, the number of state variables will be reduced from the total number of reactive elements, by the number of independent C Vs loops L I s cutsets. So, in these special cases we have what is called an extended form of state and output equations. And that I would like to discuss now, illustrate in this by through an example once again.

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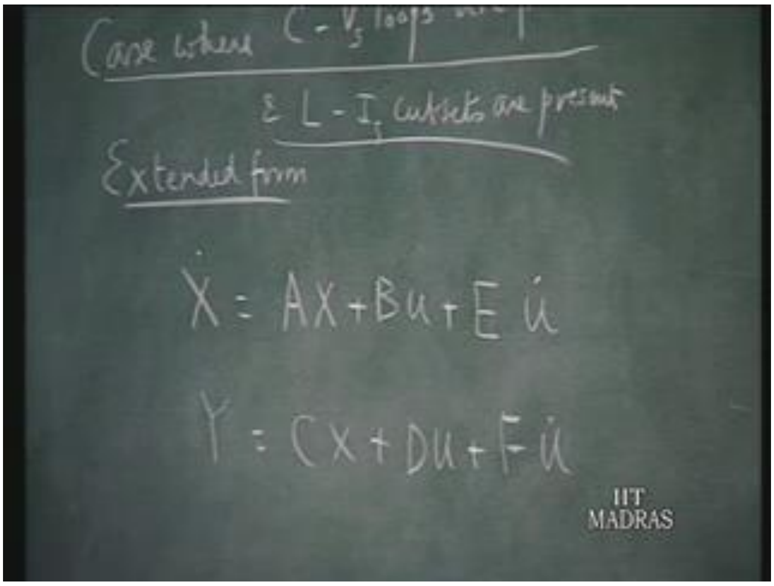

So, special case where C Vs loops are present by this what I mean; is that you have a closed circuit constituted entirely by a capacitances with or without voltage sources that is if the loop all the elements; inside the loop must be either all capacitors are some capacitors and some voltage sources. In such cases, we have extended form of the equation. The extended form will be X dot equals AX plus Bu as before, in additional you have another matrix E multiplying u dot where u dot is the lecture of derivatives of the input functions. And Y likewise CX plus Du this is the standard form plus F times u dot.

So, this new matrices E and F arise whenever you have C Vs loops or we will first talk about with this. And also we will say that similar case arise when inductor current source cutsets are present. So, let me as well put here and L Is cutsets are present. So, even in this case we have the extended form; how this extended forms arise will be illustrated by means of an example.

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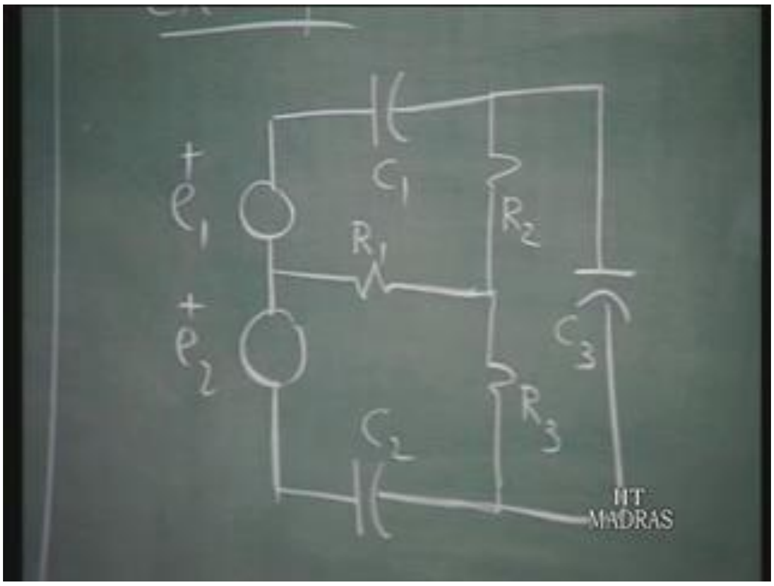

Let us consider that we have a circuit in which 2 sources are acting we have 3 resistors and 3 capacitors like this. So, let this is C 1 this is C 2 and this is C 3. And this is R 1 R 2 and R 3.

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To simplify numerical work let us take C 1 equals C 2 equals C 3 equals 1 farad. And R 1 equals R 2 equals R 3 equals 1 ohm. Now, we observed that the 3 capacitors and the 2 sources form a loop consequently the sum of the capacitor voltages and the sources must add up to 0 by according to kirchoff's voltage law; which means; that 3 capacitor voltages cannot be al independent. Therefore, 1 of them depends on the other 2 and the source voltages.

So we can straight away observe note that all 3 capacitor voltages cannot be state variables consequently. Let us take Vc 1 and Vc 2 Vc 1 and Vc 2 as the state variables and the voltage Vc 3 is depend on the Vc 1 and Vc 2.

