Networks and Systems
Prof.V.G.K.Murti
Department of Electrical Engineering.
Indian Institute of Technology, Madras

Lecture – 45
State-Variable Methods (1)

So far we have been discussing, various methods access the dynamic behavior of linear time invariant systems. The methods that we have discussed, broadly fall into 2 classes: 1 the methods of differential and equations or difference equations as may be appropriate. The second one is operational methods at the transform methods, which are the Laplace's transform method and the Z transform method that we have discussed.

Now, let us see pause for a while and see what are the relative merits of these 2 classes of solutions for a dynamic performance of a system. The differential equation and the difference equation approach is more fundamental because, the characteristics of the elements which are constitute the network and systems are given fundamentally in terms of differential or difference equations.

And therefore, this is a more fundamental approach and further these methods can be extended to a case of non-linear and time invariant systems. On the other hand, the difficulty with this differential and difference equation approach is, that you have to content with calculus or an equivalent operation in the case of difference equations. And for a complicated differential equation say for a n'th order you need to have n initial conditions, which may not be always available from the physics as a problem.

So, the initial condition and the initial values for the various derivative you have to specially found out and this goes some problems. Now look at the transform methods, the transform methods the calculus part is convert into algebra similarly, the operational difference equation is converted once again in purely algebraic process.

And therefore, it appears to be quite convenient. And further, all we need to know is initial conditions associated with the dynamic elements you do not have to find out initial values of the higher derivatives, which was the problem with the differential equation approach. But then, the problem with operational methods, Laplace's transform and Z transform methods is they are not quite suitable for computer implementation.

Because, after all computers can handle numerical analysis quite efficiently and when you have talked about, Laplace's transform and Z transform method the transform methods you are dealing with function of s or Z as the case may be and it is not convenient to have computer implementation of handling. This while it is feasible but, it is not easier to convenient to do so.

And furthermore, when you want to find out the inverse Laplace's transform of a complicated rational function with a high order denominator polynomial. To perform the partial fraction expansion you have to factories the denominator. And any way, you have to go to numerical methods and therefore, there again you have problems.

Now, what we are going to talk about from this point onwards the state variable method of analysis, which is alternative to few methods that you have talked about so far. This state variable is also a fundamental method, in that it realize largely and differential equation or the difference equation on the case may be. But these differential equation and difference equation are first order differential equation or first order difference equation.

Therefore, since we are dealing with first order differential equation or difference equation in the split variable methods. We need to know, the initial values of the appropriate variables at 1 point of time, we do not have to find out initial values for the various derivatives. Similarly, we need to have the initial value of a discrete time functions at 1 particular point not a several points for negative values of m.

Furthermore this is convenient for computer implementation unlike the Laplace's transform methods. Therefore, in a sense the state variable methods have provide you certain advantages which are not present, either at the classical differential equation approach or the Laplace's transform approach. And therefore, these are widely implemented for the computer implementation of a dynamic analysis or systems and networks.

So, with this introduction let us see what the state variable method implies. We broadly define this state of a system as the information about a system at a point of time, that needs to be known for the solution or finding out the output under the influence of an excitation specified from that point of time. This sentence may not mean immediately something for you to put in other words. Suppose, we want to find out the transient analysis of let us say RC circuit.

So, under the influence of a certain input quantity. Before you do that, you must know the initial charge in the capacitance, unless you know the charge in the capacitance on the application on the input the solution will not be unique. So, this is something which needs to know to find out the output under the influence of a given input. So, that is what is meant by a state; information that is necessary for a dynamic system. So that, from that point onwards we can find out the output corresponding to a given input.

It also trans out that the state as another important connotation. Once you know this state of a system and any given point of time and also even though the input quantities that point of time, the output quantity can be deduced algebraically without having go through any calculus operation. So, these are the 2 aspects of the state which I would like to illustrate first by means of example and then define this concepts more formally.

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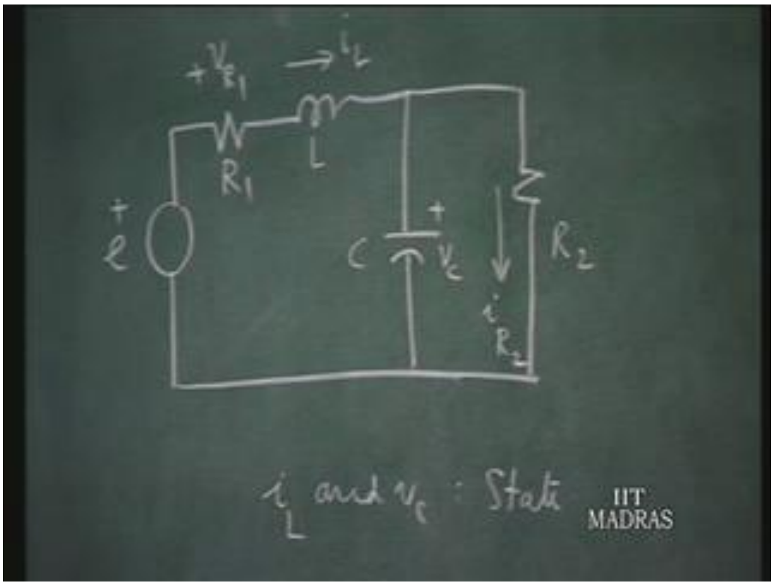

Suppose, we have a network of this 1 inductor and 1 capacitor and 2 resistances R1 and R2. Let us, say we are interested in these 2 values VR1 and iR2. These are the 2 output quantities, which we are interested this is the input excitation. So, in order to solve this particular circuit even the value of E from say t equals 0 onwards. The solution can be unique only if you know, the initial value of the inductor current and capacitor voltage.

So, iL and Vc there values a time t equal 0 on the application on the input must be known. So the 2 variables that we are talking about iL and Vc constitute what is this called state variables of this particular circuit, this is 1 choice of state variable. So, iL and Vc these 2 together constitute the state of the system.

So, we need to know the values of these 2 variables at a particular point of time so that, we can predict the behavior of the circuit and in particular calculate the various output quantities under the influence of an excitation uniquely. Unless this is specified the solution of the problem is not unique that is 1 aspect. Secondly, suppose you know iL and Vc state any particular point of time you can calculate VR1 and iR2 purely algebraic way without having any calculus.

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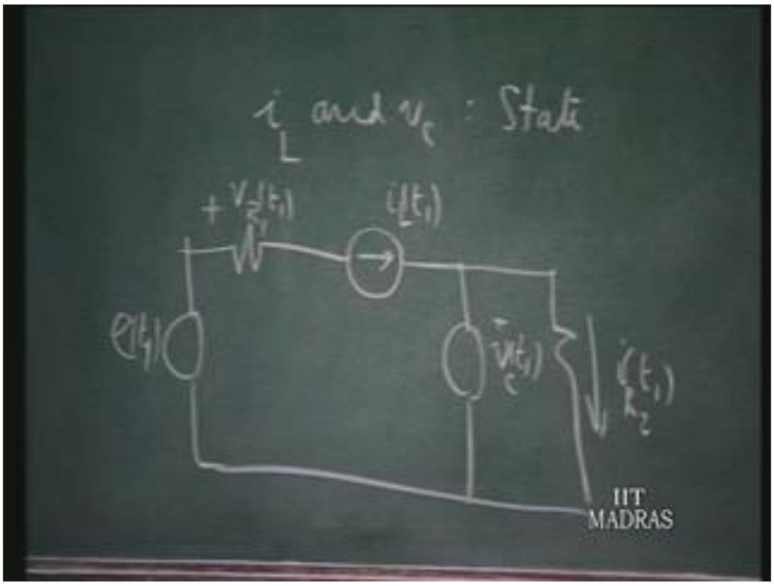

Because, you can say you know the value of iL here, I can replace this by current source of value iL. And then, the capacitor voltage is Vc therefore, I can have a voltage Vc here. And this is e so if all this values are known let a particular point of time t1.Vc t1 E t1. Then, VR1 at t1 and iR2 at that point so, if you know the state at particular point t1 and you know the excitation function also t1 these values can be calculated purely as in a non-dynamic system purely as resistive system.

