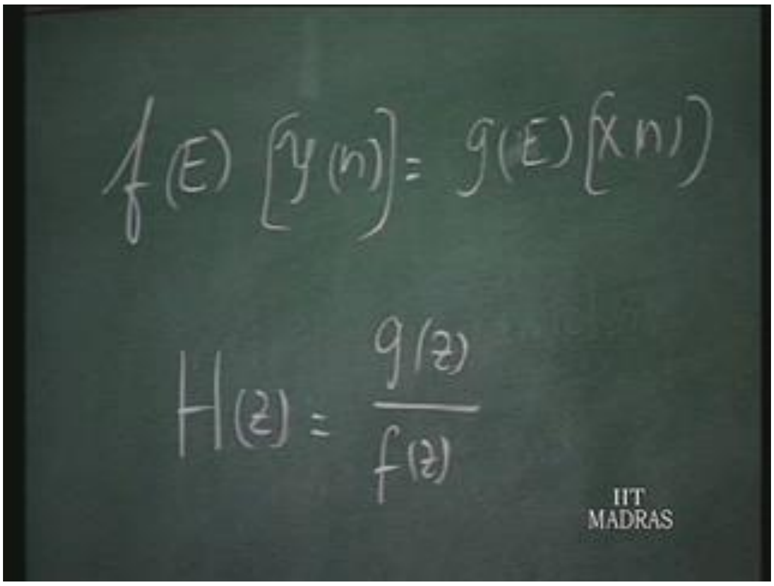
Networks and Systems
Prof V G K Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 44
Discrete – Time Systems (7)
Natural and forced responses
Frequency response
Exercise 8

We acquired in ourselves if the concept of discrete time system function H of z in the last lecture. We define this discrete time system function as the Z transform ratio of Z transform of the output discrete time signal to the Z transform of the input discrete time signal with 0 initial condition prior to the application on the input. The definition follows and closed lines to the definition of the system function in the continuous time case.

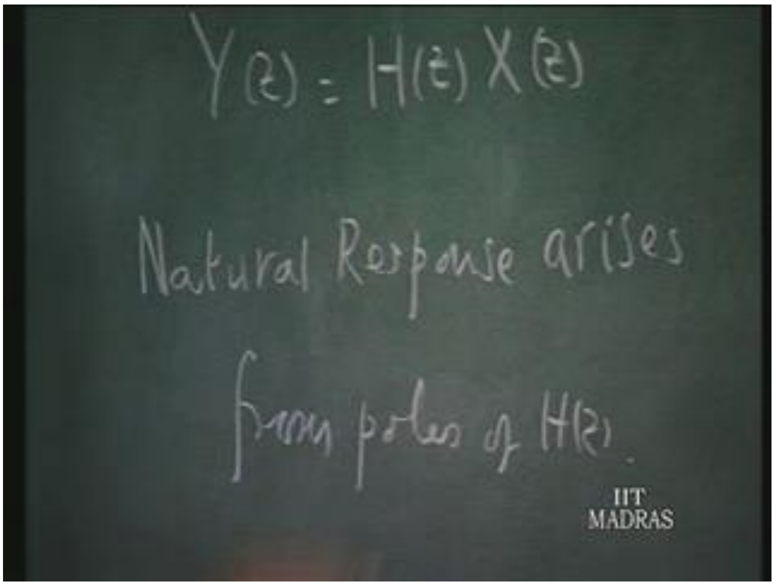
(Refer Slide Time 02:15)



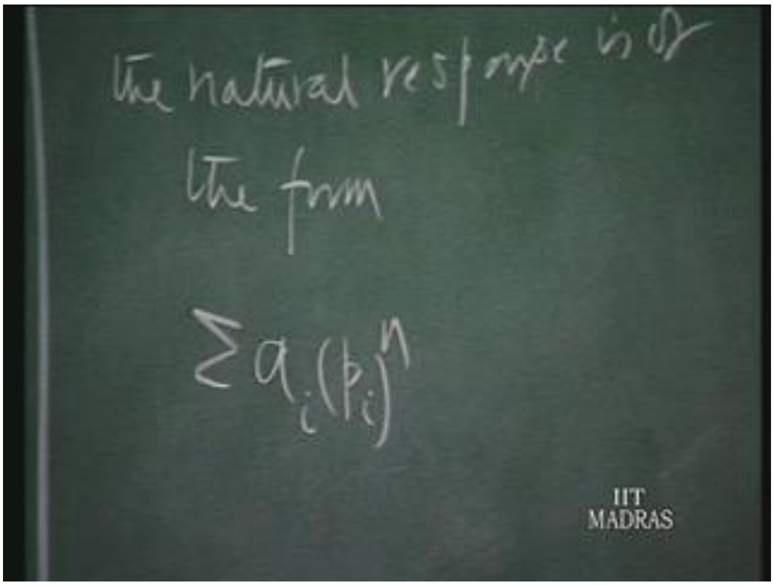
We also saw that if the difference equation pertaining the input and the output is of the form fE operating on y of n is g of E operating in x of n. Where fE and gE are operator functions. Then we said the system function H of Z can be obtained as g of z over f of z where g and f are polynomial in Z. We also observed that, H of z is the Z transform of the impulse response of the system and we also said that for any given input xn whose Z transform of it Hz. The output Z transform is obtained as Yz is HZ times X of z.

Now, when you try to find out the time function corresponding to Yz y of n you make the partial fraction expansion of that and you make the partial fraction expansion certain terms come from the poles of H of z and certain terms come from the poles X of z. The terms that are contributed by the poles of a H of z will indicate the natural response or free response of the system. The term that arises from the poles of X of Z is the contribution which is due to the forced in function and which will call the forced part as the rest part.

(Refer Slide Time 03:33)

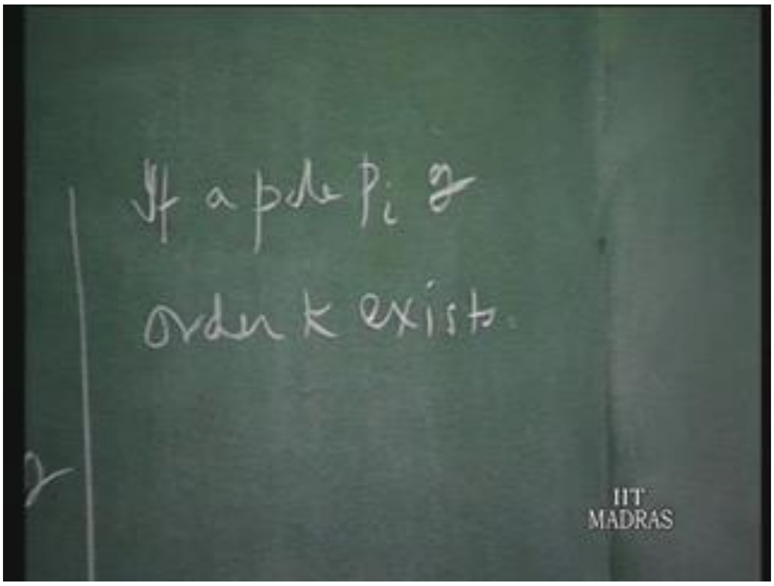


(Refer Slide Time 04:25)

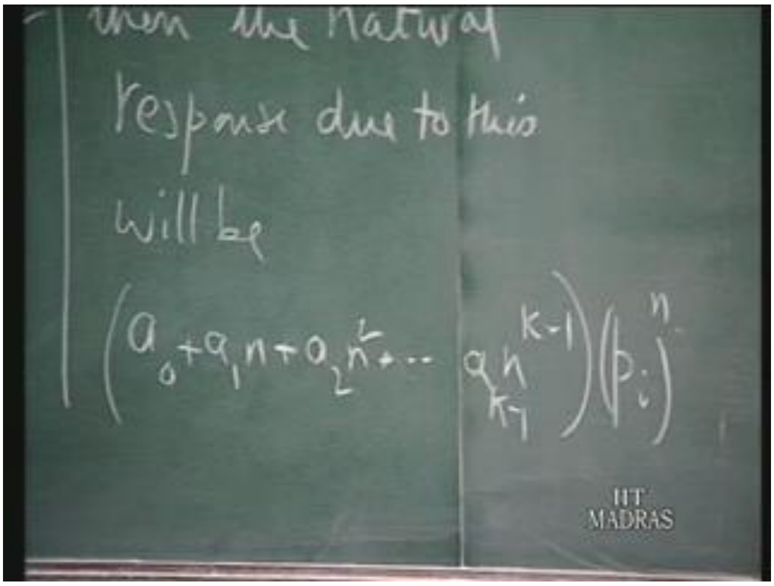


Now, let us look at the natural response in some detail. So, natural response or free response arises from poles of H of Z. So, in the natural response if the set pi are poles of H of z and are all distinct that is no pole is repeated then, the natural response is of the form ai Pi raised to the power of n summed something which you already know from our partial fraction expansion Now, in the particular pole is repeated what happens. If a pole pi of order k exist; that means, a particular pole is repeated K times; that means, Z minus pi raised to the power of k Z minus pi raised to the power of k is a factor of f of Z.