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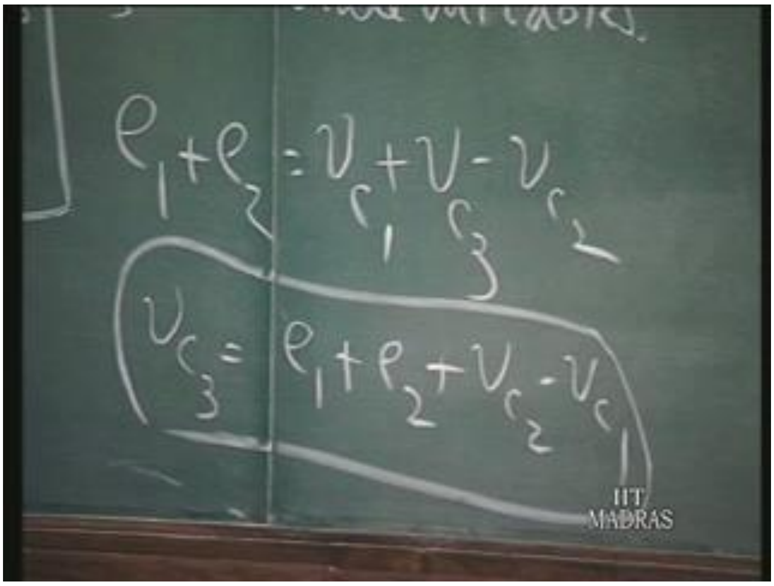

So, if you write the Kirchhoff's voltage law for this loop we will observe that e 1 plus e 2 e 1 plus e 2 equals Vc 1 plus Vc minus Vc Vc 1 plus Vc 3 minus Vc 2. Or in other words Vc 3 equals e 1 plus e 2 plus Vc 2 minus Vc 1. So, Vc 3 depends on the other Vc 3 depends upon e 1 plus e 1 plus e 2 plus Vc 2 minus Vc 1. That is what we had here.

So, in this what we do is to find setup this state equation pertaining to this we replace C 1 and C 2 by voltage sources of terms Vc 1 and Vc 2 according to our usual method; how are as far as this capacitors consigned; which will refer to excess capacitor. Because, in this loop this is an excess capacitor that will be replaced not by a voltage source but, let us say that we have replaced it by a current source which is iC3.

So the state vector now, is Vc 1 and Vc 2 is the state vector and iC 3 the current in the third capacitor which will take to be an excess capacitor that is the at the C 3 will be excess capacitor. And that means replaced by current source or strength iC 2. We will see how to handle that.

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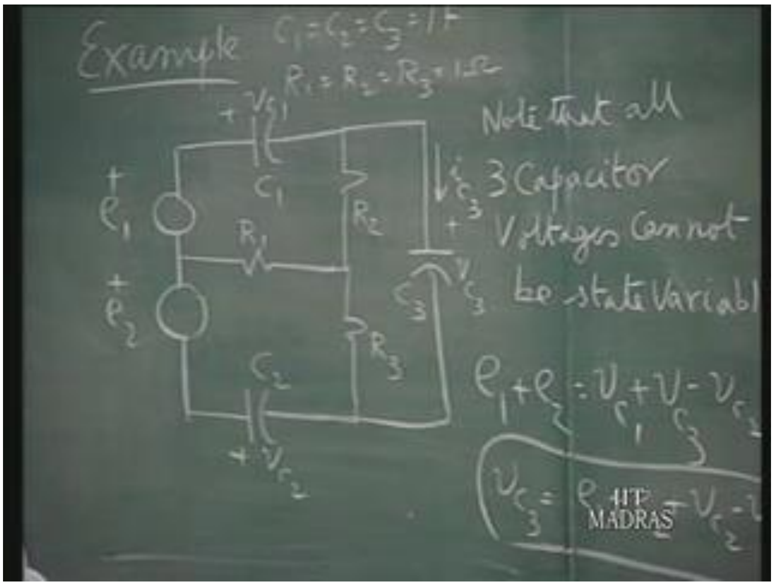

Let us imagine that we are interested at the output quantities the 2 currents I 1 and I 2 I 1 and I 2 are the output quantities.

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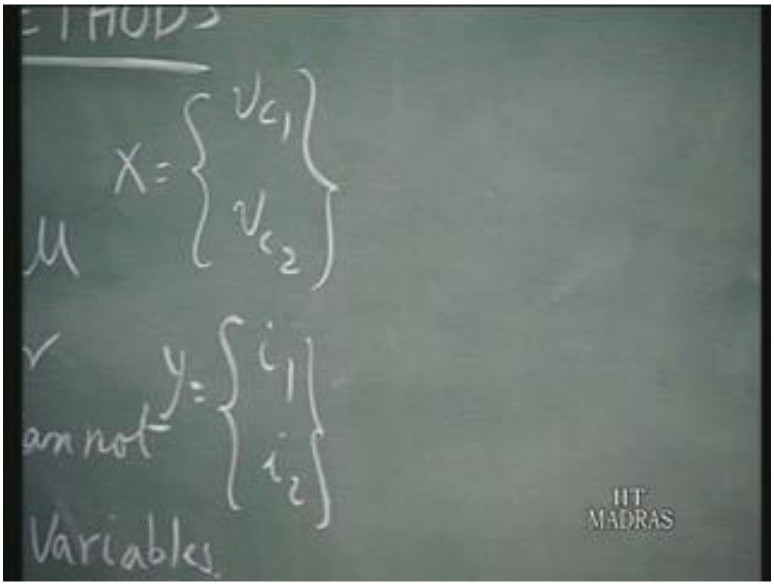

So, we have now the state vector is Vc 1 and Vc 2. And the output quantities are called I put vector I 1 and I 2 I 1 and I 2 that is what we have at the output quantity.