Because, we now have a purely resistive network under the influence of several sourced. And the solution is 1 that pertains to an instant area system around non-dynamic systems at that particular point of time t1.

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And this replacement of I inductor by a current source and the capacitor by voltage source as you know or permitted by the substitution theorem. After all whether, you have inductor carrying a current iL or a current source having strength iL this circuit behavior is a same. So, as for the state is concerned then we have 2 particular points of significants: 1 is that is the information to we need to know about a system.

So that, we can predict the feature behavior under the influence of a specified excitation function from that point onwards. Secondly, once you know the stable system that means, the values of these the outputs at that point particular point of time can be computed purely if you algebraic process involving the values of the state at the particular point and the excitation function at that time.

Now, you see the contrast between the dynamic system and non-dynamic system. A non-dynamic system which purely resistive type of system, where you have the resistances or dependent voltage sources, dependent sources and so on and so forth. And there, if you are given excitation functions are particular point of time the various outputs can be calculated purely an algebraic process.

So, in this state variable technique what we try to do is, we try to find first of all find the state of a system as a function of time through a calculus part. Then, you get an information to calculate the output purely an algebraic process. That means, you separate the problem into 2 half 1 involving calculus, the second part involving purely algebraic.

Now, as I mentioned the information regarding the state is necessary, because to solve a problem because that is a sums up in a dynamic situation the entire passed whatever information that we need to know about the past is summarized in the value of the state at that particular point of time t.

Let us say, t equals 0 we would like to find out the transient solutions from t equal 0 onwards. All we need to know about the past is the values, the value of the state at t equals 0. We do not have to worry about anything else regarding that past. So, in the sense it is summarizes the essential information about the past, that is pertaining to the future behavior of the system.

In a sense therefore, it is a link between the past and future and the essential link that is necessary for the solution of the transient problem. And certainly once you know the state you can find out the output as a function of the state variables and the excitation functions purely to algebraic process. That is essential principle of this.

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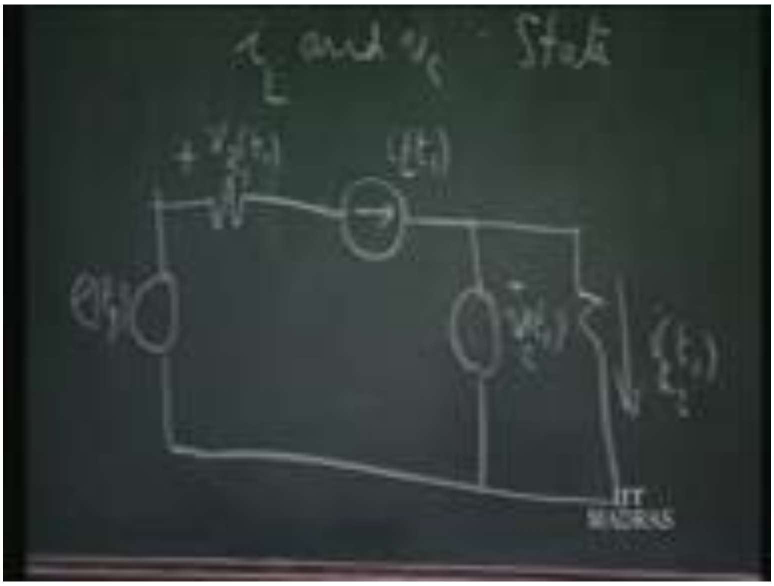

As seen in this example, the state is described by a set of dynamically independent variables iL and Vc in this particular case.

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In general, we have in a given system or network this state is described by n dynamically independent variables and these are called state variables. The state variables are set of dynamically independent variables, which we use the let us say n in number. So, we regard the number of variables as n and those n variables together constitute this state.

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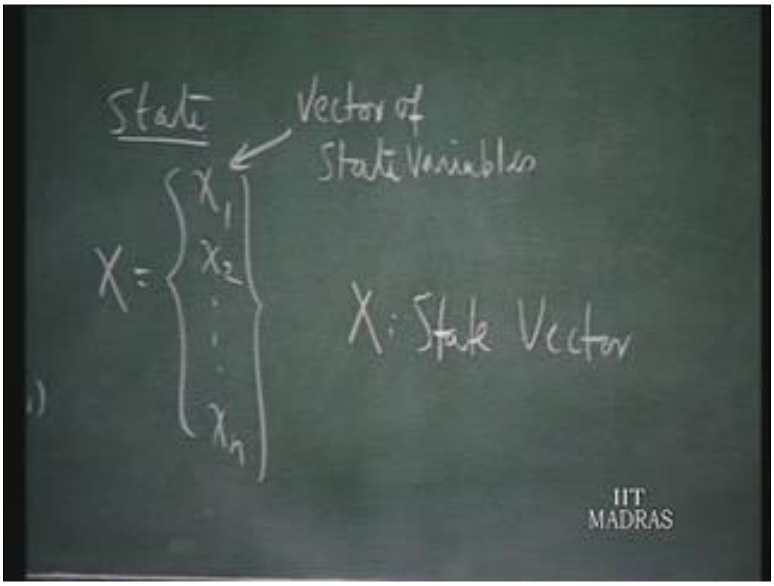

So, if the n independent variables are x1 x2 write up to xn. So, these are the set of n independent variables which together constitute state. And therefore, the state is normally represented by means of a column or a vector of state variables. And the entire vector is given the symbol X.

So, X is the state vector simply state or state vector and the various n component that we have got x 1 to xn are called the state variables. In the case of an electrical network, capacitor voltages and inductor currents constitute 1 particular choice of the state variables. It may know unique you can have several other combinations but, then this is a fundamental way of doing this and we will say a capacitor voltage and the inductor currents constitute a state variable normally we take it to that.

But as a set this is by known as unique wise but, this is the most straight forward and appropriate choice.