(Refer Slide Time 04:45)

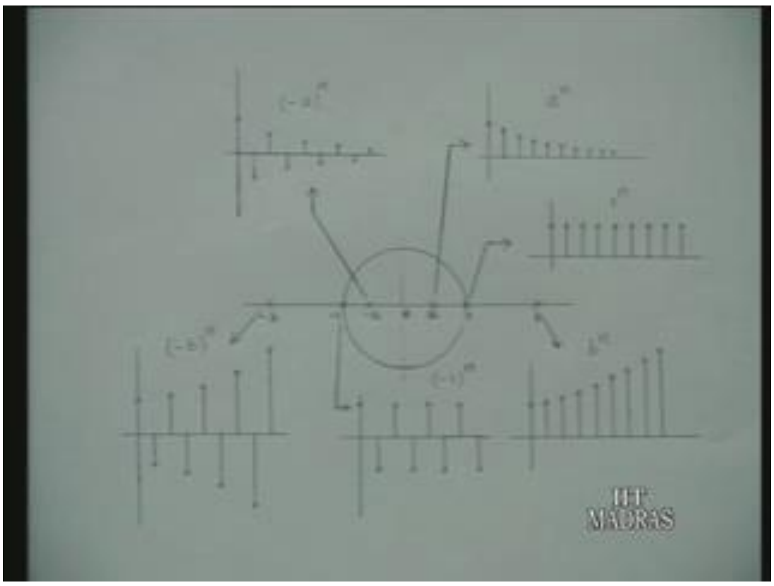


(Refer Slide Time 05:32)



So, then the natural response will be the form, due to this will be of the form a not plus a 1 n plus a 2 n squared, write up to a n k minus 1 a k minus 1 n k minus 1 multiplied by pi raised to the power of n, that will be the natural form of the response. So, to look at this natural response forms; let us look at the chats here which will give you some idea. Now, depending on the location of the poles within the unit circle you have different types of responses. Suppose, there is a pole at Z equals 1; that means, right at the 1 end at the right most corner of the right most end of this circle Z equals 1, then naturally the response will be a unit step.

(Refer Slide Time 06:40)



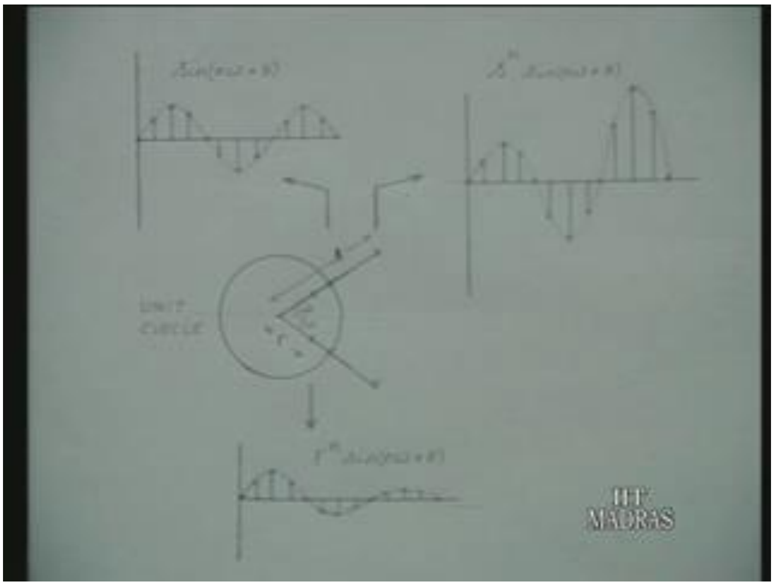
On the other hand suppose there is a pole at Z equals a; where a I s less than 1 than; that means, it will be a to the power of n it is decaying type of response that we get. On the other hand, if there is a pole which is on the real axis the ready sense greater than 1 from the origin that means; this is this rising response so, it goes up like this. On the other hand there is suppose there is a pole at minus 1 that indicates a response which is plus 1 and minus 1 alternately it is minus 1 raised to the power of n; so, it becomes alternately plus 1 and minus 1.

On the other hand, if there is a pole at say minus a where a magnitude is less than 1. Then, minus a raised to the power of n indicates a decaying response with alternate samples by positive and negative. Suppose there is a pole at on the negative real axis at a distance larger than 1 from the origin then again it is increasing the response which is alternately positive or negative. So, it is clear from the point of your stability that all the poles must be within the unit circle as we have seen here. In this case, the poles here and on the unit circle indicate table response. Let us see what happens when we have conjugate complex conjugate poles.

Suppose we have complex conjugate poles; suppose, there are complex conjugate poles on the unit circle itself then that indicates a steady response of sinusoidal; either a cosine or sinusoid it will put sin omega n omega t n omega plus theta as in general form. On the other hand, if the poles are situated complex conjugate poles at a distance less than 1 from the origin then it indicates the decaying oscillations amplitude decaying.

And that again indicates the stable response if the complex conjugate poles at a distance S from the origin where S is larger than 1. It again indicates oscillatory behavior, but the amplitude oscillation goes on increasing this clearly in unstable behavior and that is arises. Because the poles are distanced larger than 1 from the origin; let us take the third choice

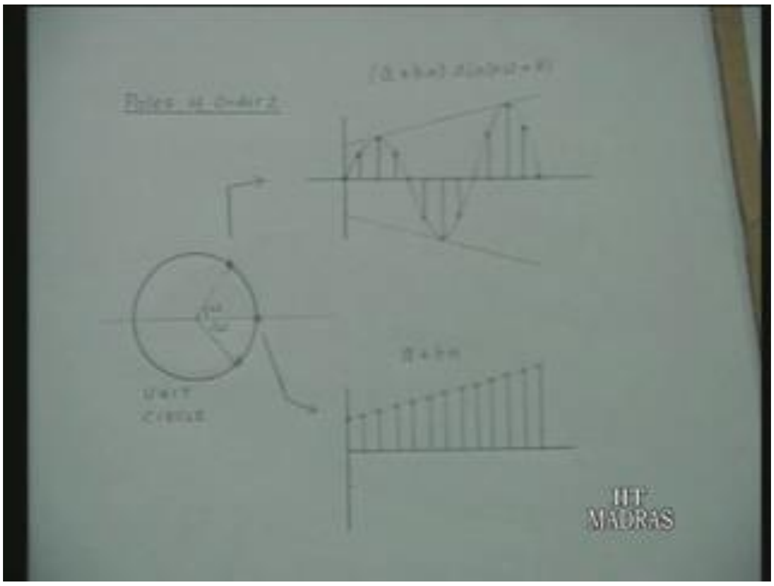
(Refer Slide Time 07:20)



Now, here we have let us take a case there are double poles there 2 poles at the origin at Z equals 1; then that will be of the form the response will be a plus bn; it indicates it increases the response increase with n and therefore, this is clearly an unstable situation. And on the other hand if you have complex conjugate poles a pair of complex conjugate poles which are order 2;then again it is a sinusoid with increasing amplitude it will be the form a plus bn sin n omega plus theta.

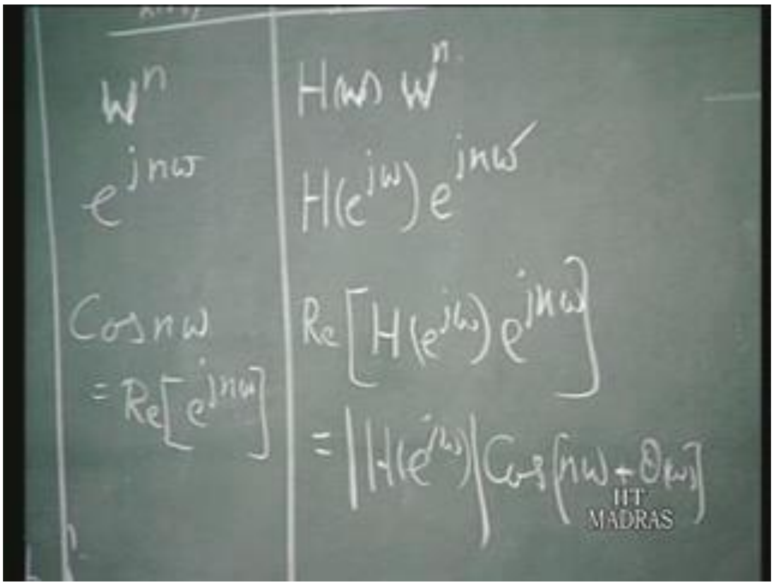
So, clearly for stable behavior we require that the poles b within the unit circle it does not matter of any order as long as there is a unit circle the poles by any multiplicity what's and ultimately the response will be decay this time.

(Refer Slide Time 08:10)



But on the other hand, if there are poles on the unit circle it that distances 1 from the origin than those poles must be simple that is sum and substance of the rest instance of the pole locations for a stable discrete time system. You recall that in the case of continuous time system we said stable system for all the poles must be in the left of real axis and if there are in imaginary axis they must be simple. In a similar way here, for a stable system we demand that the poles must be within the unit circle, if there are in the unit circle itself they must be simple.

(Refer Slide Time 11:10)

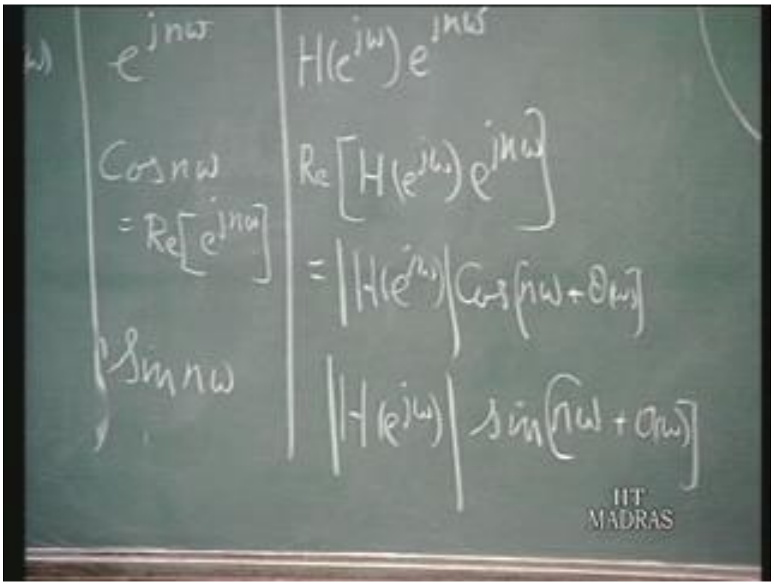


Now, let us also look at the forced response characteristics. We have seen that if the input xn is wn the forced response in the form, H of w evaluated w times wn. This can be easily worked out using the partial fraction expansion. And; that means, the X of Z will be Z upon Z minus w and find out the residue corresponding to that pole and you can work this out it can easily be shown that this is shown. This also can be seen by the point of view that the characteristics signal and the output is also a characteristics signal for a discrete time system and the coefficient will be H of w. Omega w can be any particular value it can be a complex it can be complex also its not be real.