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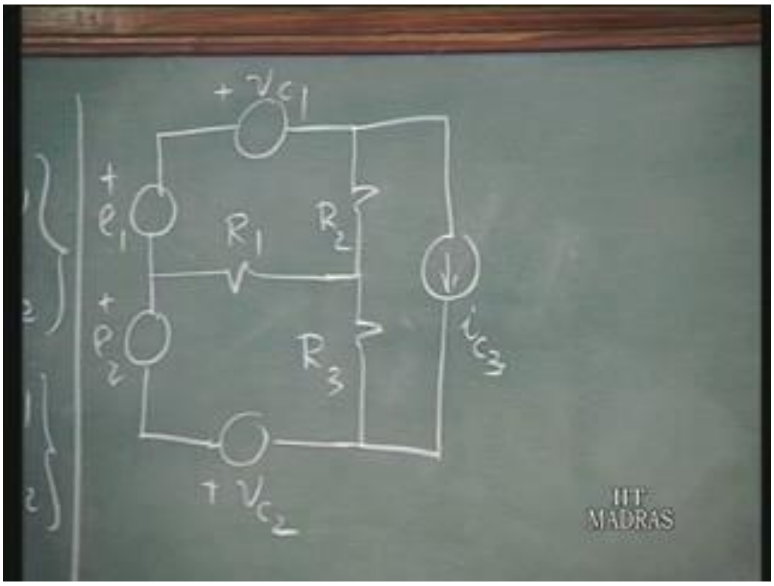

Now, so to analyze this what we do according to our usual that is we have e 1 e 2 and this voltage this capacitor is replaced by a voltage source of strength Vc 1. The second capacitor is replaced by voltage of strength Vc 2. And you have R 1 R 2 and R 3 all are unit magnitude and the third capacitor. Now, is replaced by a current source of strength iC 3. Now, we have to find out iC 1 and iC 2 interms of the state variables Vc 1 Vc 2 e 1 e 2 and also a news criticize so, that you introduce a iC 3.

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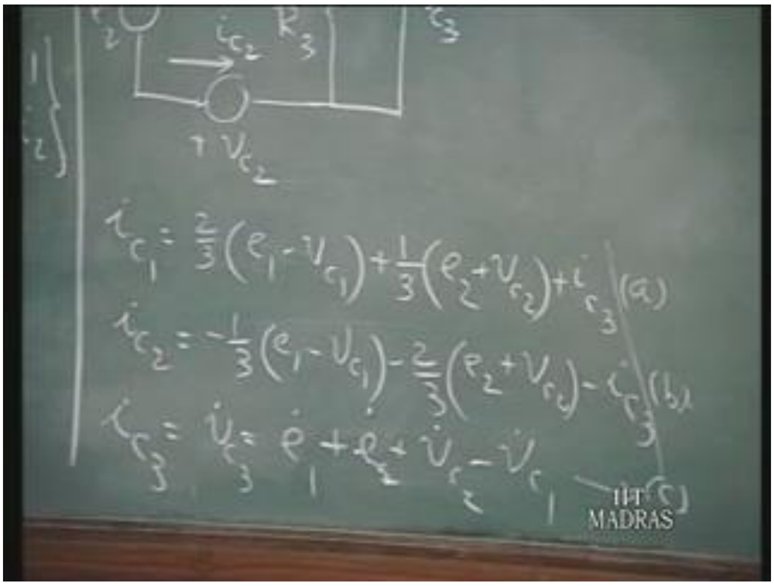

So, we expressed iC 1 and iC 2 interms of Vc 1 Vc 2 e 1 e 2 and iC 3 in the usual fashion. And if you analyze this you will find that iC 1 equals 2 thirds of e 1 minus Vc 1 plus one-third of e 2 plus Vc 2 plus iC 3. And like wise iC 2 can be shown to be minus one-third of e 1 minus Vc 1 minus 2 thirds of e 2 plus Vc 2 minus iC 3. So, let us call the sequence a like this equation b. Now, we have of course, iC 1 is C 1 times Vc 1 dot. And iC iC 2 is C 2 times Vc dot.

So, this is according to what we have except that now, we have a new quantity which we would like to get rid off iC 3 is a extraneous quantity which you should like to get rid off. That you notice iC 3 is equal to C 3 times Vc 3 dot. And we know Vc 3 dot equals e 1 plus e 2 plus Vc 2 minus Vc 1. Therefore, iC 3 can be written as C 3 times Vc 3 dot which is a but, C 3 is equal to 1 therefore, this is Vc 3 dot and from this you get this as e 1 dot plus e 2 dot where dot is the very derivative plus Vc 2 dot minus Vc 1 dot.

So, this is the third equation. So, in the equation c equation c we enables to express iC 3 interms of the derivative of the sources and the derivative of the state variables. So, if you substitute the third equation into the first 3 equations then; we can get rid of iC 3 and you have an equation which concerns this e 1 dot Vc 2 dot interms of this state variables and derivatives and the source function and that derivatives.

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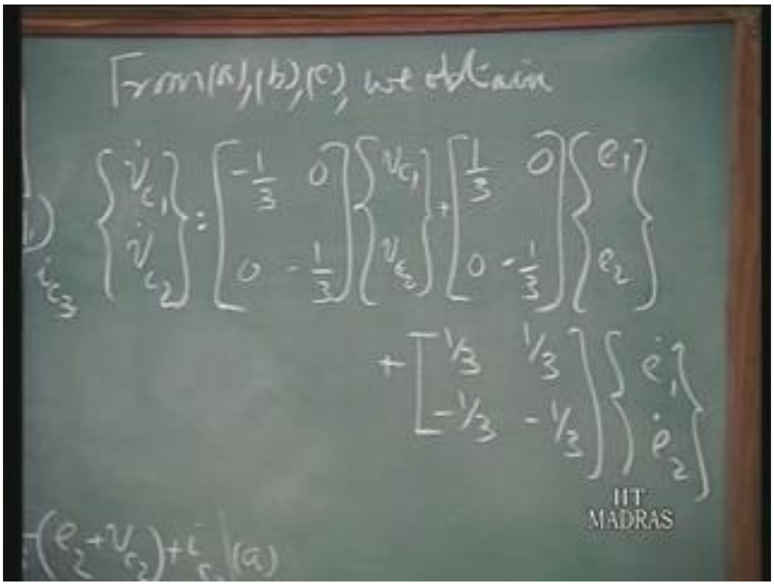

So, you work this out from a b c we can obtain Vc 1 dot Vc 2 dot derivative to the state vector equals minus one-third 0 0 minus one-third. This Vc 1 Vc 2 plus one-third 0 0 minus one-third e 1 e 2 plus a new matrix that you have got that one-third, one-third minus one-third minus one-third e 1 dot e 2 dot. So, this is the e matrix that we have got in now. That is the new 1 that we have got now. The output quantities we want the I 1 and I 2 are the output quantities.