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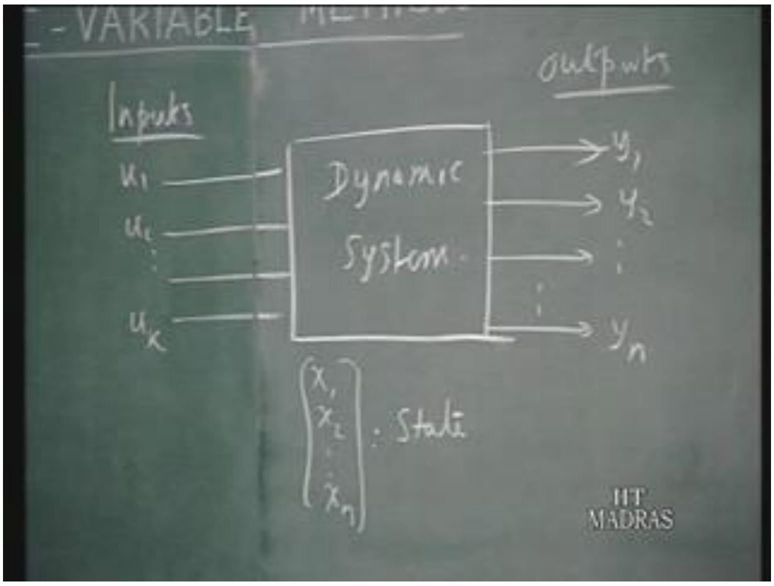

So, we have suppose in a general situation we have a dynamic system which has several inputs. Let us say, we in this particular case we have e as the 1 input but, suppose we have several inputs u1 u2 like that. Let us say uk and we have several outputs saying y1 y2 up to yn. So, we would assume that in a general situation we have several inputs and several outputs and the state in this system is described by x1 x2 up to xn this will be state.

So, we have 3 sets of quantities to deal with these are the inputs and these are the outputs and this is the state which are internal variables of the dynamic system.

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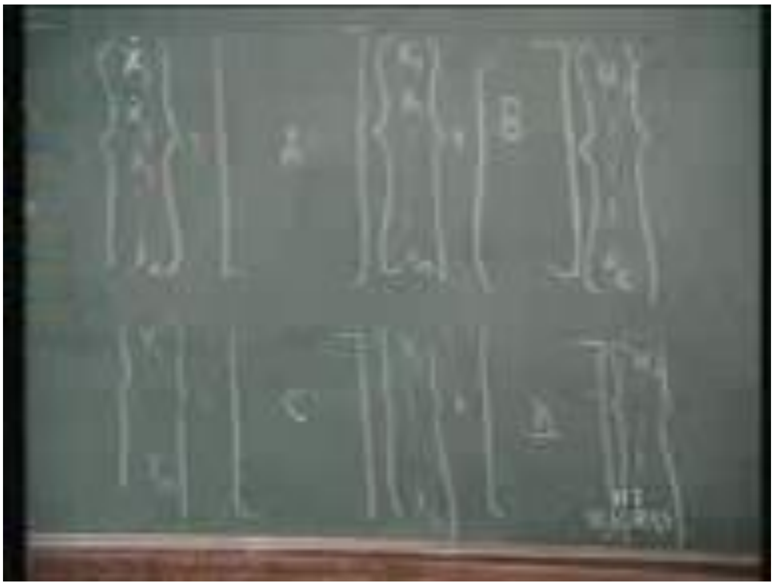

So, it turns out that you access the performance of this dynamic system. We first try to solve for the state vector, we do this by having the solution like this x1 dot x2 dot x3 dot that means, the various derivatives of the state variables. We setup as replace of this type a matrix A xn plus another matrix B u1 to I think we put uk therefore, I can write this as uk. So, to setup to solve for the state we form an equation like this.

The derivatives of the various state variables, expressed as a vector is obtained as a solution of this equation where this matrix A multiplying the state vector x1 to xn plus a matrix B multiplying the input vector u1 to uk. And once you have solve for this vector you get the output y1 on up to ym as another matrix C multiplying the state vector xn plus a fourth matrix D multiplying the input vector uk.

So, that is the how we plan to find out the dynamic performance of the system a multiple input multiple output system. So, this is the first equation this is called the state equation and this is called the output equation. The first equation is a matrix equation, we have to solving for x1 x2 x xm so, it is a all are differential equations each equation represents differential equation of first order.

Because, x1 dot is expressed that A1 times x1 plus A2 times x 2 this An times xn plus something else. Therefore, this is the first order difference equation x1 dot, the second row represents a first order difference equation x2. A third row a first order difference equation x 3 and so on. But they are all coupled so, because x1 dot involves x2 x3 xn also.

So, the first set of equation this whole thing is n first order differential equations n coupled first order differential equation. Because, the differential equation is x1 dot involves x2 and xn also.

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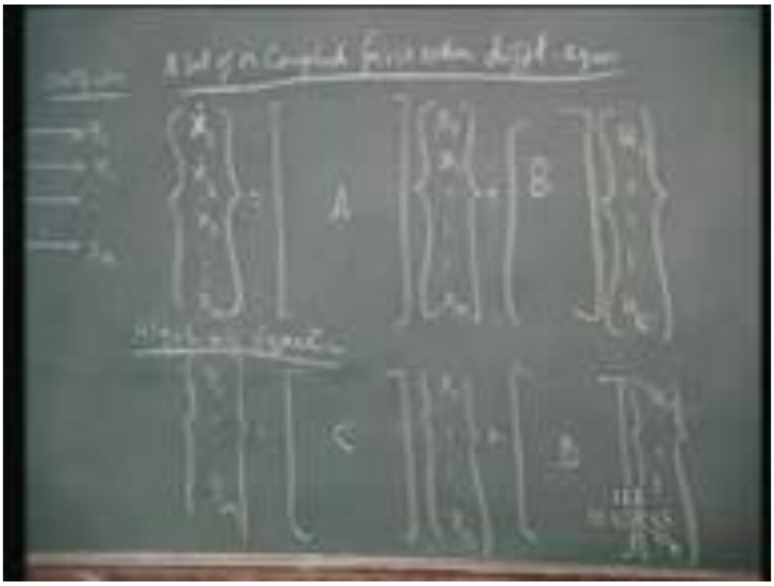

So, this is a set of n coupled first order differential equations. And the second 1 is a purely algebraic equation means once you are solved for x1 to xn and you know the input function u1 to uk. Y1 is obtained purely algebraically substitute in the values for x1 to xn. You already find out and u1 to uk which we have already known. So, these are all the algebraic equations, what I have discussed here is, what is applicable to continuous time system.

So, we will later on we will talk in the first incidence about continuous time systems only begin with at the end of this treatment you will also see, how the equations come about for a discrete time system. So, from this point onwards let us assume that we are dealing with dynamic system which dynamic continuous time system.

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Now, if the system happens to be linear and furthermore I mean this is assume the system is linear here in this example already. And furthermore, if it is a constant parameter system so I will say linear dynamic system with constant parameters. So, if the system parameters are constant that is time invariant system the entries in A B C and D also turn out to be pure constant pure numbers they are not function at any either t or anything. So, they are ABCD or matrices of constant numbers.