So, suppose you take e j n omega; that means, w is now identified; this is w is identified at e to the power of j omega. So, the force response will be H e to the power of H of e to the power of j omega times e to the power of j n omega. Now, let us take cos n omega; which is the real part of e to the power of j n omega. So, the response due to this the forced response due to this, which is in the case of steady state response this is steady state input. The steady state response due to that; assuming that, all the transients all the the natural response is a decaying pipe.

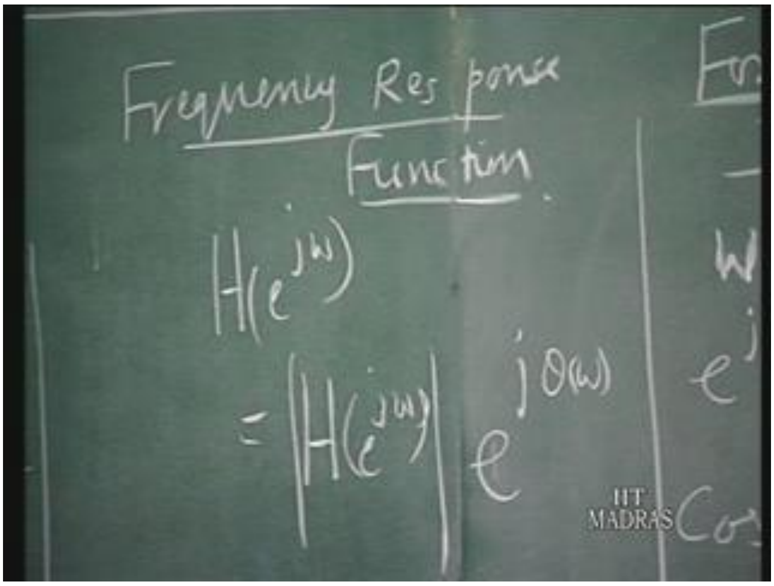
These steady states response to that will be obtain by taking the real part of this because the you are taken the input as the real part of this they will put to the real part of this. So, the real part of this of H e to the power of H of e H of e to the power of j omega e to the power of j n omega which can be written as, H e to the power of j omega magnitude; this may have a magnitude and an angle. So, H e to the power of j e to the power of j omega magnitude times cos n omega plus theta omega, where the H e to the power of j omega. Suppose; I write H e to the power of j omega as H e to the power of j omega magnitude and with an angle theta omega.

(Refer Slide Time 12:51)



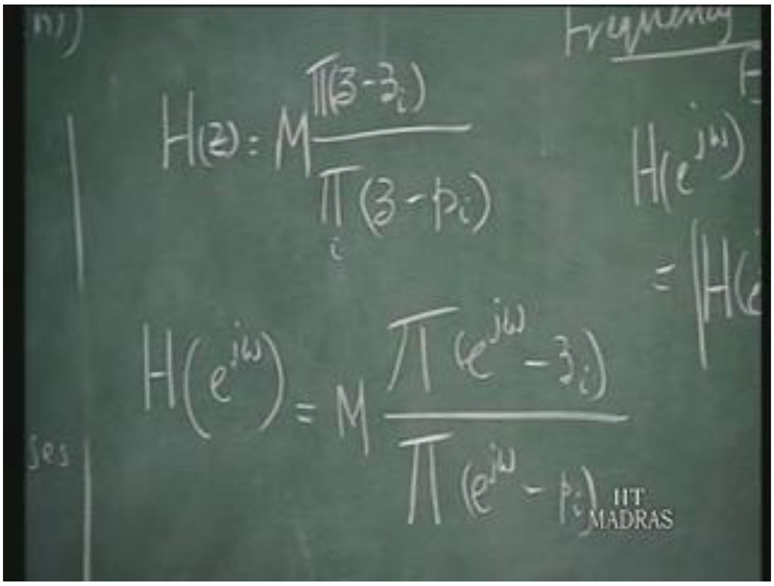
So, the angle associated with H e to the power of j omega that theta omega is the additional angle in the cosine function and the magnitude is modified by this. Similarly, you can write sin n omega you can show the output will be H e j omega magnitude sin n omega plus theta omega. So, when you are driving this with a sinusoidal input sequence then, the output is also sinusoidal and steady state condition assuming that the transients are decayed down and that time you are dealing the stable systems. The input sequence is modified in amplitude by this factor and it is modified in face by this factor.

(Refer Slide Time 13:31)



So, clearly H e to the power of j omega has the same role to play in the frequency deciding the frequency response; as what will have in H of j omega in the continuous time systems. So, this is called the frequency response function. So, in the discrete time case the frequency response function is obtained, from the system function H of z by substituting e to the power of j omega for Z. In the continuous time case we substitute j omega for S; here you have substitute e to the power of j omega for Z. So, this defines the magnitude and the face both are function of omega the particular frequency of the input sinusoid.

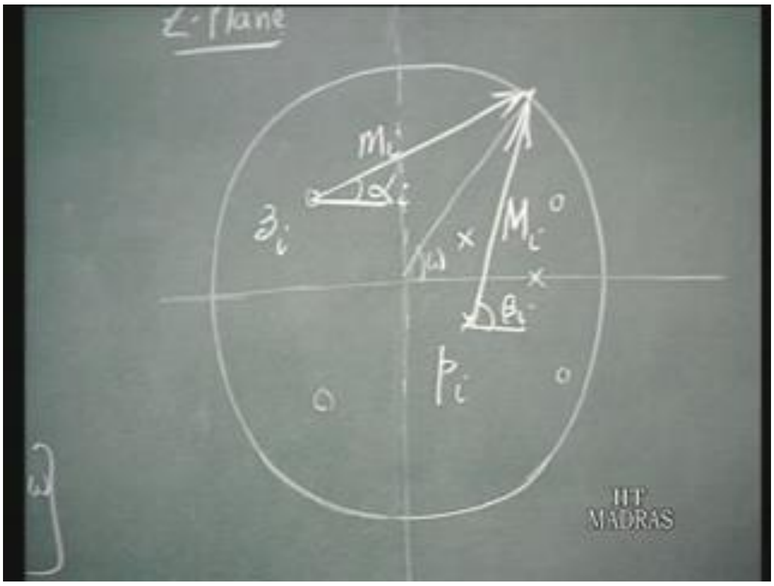
(Refer Slide Time 14:28)



Now, to obtain H e to the power of j omega you can once you know the rational function corresponding to H of z you can certainly substitute e to the power of j omega for Z. And evaluate both the magnitude and face of the frequency response function. Alternatively you can say, suppose H of z is of the from M z minus zi product. So, the numerator is factored and these are the various 0 locations and the denominator have poles and you have these are the various pole locations p1, p2, p 3 and so, on and so, forth.

So, when you want to evaluate H of z, when z takes the value e to the power of j omega H as a function of e to the power of j omega well now will be M times the product of e to the power of j omega minus zi divided by the product e to the power of j omega minus pi. So, all this means is you are evaluating H of z at a point e to the power of j omega is clearly a point in the unit circle. So, you take a point on the unit circle corresponding to the omega that, you are considering that distance when that point from the each 0. The each term represents the directed line from zi to the particular point under consideration.

(Refer Slide Time 16:00)

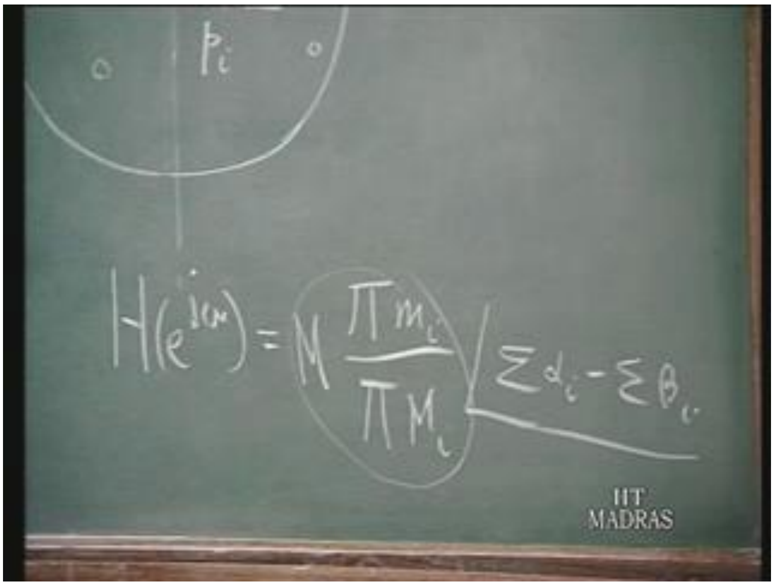


So, you take the product of those entire complex fractions divide by similar products of the lines, complex numbers representing the line joining pi e to the power of j omega multiplied by the scale factor that will give the frequency response. To put this in other words let us look at this diagram, where we have the poles and zeros locations of H of z are marked here,0 are marked by circles and the poles are marked by crosses.

Suppose this is the point at which you want to consider the frequency response; that means this angle is omega; e to the power of j omega is this point. So, the line from the particular 0 to this we will have let us say this is zi the location of this zi and let us say the line has a length Mi at an angle alpha i. Similarly, you consider per each pole suppose this is the pole pi; so, you draw a line here and this let us say, this is capital Mi the length at the angle beta i.

You repeat this at at the point under considerations you consider such lines segments for all the poles and 0s you can now say that, the frequency response functions evaluated at that point equals the constant multiplying factor M is there which we have here. That is the constant multiplying factor times the product of all this line segments going from the origin.