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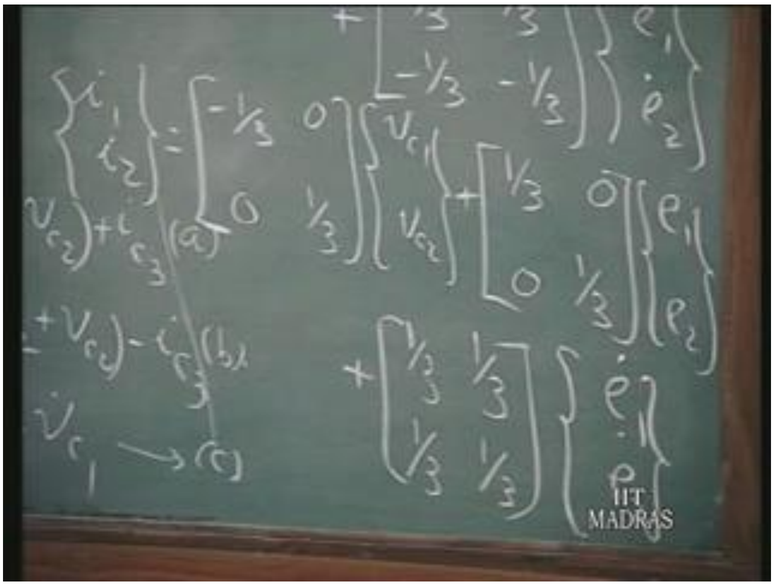

We know I 1 equals this same as iC 1 and the I 2 equals the negative of iC 2 therefore, we already have an expression for iC 1 and iC 2. Therefore, from that you can get I 1 and I 2 that is quite easy to obtain therefore, you have I 1 and I 2 equals minus one-third 0 0 one-third of Vc 1 Vc 2 plus one-third 0 0 one-third e 1 e 2 plus one-third, one-third, one-third, one-third of e 1 dot e 2, So, that is how you can obtain the extended form of the state and output equations once, we have source and capacitors loops.

So, all the principles that is involved is that you have the excess capacitors are replaced by current sources and you write the equations interms of the source tends state variable and the additional current sources. And the current sources are eliminated the rating out an extra equation for the current source strength in terms of the derivative of the excess capacitor voltage; which interned can be expressed interms of this sources and other capacitor voltages.

We have done can be extended for the wherever inductor inductors and current sources form a cutsets even in that case what we have to do is the inductor the excess inductor can be replaced by a voltage source and then you write that expression for the current in the that excess inductor of course, is related to the currents in the regular inductors whose currents are associated a state variables and the current source strength. And you take the derivative of the current in the excess inductor. And that will be the voltage across the such inductor.

That can be expressed interms of the derivatives of the state variables the inductive currents; which are state variables and these current sources and we following the same analysis will be able to write down the extended form as state and output equations wherever there are inductors and current sources form a cutset. The whole analysis follows an analogous slice to this. After learning how to write this state and output equations, it is now appropriate for us at this time to discuss ways of solving this equations.

So essentially we solve for this state vector from the state equation X dot equals AX plus Bu. So, you solve for the state vector as a function of time that means each component is known its state variable is know as a function of time. This is this kind of constitute more difficult part after having found at the state vector to substitute that, in the second equation this is the output equation Y equals CX plus Du. So, it is a mere substitution and then carrying out the necessary algebra and then; you get the various components of Y as function of time. And that constitute complete solution before, we discuss the methods of solutions of the state equations.

Let me just dispose of 1 small matter because, we saw that the state equations can come in the standard from or the extended form. However we do not have to discuss the extended form at the state equations. Because through a suitable transformation we can transform the extended form a state equations into the standard from, I will just illustrate that how the extended form can be converted into standard form through appropriate identification of a state variables. And then; we will discuss the procedures of solving the standard form of state equation.

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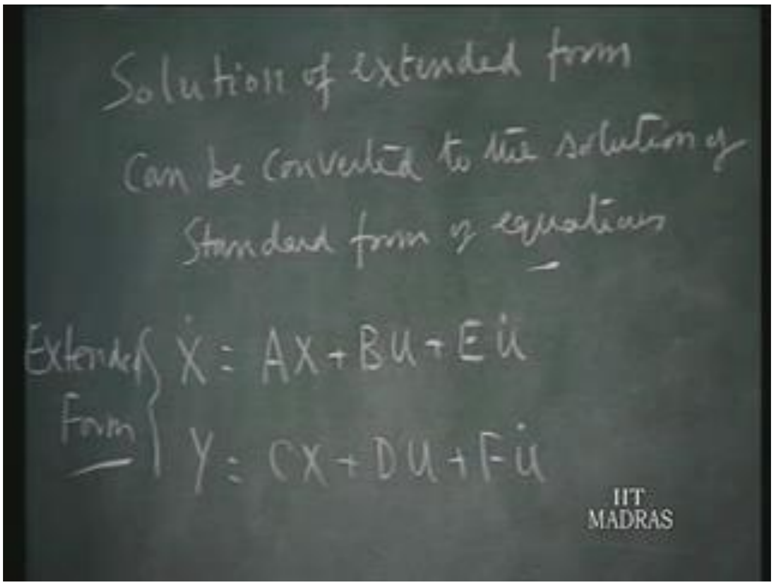

So, we will say solution of extended form can be converted to the solution of standard form how let extended form b X dot equals AX plus Bu plus E times u dot Y equals CX plus Du plus F u dot. So, these are the extended form of equation.