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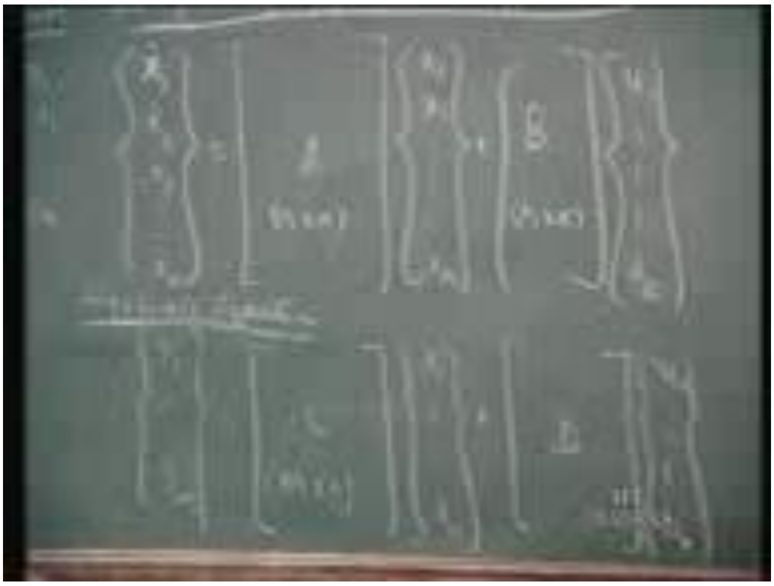

So, this whole these 2 equations can be more conveniently be written as X is X dot equals AX plus Bu and Y equals CX plus Du, where each 1 of this entries is a matrix X dot is a column matrix which is column matrices as usually called the vector also. X dot is a vector a column matrices. A is a matrix of dimension n by n as you can see A is the matrix.

Because, you have got n rows and n columns therefore, this n by n matrix. As far as the B is concerned it is coupling the input quantity is the state variables. Therefore, it must have n rows and k columns, in C you have got m rows and n columns so the size of this matrix is m by n. The size of the D matrix you have m columns, m rows and k columns. So, ABCD are matrices appropriate dimension depending on the input, output numbers as well as the state variables.

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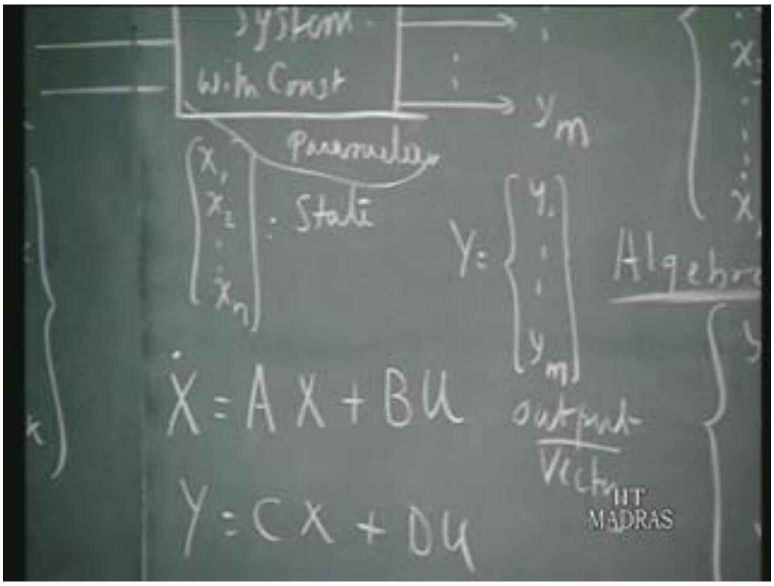

So, ABCD here in this matrix representation of this this equations or appropriate matrices of appropriate dimensions. X dot is a vector is a column X is a vector, u is a vector, Y is a vector, X is a vector. So, these are all vectors, where u represents the column u1 u2 up to so, this can be called input vector u is the input vector. And Y is y1 to I should have written ym I should written as ym because, that is what we have written here.

So, ym to ym so this is the output vector. So therefore, when you are dealing with linear time invariant constant parameters system, continuous time systems. You have equations of this type let 2 equations X dot equals AX plus Bu. Y equals CX plus Du this is called the state equation and this is called the output equation. Even though we are referring to that as equation is an effect these are n equations here, and you have n equations here.

These are matrix equations so, in terms as a matrix equation this this called the state equation, this is the output equation. So, to summarize this discussion so far what we said was what you are dealing with a dynamic system you need to have some information about the past before you can predict the future out of the influence of a given excitation.

And the essential information regarding the past is summarized the by the state of the system. This state of the system is constitute by n dynamically independent variables from the system. There are several choices of choosing this n dynamic independent variables, for a given system the number n is fixed. But the choice of the particular variables is available to you.

In the case of an electrical network, RLC network, the inductor currents and the capacitor voltages offer themselves as convenient choice for the state variables. And therefore, we expect of there are n inductances or capacitors put together then we need to we will have n state variables. We have also observed, that once we know the state variables the values of the state variables or the value of the state vector.

So, we know the excitation functions we can find out that desired output quantities purely, if an algebraic process as you would do in the case of a non-dynamic system.

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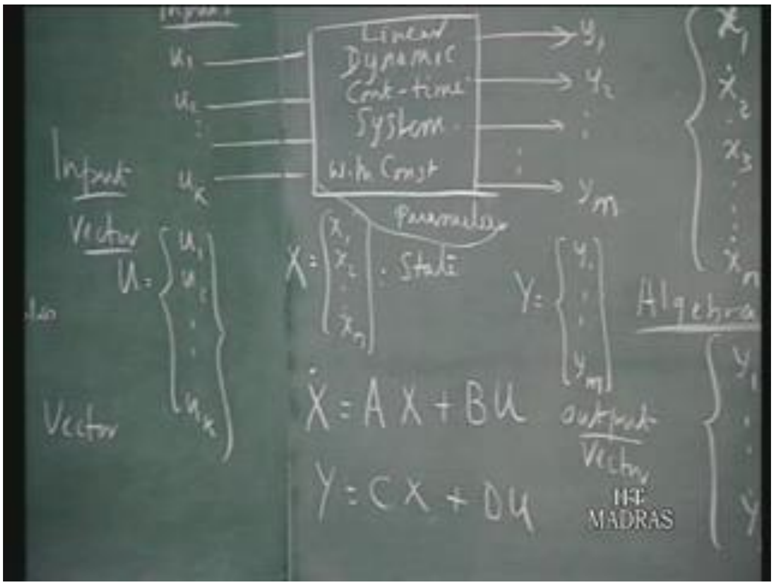

So, we will once we have this concept then the solution of the dynamics of the particular state or a particular system while downs to write in 2 sets of equations in this manner. We are talking about a general case of a k input, m output, system multiple input, multiple output system. And the inputs u1 to uk are given the symbol u this is the input vector u1 to uk.

The output vector Y is a m vector you have got m output y 1 to ym. And this state vector x 1 to xn represented by X so, we have 3 vectors u X and Y. And these are related by this equation where X dot the derivative of X with reference to time is AX plus Bu it is a matrix equation A and B have appropriate dimensions. A is of course, a square matrix of order n by n that is to be kept in mind. And Y equals CX plus Du, where for a constant parameter system A B C D or matrices a pure constant.

Now we observed that, we need to have calculus that means we must solve this first order differential equation coupled first order differential equation using various methods available for calculus. And once you have that the second part is purely substitution of that value of for this state vector that you have found out in this.

So, X is the value of X we have just found at substituted here u is already known. So, as far as computation Y is concerned it does not require any special effort in our part. The main effort would be to solve this set of equations for X. So, this is the set of coupled first order differential equations, that is what we have what we have there. 1 particular point you should like to mention here is, that earlier when you are talking about systems we said X is the input and Y is the output.