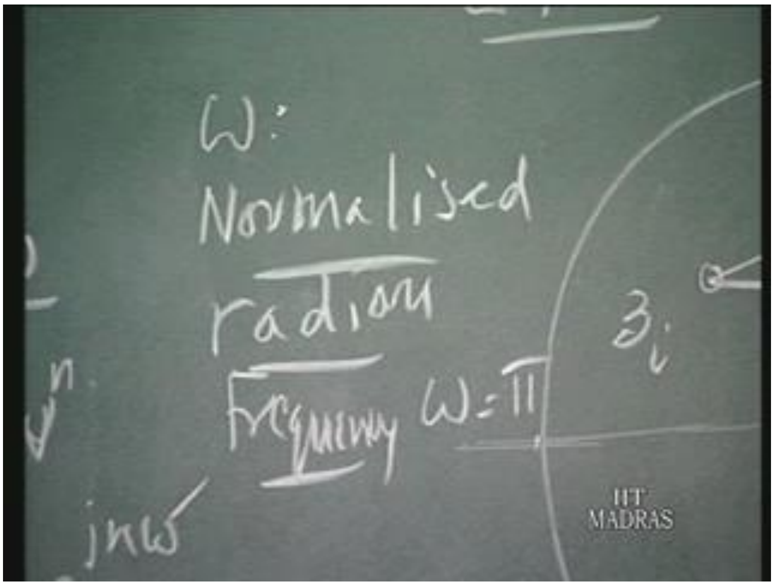
(Refer Slide Time 17:33)



Therefore, you have product of all this small mi’s divided by the product of all this capital Mi’s thus at the poles; in addition this will have an angle which is summation of alpha i minus summation beta i. So, this is the angle associated with the frequency response function and this is the magnitude of the frequency response function. So, 1 can visualize the frequency response of a particular discrete time system by looking at the poles and 0s. And as a move along this unit circle, you can pick up the nature of the various the frequency response for the different frequencies. Clearly this is point corresponding to omega equals 0 and this is equal to omega equals pi by 2 and this is equal to omega equals pi.

Now, when you are talking about the sinusoidal signal sin n omega and cos n omega we are assuming here that, the spacing between different samples is 1 second; actually we should have written sin n omega t in a general case, where if the time axis we are talking about 0 T, 2T, 3T etcetera. So, in this case you would have to sin n omega T; because the samples are placed displaced by 1 another by an angle by an amount of T seconds, but here we are talking about normalized case where T is 1 seconds. And therefore, the omega comes out to be the angle itself.

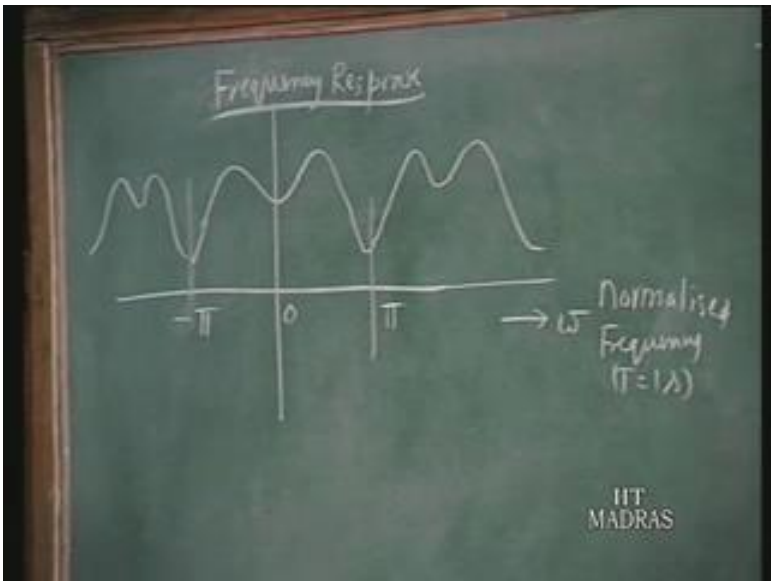
(Refer Slide Time 19:15)



So, omega T you should have actually what we have plot the angle is omega T, but T being 1 second omega itself is measured in terms of radians it is a normalized frequency variable. So, omega in this case that we call normalized frequency values normalized radian frequency. What happens if you do not have this normalization, but we will postpone that for the time being? Let assume that is T is 1 T is 1 second. What we have observed here; therefore is as you change the frequency you calculate the frequency response you can go up to pi 2 pi and so, on.

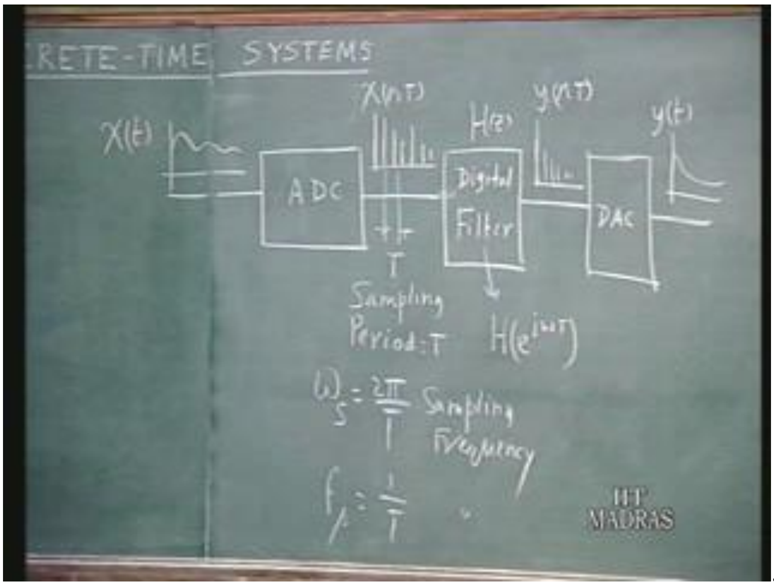
Suppose you come back to 2 pi and start all over again you must get the same frequency response. Because as you move along this unit circle complete 1 circle and come back to same point you must have the same frequency response. That means; as omega increments by 2 pi radians, whatever frequency response you have the original value will continue the same at the new value, which means; on other words that, the frequency response function of a discrete time systems periodic character. So, let me illustrate that a little more in get a detail.

(Refer Slide Time 20:08)



So, the frequency response functions suppose you plot, you want plot the magnitude the frequency response function with as a function of omega, then as you can see that, it must be a periodic and character because as you move around the unit circle, you come back to the same values after every rotation through by radians. So, this is periodic and character and that is something, which is a fact of life as far as the discrete time signals are systems are concerned. Now, I mentioned that we have taken here the normalized frequency; that means, the kept the sampling period is equal to 1 second.

(Refer Slide Time 21:01)



.

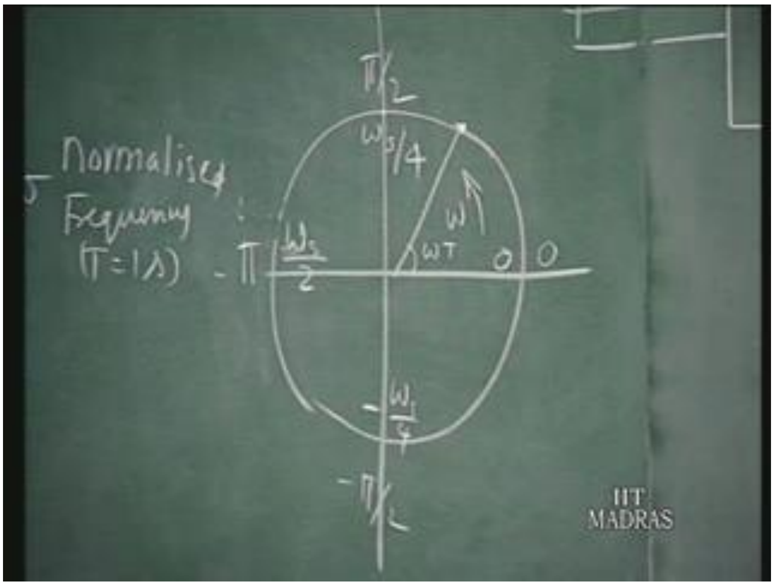
Let us look at general case, where discrete time systems are commonly used. A digital filter gives a particular example of a discrete time systems. It is a digital filter what is done is, a given analog signal continuous time signal except t it is digitized its discredited you generate from thus through a sample at whole circuitry an analog to digital converter whatever it is X nT; that means, this is discrete time signal with sampling space between 2 existence samples by T seconds.

So, this is called the sampling period T; and we also indicate omega S as the sampling frequency in radians per second or in hertz f of S 1 over T this is also called sampling frequency, this is in radians per second this is in hertz. Now, this input sequence is given to a digital system, which is we called the digital filter in this context. And you get an output sample y nT and the system function corresponding to this discrete time system is H of z.

Therefore, the frequency response for this now we will be H e to the power of j omega T instead of e to the power of j omega because we are now, taking a general sampling period T instead of where, we want to evaluate this frequency response of this system function. You must substitute e to the power of j omega T rather than e to the power of j omega. And once you have the output sequence, than you put through some filters to remove the high frequency components and so, on. This is what is called analog re constructor you can recover back your y of t.

So, given an analog input signal you get an analog output signal, but in between you convert this discrete time signal and the frequency response of the system is characterized by H e to the power of j omega T. So, you design your filters so that, this H e to the power of j omega T has the desired variations with reference to the frequency.

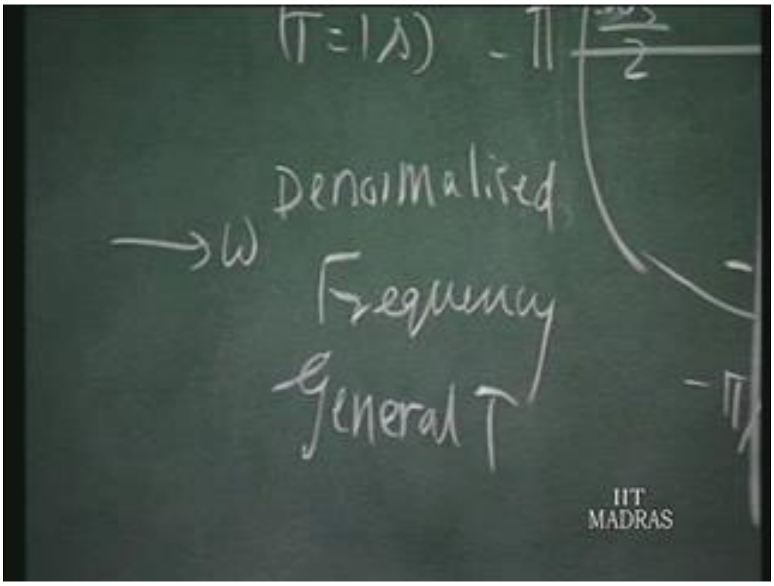
(Refer Slide Time 24:15)



Now, if H e to the power of j omega it frequency response, then our unit circle now earlier in terms of the normalized variable, we put 0, pi by 2 and minus pi by 2 minus pi and. so, on so, forth. But in terms of the actual frequency now, you can write this as this is equal to omega s, this minus omega s, and this is omega s by 2 this must be omega s by 2; I will write here pi here I will write here omega s upon 2, this is omega s upon 4, this is minus omega s upon 4 and so, on and so, forth and this is of course, 0.