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Now, you make you define a new set of state variables Xn as X minus Eu. So, this is the new state set of state variables which are related to the old state vector by X minus Eu. So, this new state variables not only depend or dependent on the old state variables. But also the function source fashion here.

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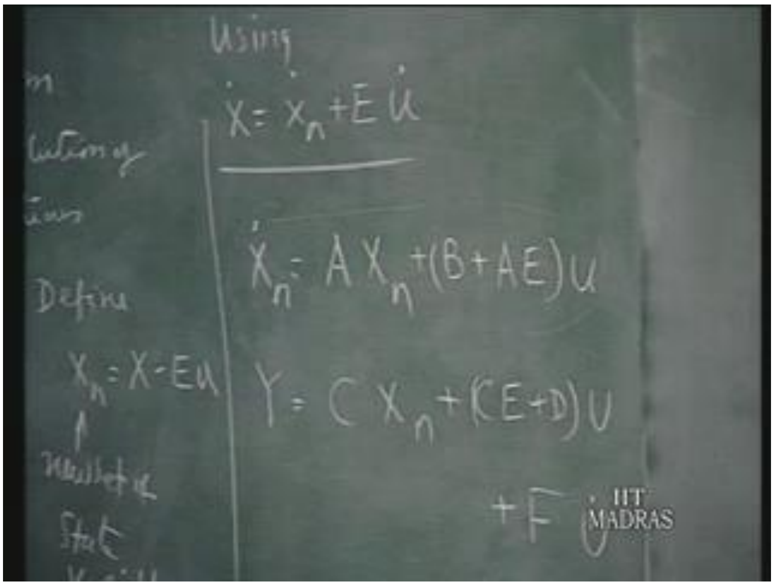

Consequently you can write X dot equals Xn dot Xn dot X dot equals X dot equals Xn dot plus E times u dot right. So, you can substitute for X dot as Xn dot plus Eu dot in this. And you can simplify you will finally, get you can show that using this relation you can finally, show that Xn dot equals AXn plus B plus AE times u. And Y equals CXn plus CE plus D u plus F times U dot. By as a set it is the solution of the state equation which is most fundamental which is more the expressed.

Now, this is the new state equation which are solved for essential solution conceive AX has to something related little bit A. So, instead of the B matrix you have B plus AE a new matrix but, otherwise the form of this the same as the old form in the standard form AX plus Bu. So, the whatever solution by that is your quantum plate for the solution of X dot is equal to AX plus Bu can be applied as well here. And once you have got the new state vector solution for the new state vector Xn to substitute here and here there is U dot does not really matter.

Because, here your role is substituting Xn and then; calculating the value of Y you are not solving this as a matrix equation or whatever it is. It is mere substitution and evaluating Y. So, you know the source function U you can find U dot and there is no difficulty. And therefore, you can get the output Y. So, here after wards therefore, we need not give any special mention or illustration of the solution extended form of state equation. Because, if you know how to solve this standard set of state equation, you can as well apply the same techniques for extended case also by converting the old state vector into a new state vector with this kind of substitution.

Now, let us discuss the solution methods themselves. The solution of the state and output equations can be done in time domain. But before that we would like to use the Laplace transform techniques and find out the solution, because it will be illustrative and it will also give us some fuel for the problem, because we already are familiar with Laplace transformation techniques. We know what is the natural response? What is the forced response, in the Laplace transform techniques. And see how is this related in Laplace transform solution. Then later on, we will talk about time domain solution where Laplace transformation technique is not at all used.

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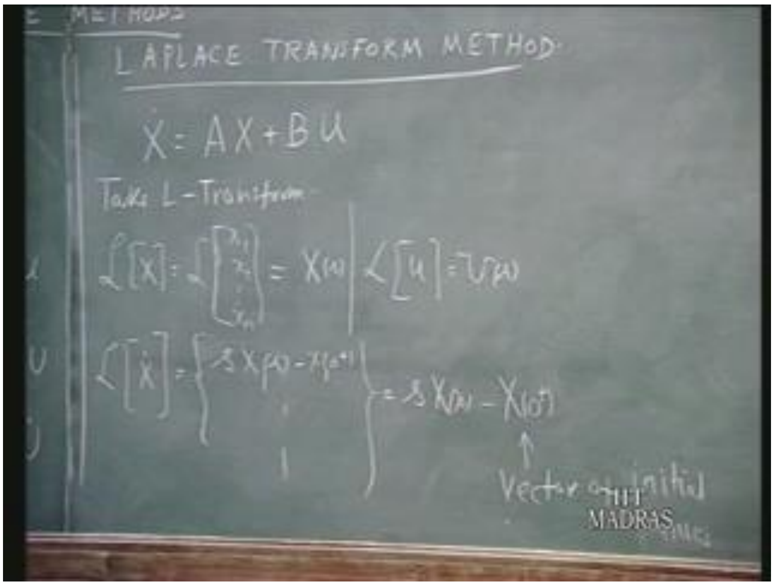

So, the first method of solution that; we talk about the Laplace transform method, what we meant by this is. The coupled first order differential equations X dot equals AX plus Bu will be solved to the Laplace transform technique. So, we have X dot a vector equals AX plus Bu. We call that X dot is a vector a number of variables X is also a vector and u is also a vector. Take the Laplace transform of this equation of this entire set of equation. So, after all when you want to take the Laplace transform of X dot before that let the Laplace transform of X what we meant by Laplace transform of X is Laplace transform of various quantities x 1 up to xn.