But in literature it is conventional to take u as the input whenever talking about state variable theory it is conventional to take u the symbol u for input and y for the outputs. Therefore, we have deviated from our standard practice and taking X as the input and Y as output. In the state variable theory by literature you will find normally X being the reserved for the state. Therefore, we are following that particular notation in dealing with state variable techniques.

So, X is not the input quantity it will be state input is given u and we should not confused this with the u that we use when we deal with unit step functions. Here it is differently simply the variables u 1 to uk function of time. Let us, workout on example to illustrate what we have discussed so far.

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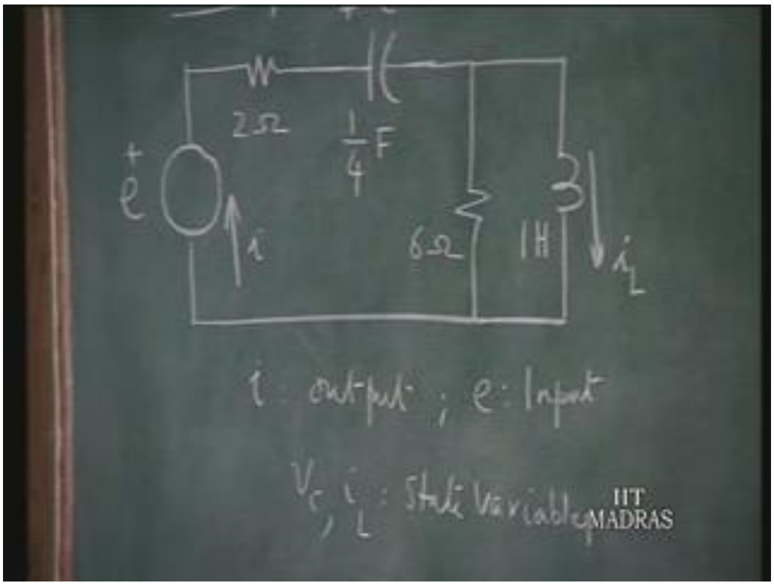

Let us, consider this example in which we have a capacitor and inductor. Let us, assume that this particular circuit is excited by a voltage source e and we are interested in finding out I as the output quantity. That means, our output vector 1 y consist of just 1 variable that is i. And let us take Vc the capacitor voltage and inductor current iL as a state variables.

So, Vc, iL are state variables so, this is x1 and this is x2 could be. And the input is e so this is u vector is only 1 variable that is e. So, with this identification let us try to form the equations X dot equals AX plus Bu and Y equals CX plus Du in this standard part.

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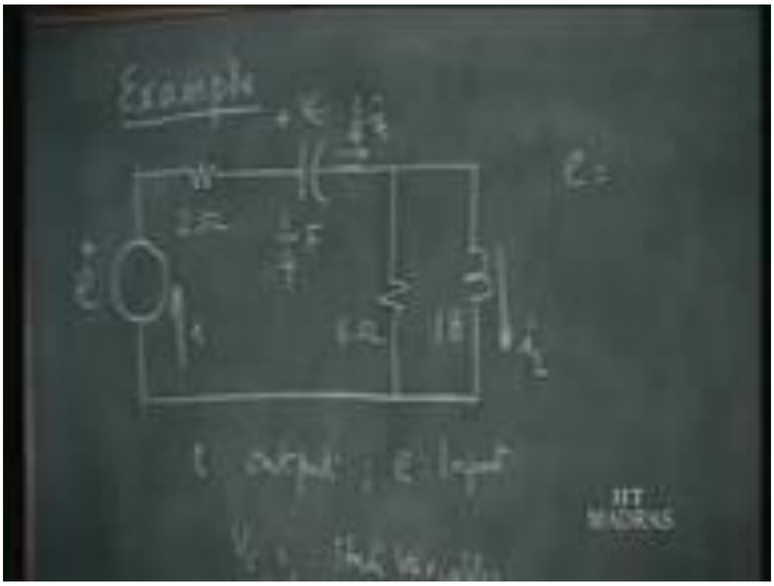

Now, we have e equals the current this is Vc therefore, the current in this branch CD Vc dt that is 1 fourth Vc dot. Since this is capacitor of 1 fourth farad and the voltage is Vc we will like to express everything in terms of Vc and iL and the input and the output quantity therefore, I would like to express the current in terms of Vc therefore, this 1 fourth Vc dot.

So, that is the current we passes through this therefore, the voltage drop across this is 2 times 1 fourth Vc dot. So, if you write the Kirchhoff's voltage law around the circuit we have to e equals this voltage plus this voltage plus this voltage. And what is this voltage? This voltage depends on the current here, this is iL and this is 1 fourth Vc dot. Therefore, the current here is 1 fourth Vc dot minus iL.

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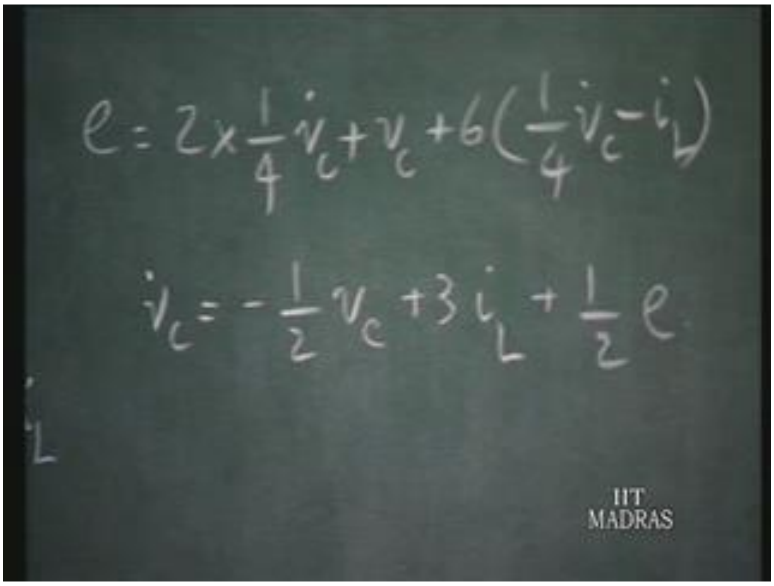

Therefore, I can write e is 2 times this is 2 times the current in that 1 fourth Vc dot plus that is the voltage across this drop, voltage drop across the resistance plus this voltage which is Vc plus the voltage drop across 6 ohm resistor which is 6 times the current through this 1 fourth Vc dot minus iL. And that if you simplify this you will get, Vc dot equals minus half Vc plus 3 iL plus half e this is 1 equation. Similarly, we must also have the equation involving iL dot.