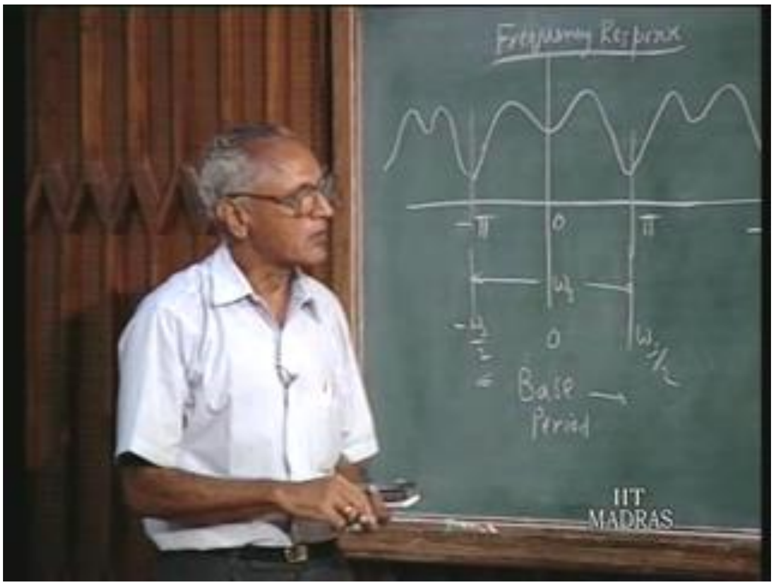
So, we graduate this unit circle if you want to do it in in terms of frequency, actual frequency omega. So, when we want to evaluate the frequency response at omega s upon 4 this is the point, which you consider in the normalized case it becomes pi by 2. So, the connecting between that is omega T is now, in the angle that you have to use instead of simply omega . So, at any particular frequency if you want, what is the angle this must be omega T at that point we must evaluate the frequency response.

(Refer Slide Time 24:55)



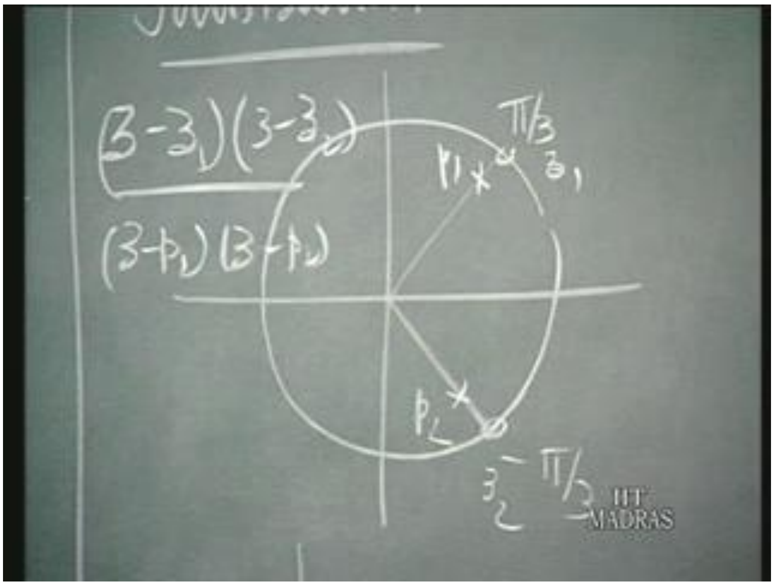
So, which means that; in terms of the frequency response characteristics here interms of actual frequencies this will be 0, this will be omega s upon 2, and this will be minus omega s upon 2? You can say demoralized frequency; so, if you do not normalize it and consider this omega s such in the raw form general T then this would be how it is.

(Refer Slide Time 25:23)



So, the frequency response of digital filter or any discrete time system in general is periodic and character and that period is omega s is depend upon the sampling frequency. That means; if the period sampling period is T 2 pi by T is the sampling frequency omega s and this is the base period. So, this period whatever occurs in 1 period repeats itself identically as we go long; just as illustration of how you can obtain the frequency response from the pole 0 plot.

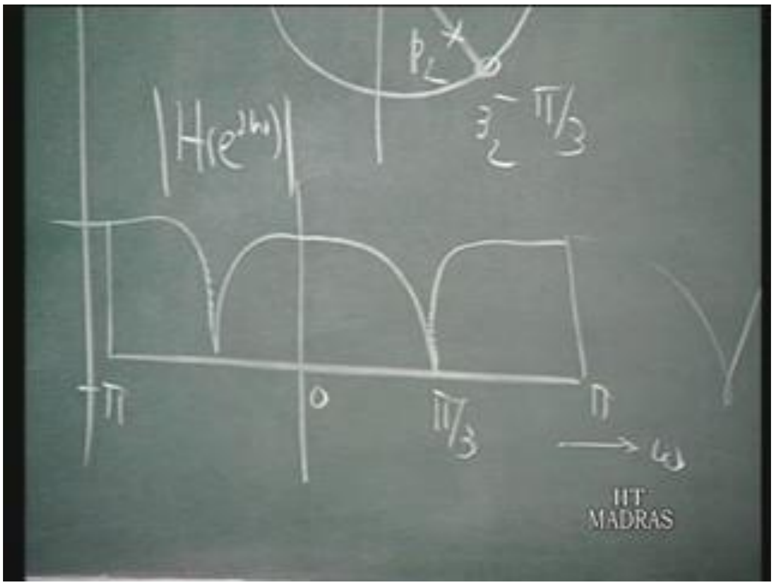
(Refer Slide Time 25:30)



Let us take a particular discrete time system function which has got a 0 at an angle pi by 3 and minus pi by 3 at pair of 0 complex conjugate 0s and poles, which are very close to them at the same line. So, the system function let us say is, this is z1 this is z2; this is p1 this is p2; that means, the system function is z minus z1 times z minus z2 divided by z minus p1 times z minus p2 that, we have taken a complex multiplying factors 1.So, what will be the frequency response of this; suppose you ask, then the answer will be.

So, H e to the power of j omega magnitude is what we are plotting; now, at this point that is the dc value now omega equals 0 that distance between at that point from z1 and p1 are they are all same, because this very close to this. Similarly, p 2 and z 2 are very close. So, in as a matter of that; as you move along this line at almost most of the point that distance between the row in point and the distance in the row in point p1 and z1 will be the same and the distance in row in point from p2 and z2 all be nearly the same. How we are as you approach this point z 1 then the distance becomes 0. So, clearly if you are sitting here the let the line segment from the row in point z1 is going to be 0; that means, the numerator is going to fall down to 0.

(Refer Slide Time 27:50)



So, as a function of omega it will be like this and this is Dc and this point corresponds to suppose you call this sixty degree pi by 3 I have mentioned already. Therefore, this is pi by 3 and then this is pi and this is minus pi the same thing repeats itself again and again So, this is what is by call natch filter; that means, the output is reduced to 0 at a single frequency, while trying to maintain the output that the same level as in the input for the rest of the frequency.

So, if this is 1 almost it is 1 everywhere except at this particular frequency this is what is meant, by call the natch filter. And in terms of the sampling frequency this will be omega s by 2 this will be omega s upon 6. So, such filters are designed to blank out some particular undecided frequencies in a given systems. This is what may call as natch filter. So, what we have discussed; so, far in the discrete time systems the treatment of discrete time systems we have started with the concept of discrete time signals.

These are signals, which are defined specified at distinct points at the independent variable, independent variable can be the position a length or time you take time to the commonly at the common independent variable, commonly you take that with independent variable that is why we call this discrete time signals. And systems which operated and discrete time signals, which take a input sequence of this sample values and then, out this as corresponding output or called discrete time systems. And these systems of the various kinds of the discrete time system that are possible we can confined and our attention to the linear time invariant case, where the systems are described by linear difference equation with constant coefficients.

So, then we saw how to analyze this system based on the difference equation approach we looked at the briefly at the classical method of solution of difference equations. And then, after wards we studied the operational methods, transform methods applicable to a discrete time system. This particular transform is called the Z transform, which is the counter part of Laplace’s transform in the continuous time case.We saw the various properties of Z transforms and we saw how the Z transform of enables as to convert a difference equation in to pure algebraic equation.

So, then we worked out a few examples showing the efficacy of the Z transform method in the solution of the transients associated with the discrete time system. We also look that, the discrete time system function H of z, which is the counter part of the H of z the continuous time system function all the properties more or less same; H of z is the Z transform for the impulse response and we saw that, the poles of H of z will specify the will indicate the character of the natural response of the system. And stability, these poles must be within the unit circle; if there is the unit circle those poles must be simple of unit multiplicity.

We have also saw that, H evaluated at a point of the imaginary at a point in the unit circle will specify the frequency response. There is a correspondence between the point and the unit circle and the particular frequency, which want find out the frequency response. You saw the various issues relate in to that, importantly we have found noticed that; the frequency response of discrete time system is peridican character there is base period, which is equal to 2 pi by the sampling time period T 2 pi by T or omega s the sampling frequency. And whatever is happening within the base period gets repeated a subsequent intervals.