Let this we called Xs so, it is a new vector where instead of time dependent quantities time variable quantities we have transforms functions of s. So, Laplace transform of X vector let it call Xs. Now, what will do the Laplace transform of X dot you are taking the Laplace transform of X 1 dot X 2 dot Xn dot. So, if x 1 is the Laplace transform of X 1 of s. The Laplace transform of X 1 dot is s times X 1 of s minus X 1 0. In line with what we do in the differential equations we take the initial conditions pertaining to 0 plus. Therefore, we get Laplace transform of X dot will be X 1 s minus X 1 0 plus like that it goes on.

So, I can write this entire thing s times X 1 of s sorry, because we are we are taking the Laplace transform of x 1 is X 1 of s Laplace transform of X 1 dot will be s times X 1 of s minus X 1 0 plus like that it goes on. So, this entire lecture which consists of s times X 1 of s minus X 1 0 plus s times X 2 of s minus X 2 0 plus and so on and so forth; can be written as: s times Xs. Because, after all it is a vector which is multiplied by a scalar s every 1 of the component is multiplied by s minus X 0 plus this is a vector of initial values.

This is the vector of initial values that is the value of the state at time t equals 0 or plus X is the state vector and as a function of time X 0 plus is the state vector at time t equals 0 or plus. So, we have the Laplace transform of X is X of s. The Laplace transform of X dot is s Xs minus X 0 plus. All are n vectors. And the Laplace transform of u we will call that U of s that is also a vector.

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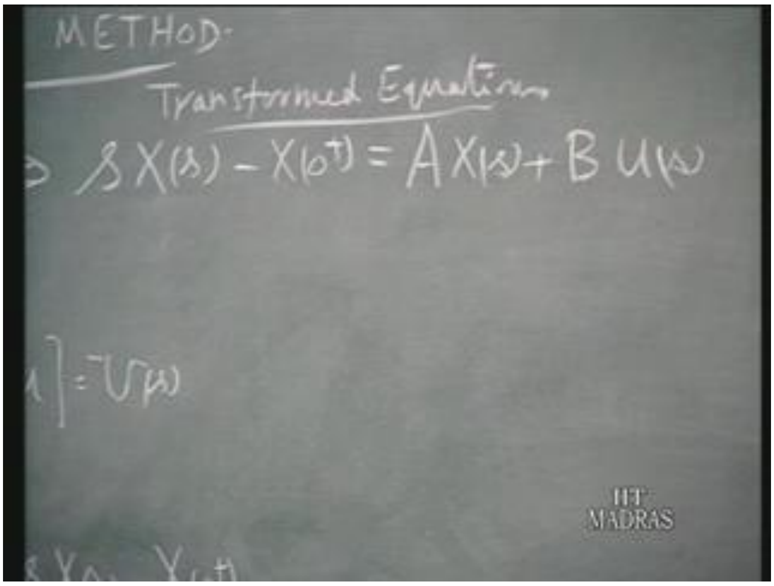

So, this entire set of equation can in time domain in the transform domain, we look like s times X of s minus X 0 plus equals A times X of s plus B times u of s. So, this is the transformed set of equation transformed equation. Now, what we really having here is X 1 of s is multiplied by s X 2 of s is multiplied by s. So, here we would like to have X of s term here X of s term here; we would like to combine these 2 we have X of s there and X of s here.

We would like to combine them suitably, here it is multiplied by a matrix here it is multiplied by a scalar s.

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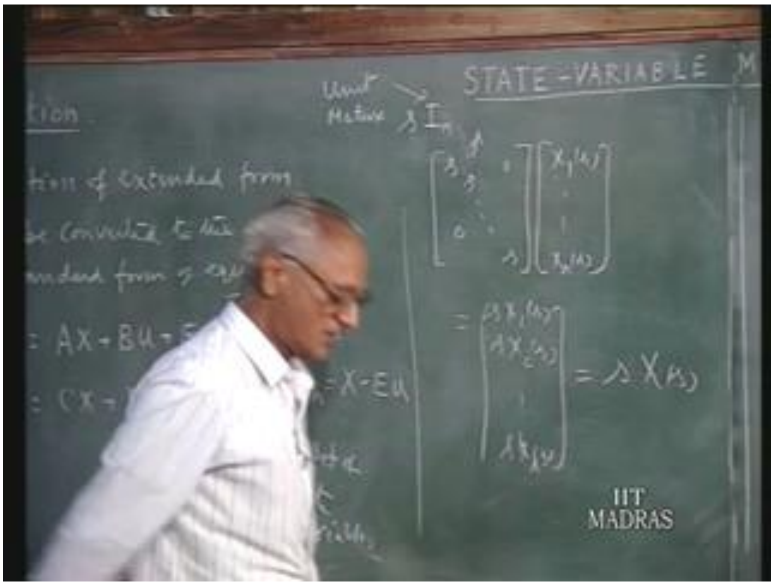

So, to get overcome this problem after all. If I multiply all 0’s X 1 of s down the line to Xn of s; if I multiply this the vector X 1 to Xn of s by a matrix; which has s write along the diagonal. This will of course, the s times X 1 of s s times X 2 of s and so on and so forth; s times Xn of s. And this particular matrix can be written after all we have instead we have a only diagonal entries and all are them are equal to s. And this matrix is s times the unit matrix I where I is the unit matrix; where unit matrix which is ones all along the diagonals. You find the literature unit matrix is also indicated as u.