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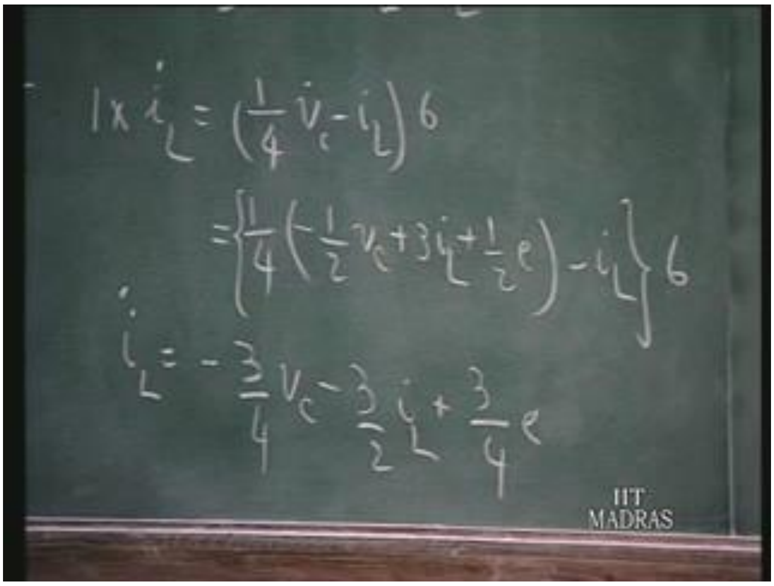

The voltage across this inductor is L di dt so, 1 times iL dot that is the voltage across the inductor. And that voltage is also equal to this voltage which is 1 fourth Vc dot minus iL times 6. And Vc dot is known to us already therefore, 1 fourth of for Vc dot is substitute minus half Vc plus 3 iL plus half e. That is Vc dot which we already known minus iL and the whole thing multiplied by 6.

And on simplifying this you can show that iL dot equals minus 3 quarters Vc minus 3 by 2 iL plus 3 quarters e.

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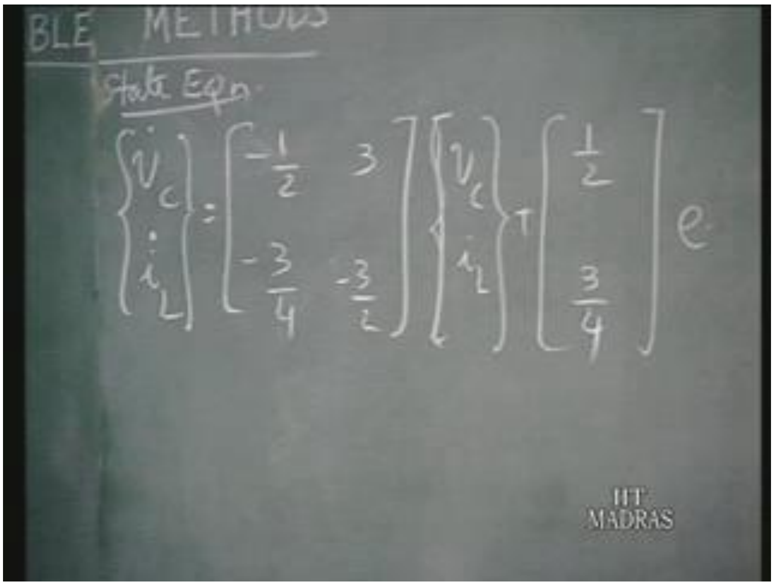

And therefore, we can now use these 2 and write Vc dot iL dot equals Vc iL plus another matrix times e. And using this Vc dot is minus half Vc plus 3 iL therefore, minus half Vc plus 3 iL plus half e therefore, similarly, iL dot equals minus 3 quarters Vc minus 3 by 2 iL plus 3 quarters e. So, this is the state equation in standard form X dot equals AX plus Bu minus half 3 minus 3 quarters minus 3 by 2 etcetera.

Now, for the output your output i this is the current we are we are interested in i equals 1 fourth Vc dot after all the same current that goes through. So, i can be obtained as 1 fourth Vc dot and we have the equation from Vc dot already.

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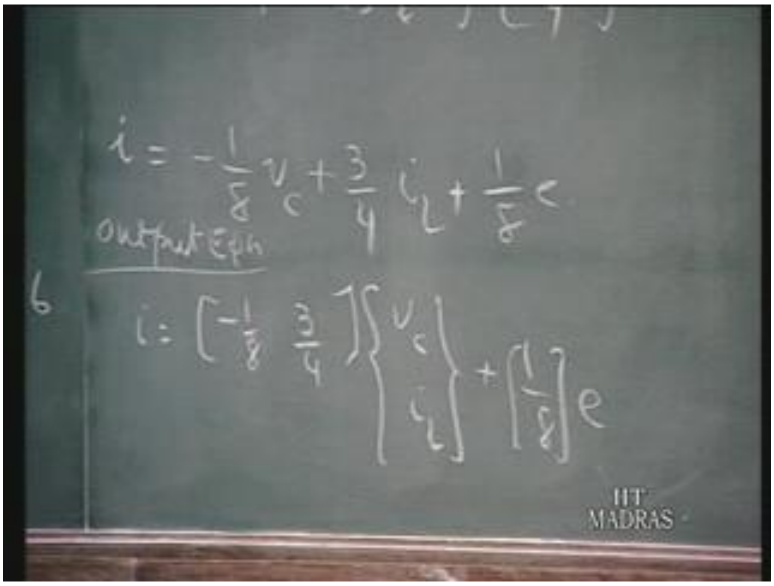

Therefore, i equals 1 fourth Vc dot that means minus 1 by 8 Vc plus 3 upon 4 times iL plus 1 by 8 e or to put this in the standard matrix form i equals minus 1 8 3 quarters Vc iL plus 1 eighth e. So, in this this is the output equation, in the standard form.

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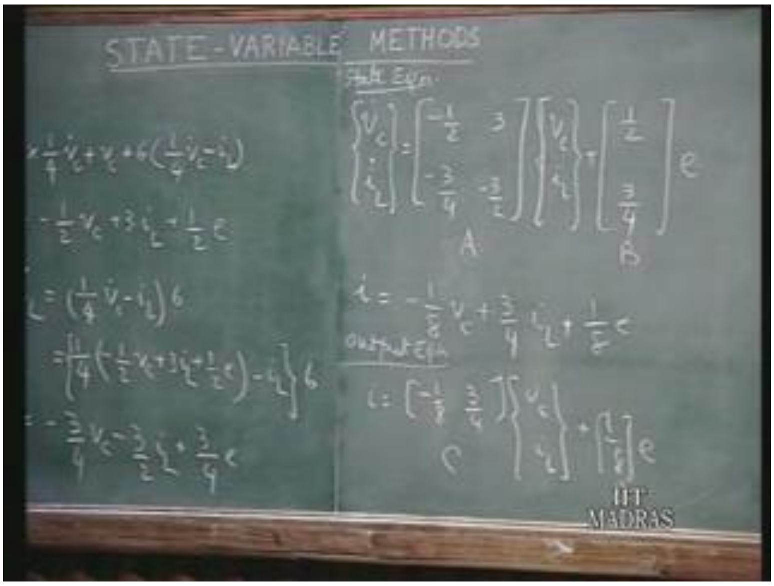

So, in this case the matrix A is a 2 by 2 matrix this is the B matrix, this is A matrix here, this is B matrix, this is C matrix and the D matrix is a pure scalar for migrate. Because, we have got only the single input and single output. So, this example illustrates how the state and output equations are formed choosing the appropriate set of state variables. And then, you got this situation in the final result in this form.