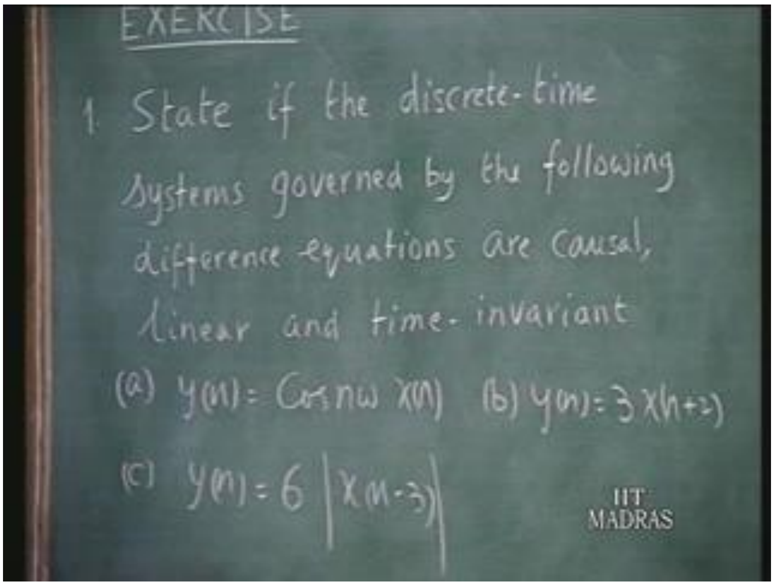
So, any filtering you want to do; you must make sure that you do not have a components outside that or if you have very components outside that, you must somehow hope up with the periodic character of the frequency response function. So, usually what is done is in you would confine all the filtering that you need to do, within the base band of that and you see that, you do not have to content with components outside that base band. And discrete signal processing discrete time signal DSP digital signal processing; these techniques play a weightier role or the Z transform methods and the frequency response and so, on.

What we have not done in this particular course of lectures, particular set of lectures we have been discussed. The concept similar to Fourier series or Fourier transforms in the continuous time case, here also we have discrete Fourier series and discrete Fourier transforms. These follow and analog the slice as the continuous time systems and we can thing of discrete Fourier transform as a special case of the z transform. When z is e to the power of j omega, but that is something which is outside the scope of our lecture; we did not discuss that in this particular set of lectures.

The importance of discrete time system is going day by day particularly with digital signal processing. And this technique we have discussed here will enable you to study the digital signal processing and alley topics with once you have a good foundation for studying these topics, which are recommend more and more important today with the computer hardware available and digital signal processing hardware available to you. And these are becoming very important not only communication, but also in control and instrumentation.

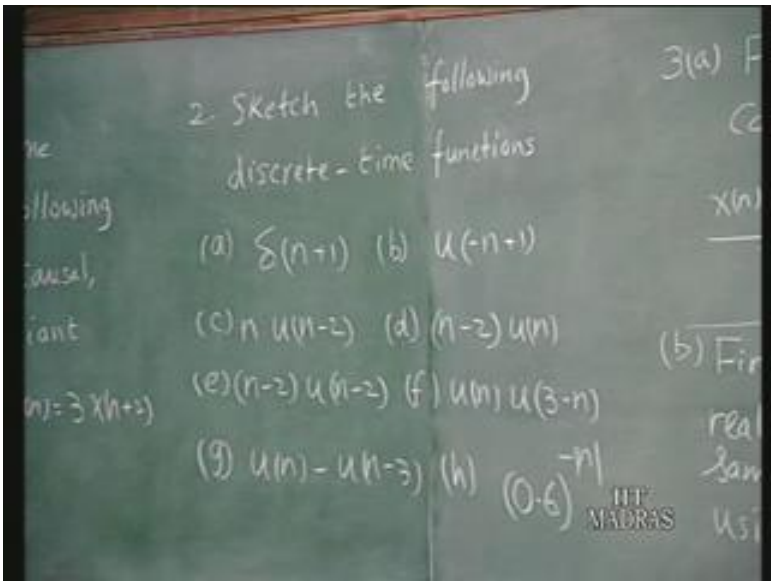
So, what you have studied here will be quite useful for you take up advance topic in DSP and similar related subjects. At this time now, we will put end to our discussion of stop our lectures on discrete time systems at this point. I will give you an exercise for you to work out on the topics that is you are studied and discrete time systems.

(Refer Slide Time 33:52)



First problem in this exercise; I am giving you certain different equations state if the discrete time systems governed by the following difference equations or causal linear and time invariant. So, examine each of this difference equations state, if they are causal if they are linear and time invariant; 1 or this other attributes may be absent. First 1 yn cos n omega xn. The yn is always the output and xn is the input; b: yn equals 3 times x of n plus 2. c: yn is 6 times the magnitude of the signal x n minus 3. So, take these 3 and for each of this examine, if they satisfy the property of causality, linearity and time invariance. This is the first example; the second question is sketch the following discrete time functions.

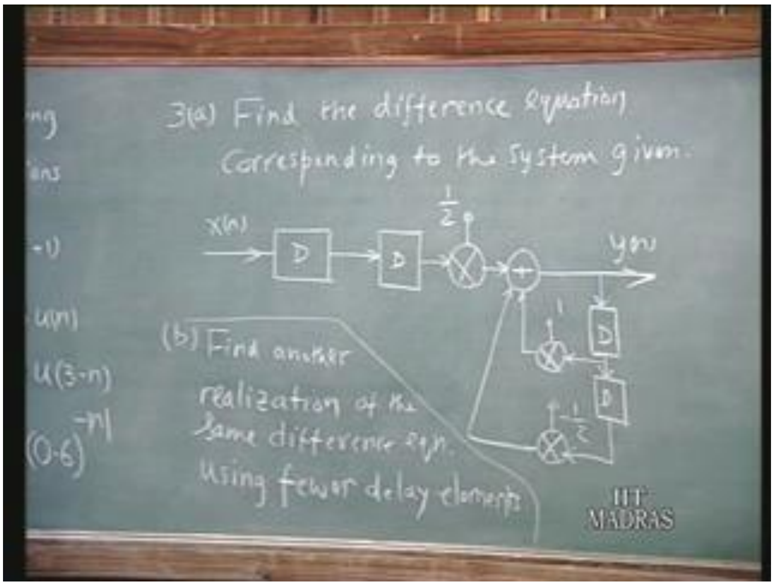
(Refer Slide Time 35:42)



So, we give you a number of discrete time functions symbolic fashion you try to sketch this samples the sequence of samples showing the waveform of the signals .A: delta n plus 1.B: u of minus n plus 1.C: n times u of n minus 2o. It is understood u n is the unit unit step functions. So, you must find what u n minus 2 is. D: minus 2 multiplied by un. E: n minus 2 multiplied by u of n minus 2. You must find see the difference between c d and e. F un multiplied by analog discrete time signal u of 3 minus n. G: u n minus u of n minus 3. And h: 0.6 raised to the power of minus magnitude of n.

So, for positive n it is minus n; for negative n is of course, minus of magnitude of the negative n; that means, it will be symmetrical for both directions 0.6 raised to the power of minus the magnitude of n. So, you sketch the discrete time function pertaining in to this various functions given the symbolic form in this fashion; 3a this is a discrete time system, given as a assembly of delay units coefficient multipliers and adders find the difference equation corresponding to the system given difference equation joining the output yn with the input xn.

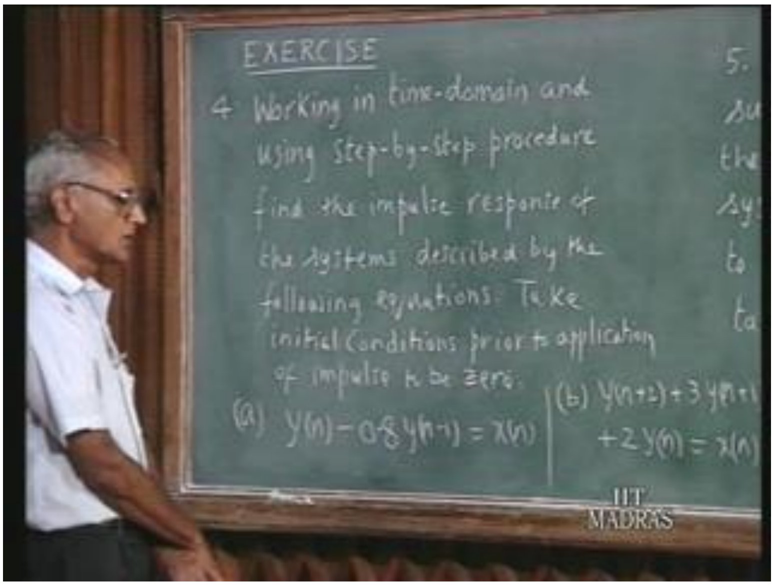
(Refer Slide Time 38:15)



So, the first equation in terms of yn and xn yn is the output and xn is the input. So, xn is feed to delay unit 1 unit delay, but another delay unit and this is given to a coefficient multiplier with a coefficient half multiplication factor half. This is given to as 1 input to an adder, the output of the adder goes to yn that is the output; the output is delayed by 1 unit in this another 1 unit in this and the 2 delayed versions are multiplied by coefficient 1 and minus half respectively and the outputs of this coefficient multipliers are given as an additional inputs to the adder here.