But, since we are using u for something else we use I unit matrix that is the matrix; which has got ones all along the diagonal. And the further say that it is union matrix of dimension n sometimes people write In. But, we simply write I because after all we are not writing the dimensions for the other quantities as well. Therefore, this is s times In is gives this quantity and that is exactly s Xs this is sXs this is what we have here.

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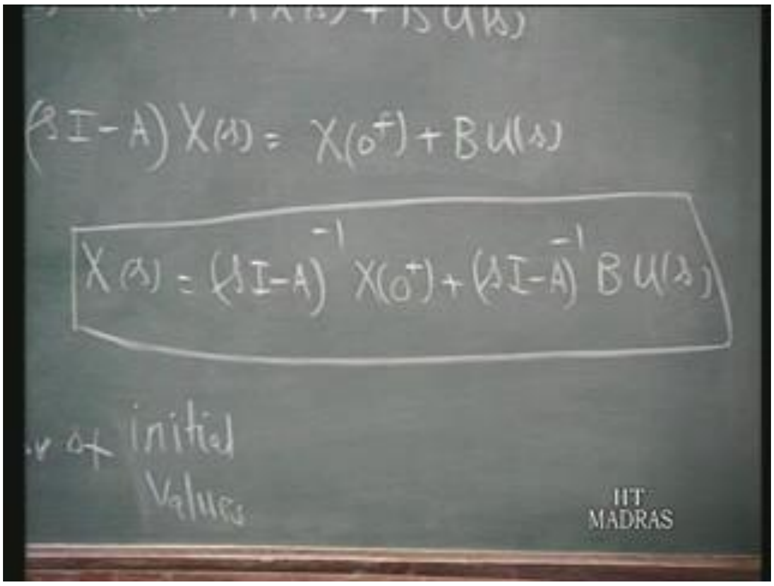

So, what I would like to do is I will write this equation s times I times Xs minus X 0 plus equals AXs plus B us. And this matrix at the dimension n by n and so, is this matrix. So, if you transfer this to the other side I can write this as sI minus A times X of s equals X 0 plus B us that is what get SI minus A times X of s is X 0 plus B us. This is now a matrix of dimension n by n. So, this is the square matrix of dimension n by n and we have to solve for X of s. So, you can think of multiplying this entire set by the inverse of this matrix.

We will assume that it exist and then we got the solution for X of s. Therefore, I can write X of s as sI minus A inverse multiplying X 0 plus the initial vector source initial state vector times sI minus A inverse B times us. So, that is the solution for the Laplace transform of the state vector, in terms of the initial state X 0 plus and the Laplace transform of the forcing functions u. Therefore, in this solution the what involve what is the involved is the inversion of n by n matrix where n by n matrix is not a constant it is also a function up to s .

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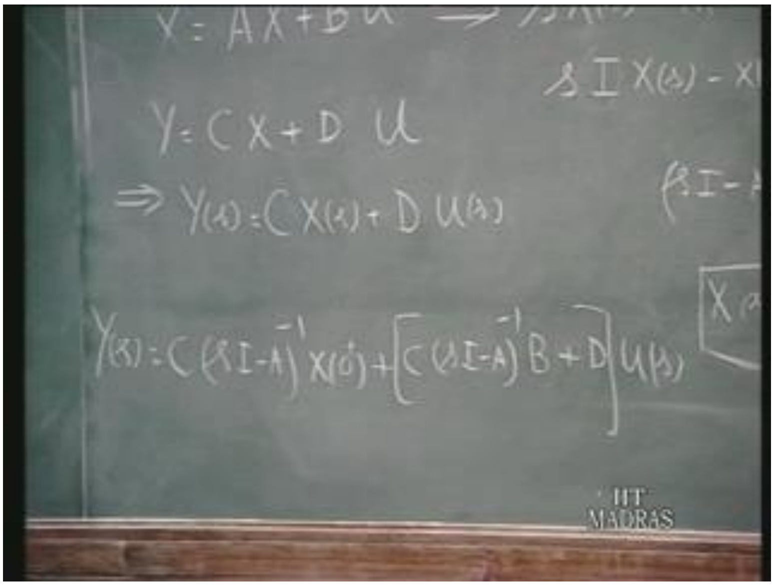

Once we have that then; we can find out the output quantity after all output equals this is the state equation output Y equals CX plus Du. Therefore, taking the Laplace transform of this becomes Y of s is Laplace transform of output quantities is C times X of s plus D times u of s. We know the solution X of s from here. Therefore, we now have the final solutions are the output in the Laplace transform domain as Y of s equals C times sI minus A inverse X 0 plus plus C times Si minus A inverse B plus D times u of s.

So, we have Y of s in 2 parts 1 which depends upon the initial state X 0 plus and other which depends on the forcing function u s that is what we are having. So, Y of s can be put in this form C times sI minus A inverse times X 0 plus C times sI minus A inverse plus B plus D. Suppose: we have initial conditions 0. We assume that the capacitor and inductor currents are initially, at 0 levels and they do not change suddenly, that means; you are applying an input. Therefore this becomes 0 and Y of s is equal to this times u of s.