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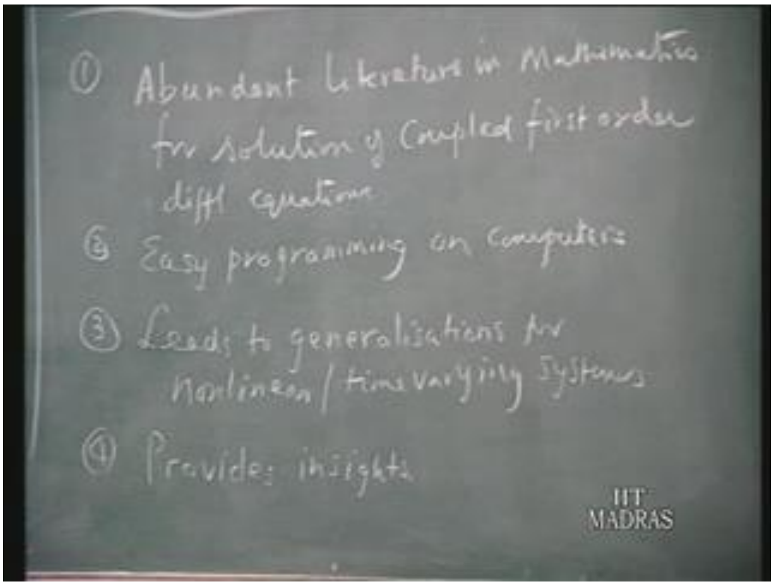

Let us now, look at the merits of state variable technique. I mentioned 1 merit already earlier but, let us summarize the various merits the state variable techniques. First of all there is abundant literature in mathematics on the solution of coupled first order differential equation which we had to deal with in a state variable technique. So, we mentioned that abundant literature in mathematics, mathematician of several techniques for solution of coupled first order differential equation.

Secondly, first order differential equation are very quite convenient for implementation for programming and a computer. So, easy programming on computers, either analog computer or digital computers. Third, leads to generalization for non-linear time varying systems. So, the approach of state variable techniques as we discussed for linear time invariant situations.

So, these the methods can be generalized to made applicable to non-linear and time varying systems also in more or less straight forward manner. Even though the solution techniques will be certainly more complicated but, the formulation of equation and so on will be can be made to follow this analogous slice. Therefore, it leads to convenient generalization as an applicable to non-linear time varying systems.

And finally, the state variable technique also provides such an insight later on we will see, we will talk about 0 input response, 0 state response, where principle of super position can be used where it can be used and thinks like that it provides such an insights into the system behavior like controllability, absorbability and several stability and so on and so forth.

So, it provides insights in such the system behavior in a striking way when you take the state variable approach compare to other techniques. So, this is an another state variable techniques. And because of that in more particularly because of easy in programming in computers the state variable technique as gained lot of popularity in reason times as compare to other methods of solution for dynamic performance of a system.

The next part that we would like to ask ourselves is, what is the in appropriate choice of state variables and what is the member of state variables that 1 would have in a particular system or a network. As I mentioned the state variables should be show chosen there dynamically independent of each other. And the same time, once you know the state variables the various quantities pertaining to the system can be deduced algebraically from the knowledge of state variable.

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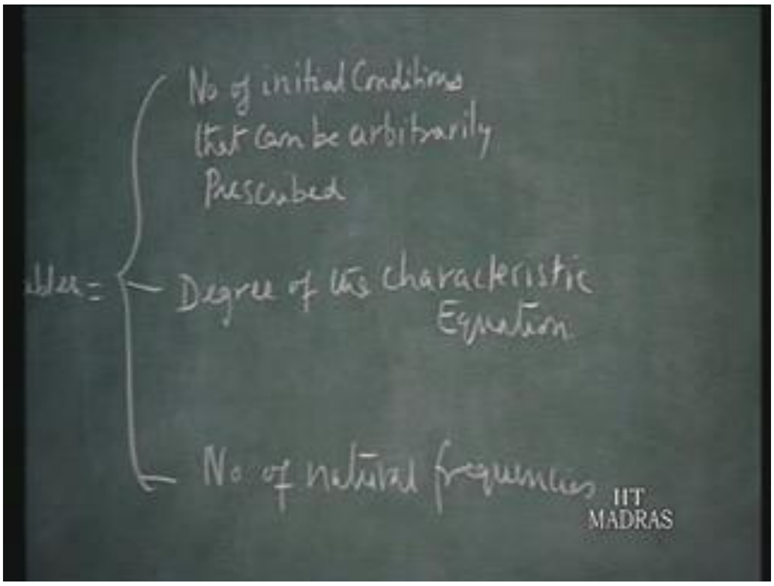

As for the number is concerned number of state variables we can let this is the value n this times out to be equal to the number of initial conditions that can be arbitrarily prescribed. So, when we are talking about a system characterized by differential equations we have to give some initial conditions. Number of independent initial condition that can be arbitrarily prescribed. So, this also turns out to be the degree of the characteristic equation, this also is equal to the number of natural frequencies. As you know, the roots and characteristic equation and natural frequencies.

So, all these are 1 at the same so, the state of a system is characterized by n variable where, n is the number is degraded the characteristic equation. If the number of natural frequencies and is also equal to the number of initial condition that you can arbitrarily prescribed. That means, the other initial conditions if it is there any can be derived from this n independent initial conditions.

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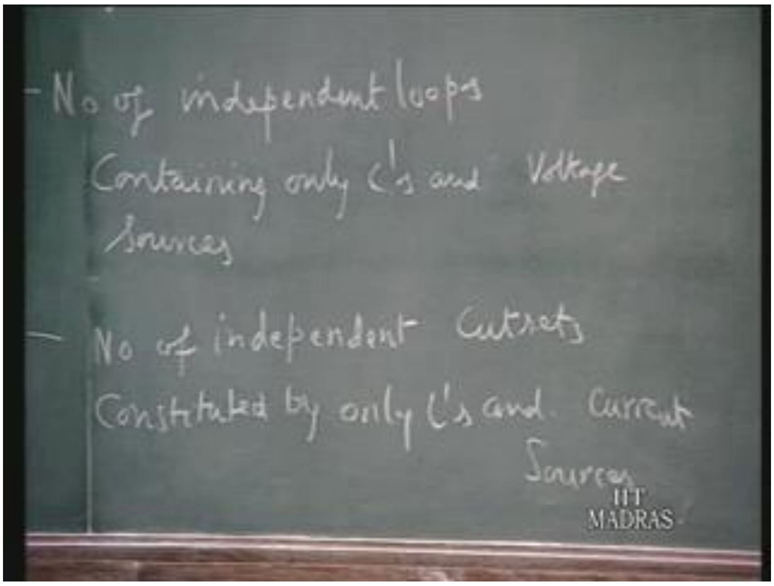

Now, coming to the particular RLC network the value of n equals after all we can associate an initial conditions with capacitors and inductors. Suppose, you have 4 capacitors and 3 inductors, you can have 4 capacitor voltages and 3 inductor current therefore, the number is 7. Therefore, for RLC network we can say n happens to be number of reactive elements that is L and C.

However, it may turn out that 3 capacitors form a loop all the 3 cannot be independently specified. Therefore, only 2 can be done because the third becomes dependent on the other 2. Therefore, you must deduce this by a number of independent loops containing only capacitors and or voltage sources we will explain that in a moment. And furthermore, it may be that as your node you have 3 inductors and therefore, all the 3 inductors current cannot be prescribed independently.

If 2 are known the third can be fixed therefore, it says by the number of independent I will say cut sets. I will explain this in a moment cut sets constituted by only inductances and current sources. I will remove this or and current sources, current source may or may not be present. I will explain this by means of examples.