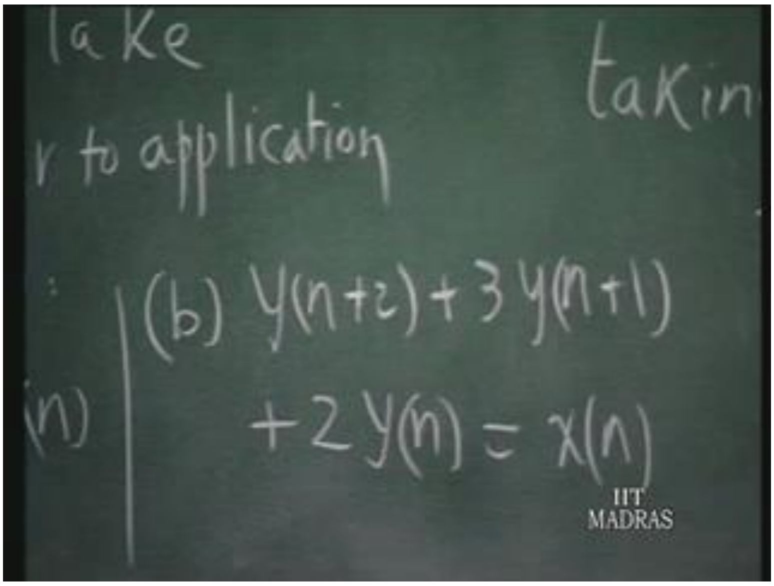
So, the sum of these 3 inputs gives to yn. So, you find the difference equation corresponding to the system joining yn and xn. Find another realization of the same difference equation; using fewer delay elements you observe that this is going to be a second order difference equation. So, in principle as a second order difference equation can be realized with 2 delay units you have used 4 units here. So, you try to find out another discrete time system, which realize the same difference equation, but using fewer delay elements.

(Refer Slide Time 38:25)



Fourth question; working in time domain and using step by step procedure find the impulse response of the systems described by the following equations. Take initial conditions prior to application of impulse to be 0.So, you take the output yn to be 0 for n less than 0. The 2 equations are yn minus 0.6 y n minus 1 equal's xn. According to usual conjunction xn is the input and yn is the output. You have to take xn to be impulse, impulse in this case workout to the iterative procedure and find out the solution for yn.

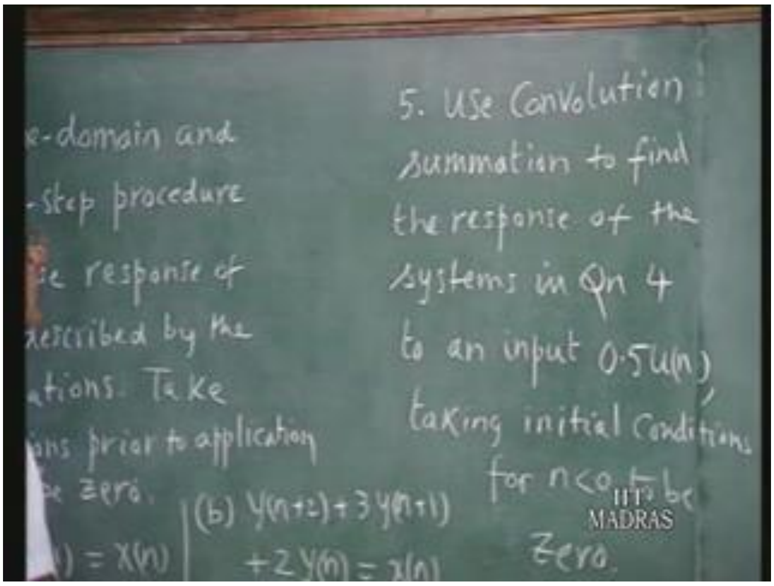
(Refer Slide Time 39:30)



The second equation is yn plus 2 plus 3 yn plus 1 plus 2 yn equals xn. So, this is the second order difference equation. So, for these 2 systems 2 difference equations to take xn to be the impulse and find out the yn, as a sequence of samples taking yn to be 0 for value n less than 0. Second example: the second next question example number 5 questions number 5; use convolution summation to find the response of the systems in question 4; these 2 earlier difference equations, but this time to an input 0.5 un taking initial condition once again to be 0 for n less than 0.

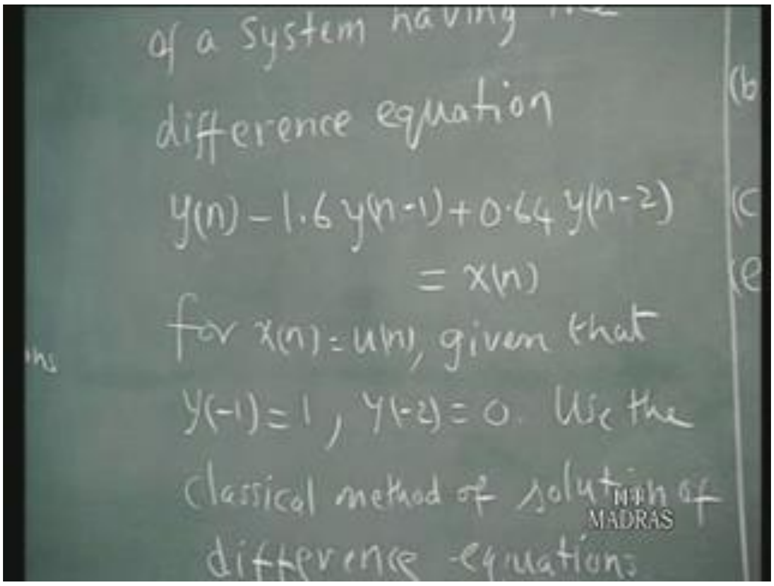
So, for the same 2 equations taking the input now to be 0.5 un and use the convolution summation because, you know the impulse response already use the impulse response and then, the new forcing function 0.5 un use the convolution principle to find out, the u response for the 2 systems.

(Refer Slide Time 39:50)



So, sixth question is find the complete response of a system having, the difference equation yn minus 1.5 yn minus 1 plus 0.4 yn minus 2 equals xn. This is once again, the second order difference equation for an input xn which is the step un, where given the 2 initial conditions y minus 1 equals 1 y minus 2 equals 0. Now, solve this using the classical method of solution of difference equations. So, you find out the auxiliary equation, find the complementary solution and find the particular solution for the given input function un. Use the classical approach, you find out the solution for this using the 2 initial conditions.

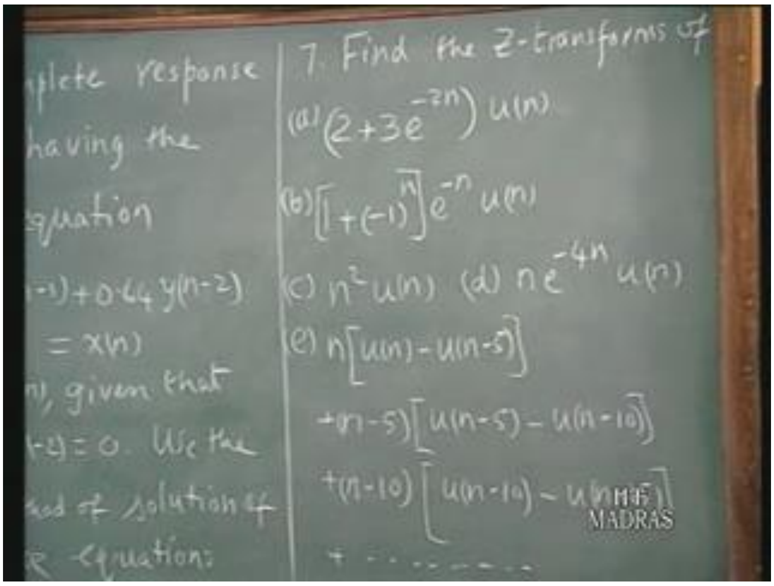
(Refer Slide Time 40:45)



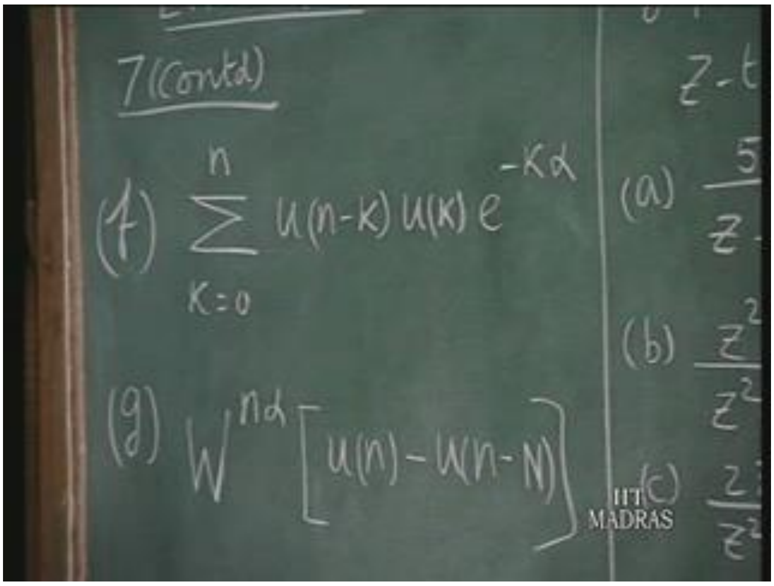
Seventh question is; you are given a sequence of discrete time functions find out the Z transform. A 2 plus 3 e to the power of minus 2 n all multiplying un; 1 plus minus 1 raised to the power of n e minus n un; that means, alternatively this will plus 1 and minus 1 when, this is minus 1 the whole thing become 0, when this is plus 1 this becomes 2. So, you get alternative samples 1 plus minus 1 raised to the power of n whole thing multiplying e to the power of minus n un. C:n squared un. D: n e to the power of minus 4 n un. E: n time un minus u n minus 5.

So, that means; this is the kind of gate function; that means, un minus un minus 5 means that, you have values only within the from n equals 0 to n equals 4 as you conceive that, multiplied by n plus n minus 5 times u n minus 5 times un minus 10. This again, a set of samples plus n minus 10 multiplying by u n minus 10 minus un minus 15 and this pattern repeats itself endlessly. So, I put dotted lines; so, you go on like this, next is n minus 15 times un minus 15 minus un minus 20 and so, on and so, forth.