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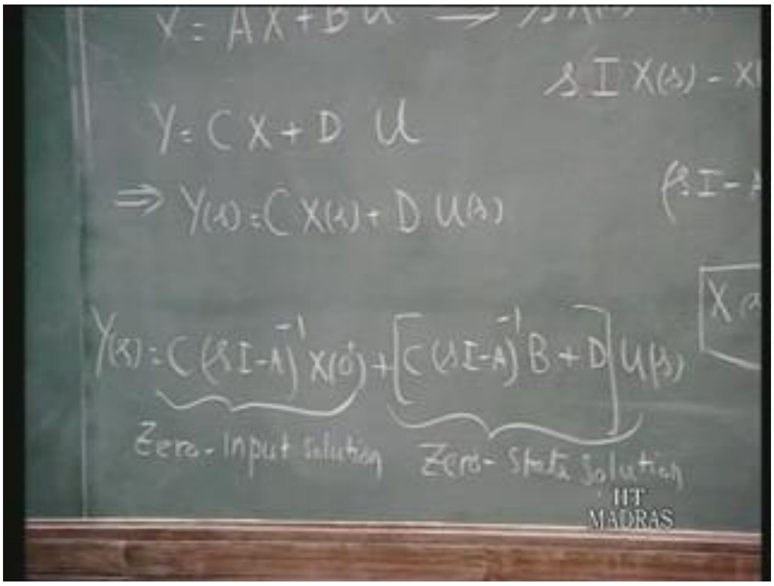

So, what we can say is this part of the solution is obtained when the initial conditions are 0 that means; initial state of the system is 0. So, this is called the 0 state solution 0 state solutions, what it means; the initial conditions are 0 that means; the initial state of the system is 0 all the state variables are all 0 values. This portion will be the solution. Suppose, the input is 0 you are given some initial conditions for the system and let the system go on it shown.

Then; the input is 0 therefore, this whole thing become 0 and you only this. So, this is called the 0 input solutions. So, we have the solution which tends into 2 parts 1 is the 0 input solution; which arises when the input is 0 and the system is going it on so under the influence the initial conditions. The second part is that 1 could be obtained, if the initial conditions were 0 which means; the initial state system is 0 so it is 0 state solutions. We have to find out the inverse Laplace transform of this quantities to get the output Yt, you recall that this is a matrix equation.

So, Ys will be whatever, you are getting here will be a vector of function of s both in this term and this. So, to find out the inverse Laplace transform; we have to find out inverse Laplace transform of each 1 of those quantities that what you are having. Now, we will talk about this in great a detail, in the next lecture because find out sI minus A inverse and related aspects take some time. But, what we can see is suppose you have a single input and single output system there is only 1 input and 1 output.

That means Y of s is a scalar and u of s is just scalar just 1 quantity. And the initial conditions are 0 therefore; the Laplace transform of the output to the Laplace transform of input is just this quantity. So, in this case our single output single input system the Laplace transform of the output and the Laplace transform of the input a 0 initial condition is equal to this.

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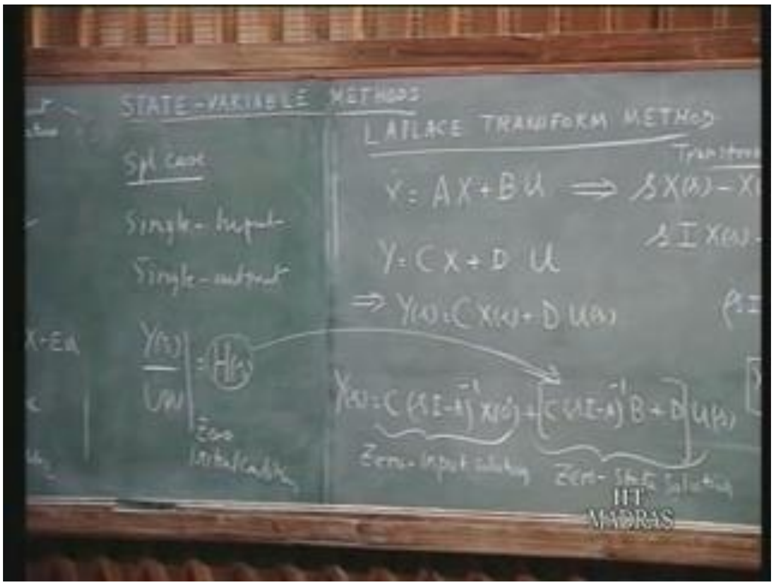

Therefore, we can observe straight away that special case single input single output system, we have Ys over Us 0 initial conditions is our system function H of s and the system function happens to be equal to this. So, that system function C times sI minus A inverse B times D. So, H of s is equal to C times sI minus A inverse times B plus D. So, we have found out an alternative way of find you H of s in terms of the ABCD matrices. So, in this lecture, we continued our discussion of the state variable concepts.

In the last lecture, we learned the concept of state and the number of state variables that come about in a particular network. Today, we will learn how to write the state and output equations we discussed 1 general method where, the capacitor volt capacitors are replaced by voltage sources. And the inductors are replaced by current sources. And then; we write the equations of performance of the resulting non dynamic network excited by various sources. And use that information to find state and output equations.

We also saw the extended form of state and output equations; when C Vc s loops are present or L current source cutsets are present. We said that is not necessary for us to consider separately, the method of solution of the extended form of equations. We call the form of equation can be converted in this standard form for suitable transformation of the state variable state vector. Then; we started discussion of the Laplace transform method of the solution state equations.

We saw that the suitable Laplace transformation of the various quantities. We arrive at the solution for the state vector in this form, in the Laplace transform domain. And for the output vector, in this form and we set the output solution can be considered to be 2 parts; 1 which arises when input is 0 0 input solution and the other where, the input the initial state is 0 this is called the 0 state solution. These 2 have special significance, as we will see later on in our next lecture; will continue this discussion at this point, in our next lecture.