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Suppose, I have a network like this you have 2 capacitors and 2 inductors therefore, n happens to be 4 we know the problem about this. Because, there is no loop or a closed circuit containing only C's independent loops containing only C's and or voltage sources. There is no loop which contains only C's or and voltage sources because, this loops as this circuit has got an inductor.

Therefore, there is no such thing and about the cut set I will explain in a moment later. But the take it for there is no cut set of inductors in this particular example.

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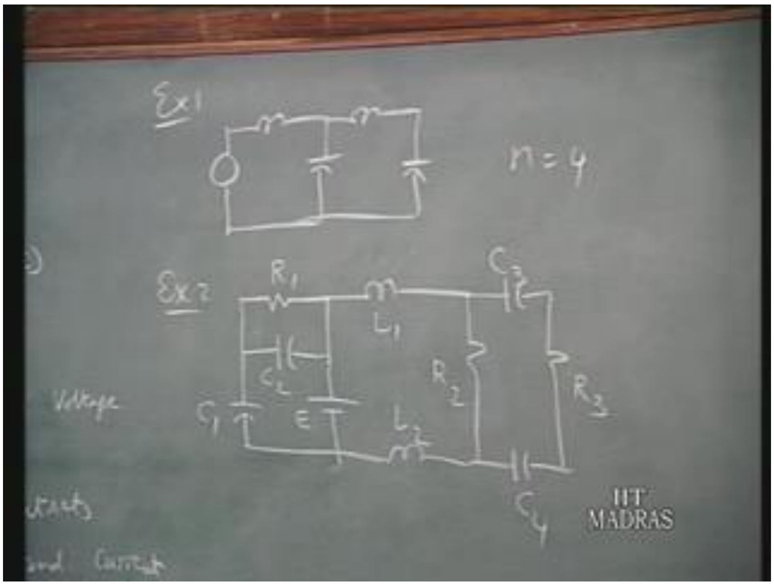

Second example, suppose I have this call this C1 C2 E R1 L1 L2 R2 C3 C4 R3. Now, in steadly quietly complicated situation, you observe that there is a loop here constitute by C1 C2 and E. So, if I know some voltage across the capacitance I know the voltage E therefore, this capacitance voltage gets fixed. Therefore, you cannot arbitrarily assign in the initial condition for both C1 and C2 only 1 of them you can independent you can assign.

Therefore, you have 1 loop containing only capacitance and voltage sources. Now, secondly these 2 inductances have the quality that there remove this if you remove this 2 inductances as from the circuit then the network falls in the 2 pieces. Or in other words we can say if there is a current i 1 here this current i 2 must be returned here. Because, after all i 1 enters the whole 1 part network and that current must come back.

Therefore, i1 and i2 are equal to each other in other words, these 2 currents in L1 and L2 are not independent of each other. If I know this current I know this current. Therefore, you cannot independently described the initial currents of the 2 inductors L1 and L2 both of them you can prescribe the initial current for only 1 of them.

So, that is what is meant by a cut set, A cut set is really a set of elements and removal of which the network falls into 2 pieces, there no connection between 1 part and the other. And if I have 3 inductors coming to together this is also a cut set because, the removal of this particular node gets a separated from the rest of the network that is also a cut set.

So, in this example now the n the number of state variables at we should content with in the solution of this problem.

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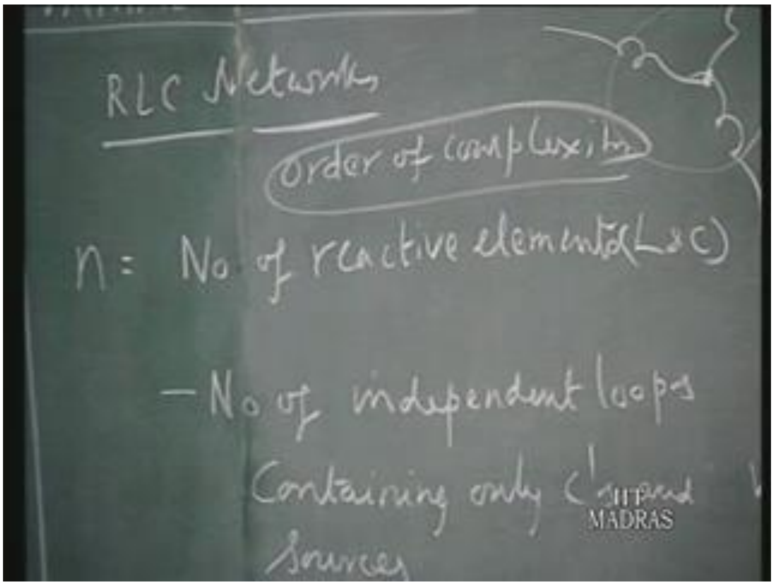

The number of reactive elements 1 2 3 4 5 6. 6 is the number of reactive elements, number of inductors and capacitance put together minus the number of independent loops containing only C's and voltage sources there is only 1 such loop minus 1. The number of independent cut set constitute by a inductances and current sources. There is only 1 such minus 1 therefore, its equal 4.

So, even though you have total of 6 reactive elements the number of state variables turns out to be 4. And this number of state variables is also referred to as a order of complexity. So, this is another name for it the order of complexities of this network id 4 that is equal to the number of state variables that are required for this review for this problem. So, in this lecture we introduce ourselves the concept of state of a system and mentioned that is the minimum information that is needed about the past of the system.

So that, with this knowledge and the knowledge are the input that is applied from that point onwards we can deduce the output. Secondly, we also found that if you state of the system is known as a function of time and the input also known as function of time, whatever output you need to know about the system can be deduced from the 2 sets purely it is an algebraic process without having to solve any differential or difference equation.

So, we found that a way of representing this state the equation formulating the equation for the solution is to write X dot equals to AX plus Bu, where X is the state vector n vector with n components and X dot is the vector of the derivative of the state variables. So, X dot equals AX plus Bu when Bu is the input vector and the output can be once you solved this state equation for X you can get the output Y various output Y output vector Y as CX plus Du, where C and D are matrices of appropriate dynamics.

The basic problem the solution at the state equation once you have got that the output can be obtained quite in a straight forward manner. We worked out in an example illustrate, how the state equation and the output equation can be formed. And then, we set the choice of the state variables should be decide that the dynamically independent and once they are known various quantities can be deduced algebraically.

The number that is associated with this that n times out to be the number of initial conditions that can be prescribed the given system independently. This is also the degree of the characteristic equation which also happens to be the number of natural frequencies.

So, applied this this criterion apply to a particular case of RLC network turns out to be that this number n equals the number of reactive elements minus the number of loops which are constituted purely by capacitances and voltage sources or capacitances alone. Or minus the number of cut sets constituted by inductors alone or inductors and current sources.

So, this example illustrated that particular case, where we have total 6 reactive elements but, the number of independent dynamically independent variables which constitute this state value will be only 4. And this particular number you also called order of complexity of the state value. So, in the we have introduced ourselves to the concept of the state forming the state equations. And how and the number of variables that are required in a given situation.

In the next lecture, we will look at a formal method of writing down the state equations and the output equations; what we have shown in the case of example is narrow approach it is not in any systematic way we have written down the state equation of the particular example.

In the next lecture, we will discuss methods of writing the state and output equation in a more formal manner.