(Refer Slide Time 42:48)



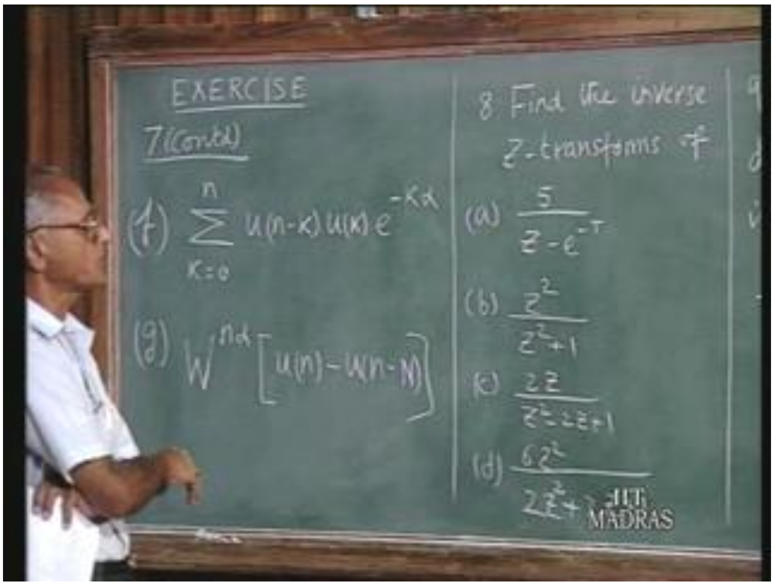
(Refer Slide Time 43:00)



So, find out the Z transforms of all this discrete time signals; I will give a few more I will write that down after wards. This problem will continue in the next presentation. This is a continuous of problem 7, where you are just find Z transforms; next part is summation from k equals 0 to n of u n minus k uk e to the power of minus k alpha. You recognize this to be in the form of convolution summation. So, you should use the appropriate properties.G: W raised to the power of n alpha multiplied by un minus u of n minus N. So, this is the pulse function multiplied by wn to the power of n alpha. Find out the Z transform of this.

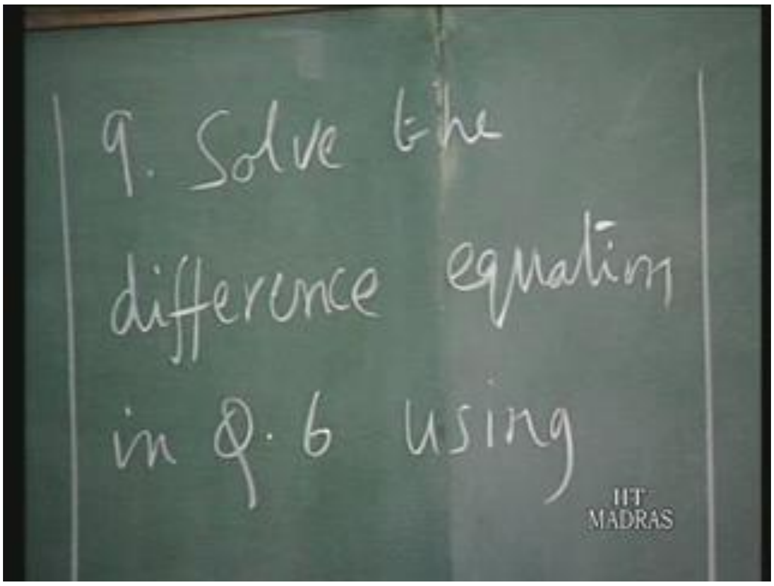
The next question is you are given a series of Z transforms you are asked to find out the discrete time functions which have this Z transform; that means, you have to find out the inverse Z transform. A: 5 upon z minus e to the power of minus T.B: Z squared upon Z squared plus 1. C:2 Z upon Z squared minus 2 Z plus 1.D: 6 Z squared upon 2 Z squared plus 2 Z plus 1.I will read this again 6Z divided by 2Z squared plus 2Z plus 1.So, you have to find out the inverse Z transforms of these 4 functions.

(Refer Slide Time 44:18)



Question number 9 solves the difference equation question 6 using the Z transform method. Earlier in question 6 you are given a difference equation and you are asked to solve that, by the classical method of solution difference equation.

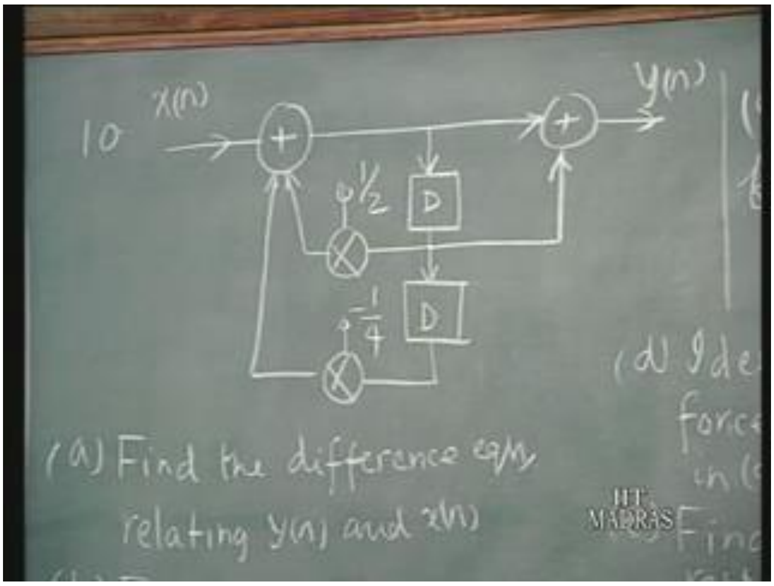
(Refer Slide Time 44:35)



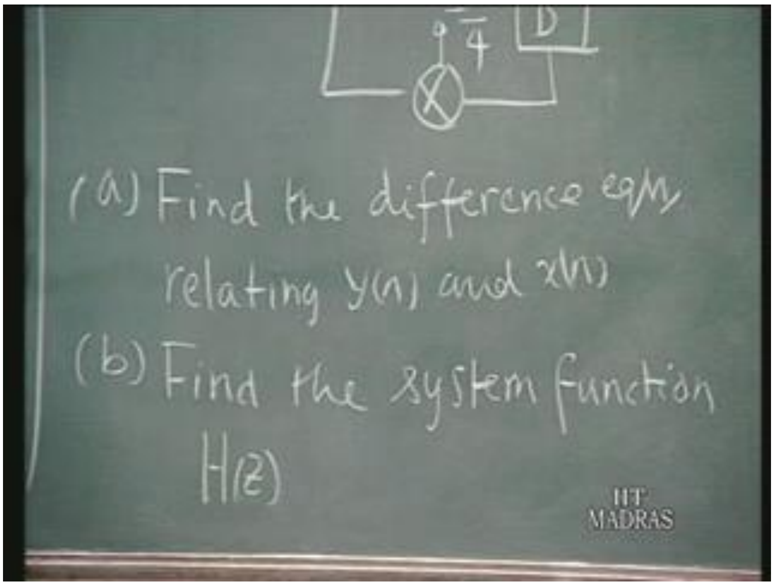
Now, you repeat the solution this time using the Z transform method and compare the answers both must be the same. This again pertains the same difference equation as given in the question number 6. The last question in the exercise, you are given a discrete time system model as in this diagram. You have in xn feeding fed as 1 input to an adder these are the other 2 inputs. And that goes out to another adder with additional input that points the yn the output sample. And whatever signal is coming here is, delayed by 1 unit in this delay unit and another 1 unit in this delay unit.

The 2 delayed versions are multiplied by coefficient multipliers using coefficient multipliers by coefficient half and minus one-fourth and the outputs are fed to this adder. And the signal at this point is fed to this adder these 2 give that, will contribute to yn. So, using this model of discrete time system find, the difference equation pertaining to the output yn and how it is related to xn

(Refer Slide Time 45:05)

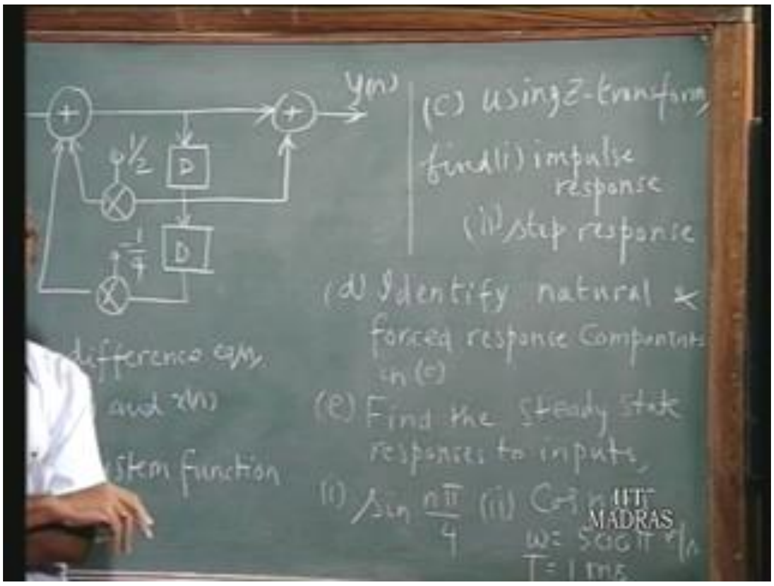


(Refer Slide Time 46:00)



So, relating yn and xn find the difference equation from this circuit representation. Find the system function for the same system H of z that is the Z transform of yn. The ratio of Z transforms of yn to Z transform xn the 0 initial conditions. Using the Z transform find the impulse response of the system and find the step response in system.Whenever we normally talk about, impulse response and step response we assume 0 initial conditions prior to the application of these 2 inputs. So, that is assumed you are not scatting just specifically find the impulse response of Z transform step response, same system using the Z transform methods. And in the solution here, identify the natural in forced response components and c for each of the solution; find out what is the natural component that is solution or the free component to the solution. And what is the force response component to the solution.

(Refer Slide Time 46:24)



E: find the steady state responses of to the following inputs of the same system sin n pi upon 4, 2 cos n omega T; where omega is given as 500 pi radius per second and T is 1 milli second. So, cos n omega T omega is 5 hundred pi radius per second and T is 1 milli second. So, for these 2 sinusoidal inputs find out the steady state output; that means, it will also be sinusoid and you because the natural response is expect to the kv times. And using the frequency response function find out the output to the corresponding to the 2 inputs.