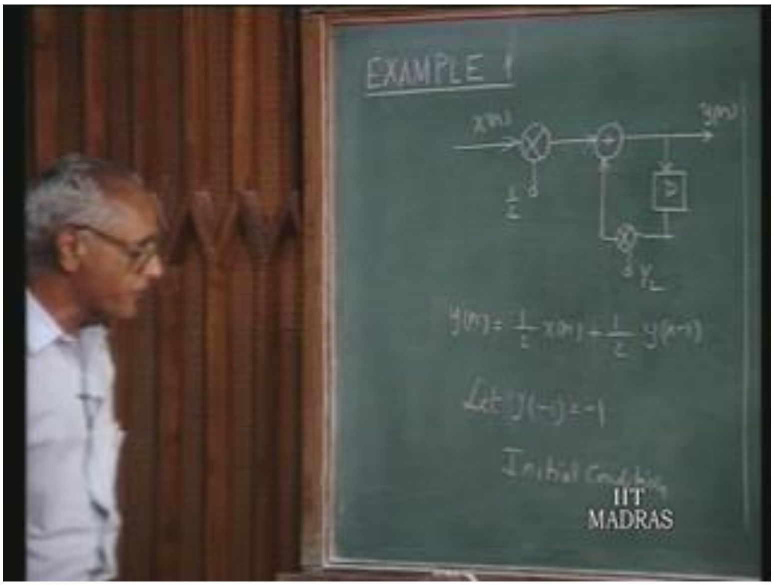
Networks and Systems

Prof V G K Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture –43
Discrete – Time Systems (6)
Examples of application of Z-transforms
Discrete-time system function H(z)

After having discussed the Z transform techniques in the last few lectures, we shall now look at a few illustrated in the typical applications of Z transforms to the solution of difference equation, pertain into discrete time systems.

(Refer Slide Time: 01:39)



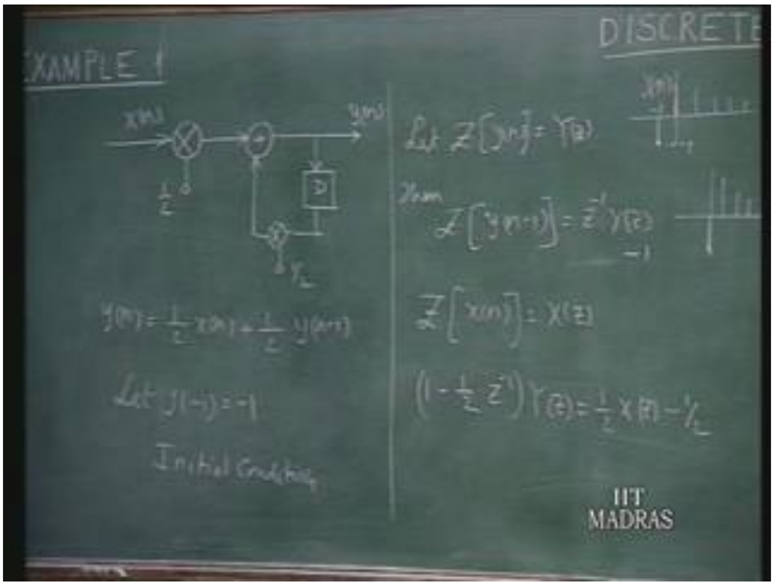
Let us first take an example which pertains to the model of discrete time system shown here. This is an input x n that is your multiplier half, given to an adder and that is the output what y n and the output is delayed by 1 unit to through by a multiplier again coefficient multiplier, again coefficient multiplier as a value half and then this is given to the adder here. So, this then is the model of a discrete time system.

Now, let us form the difference equation for this and then find out the solution using the Z transform approach. If this is y n, this is y n minus 1. Therefore, the signal that is fed to the adder here is half of y n minus 1. The other signal that is fed to the adder is half of x

n. So, very clearly the output y n equals 1 half of x n; that is the signal coming here plus 1 half of y n minus 1; that is the signal coming here.

Now, this is the difference equation and as we already observed, we need to have an appropriate observe the condition for the solution of the difference equations. Let the conditions being given let y of minus 1 be given as minus 1. So, this is the observate condition, whether we call at initial condition or observate condition, we will call that initial condition. So, see this is the first order difference equation; we need to have 1 initial condition this is what is doing.

(Refer Slide Time: 03:45)



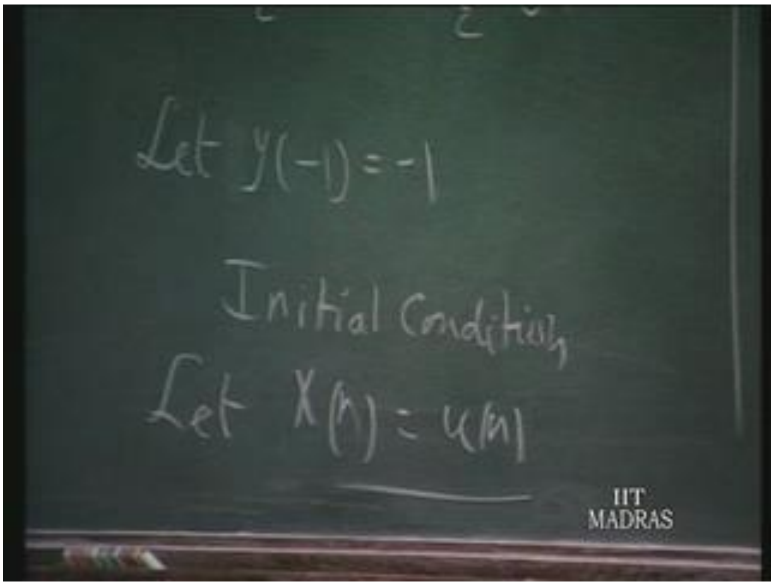
Now, to use the Z transform technique for the solution of the difference equation. Let us say the Z transform of y of n that is indicated as Y z. Then what is the Z transform of y n minus 1? y n minus 1 is signal is delayed by 1 instant of time so, you would have written Z minus 1 Y z. This should be true if it is a purely causal signal, but then we are given this specification that y minus 1 is minus 1. So, if I have y n; this is y n, we also have a value here at minus 1 its value is minus 1; that is also given to us.

Therefore, when you delay this by 1 unit, you have this minus 1 sitting here plus the other sit values. Therefore this also must be taken into account and therefore, you must add to this the value of the signal multiplied by z power minus 0 which is 1 of course. Therefore you must the value signal is minus 1 therefore, minus 1 which is y minus 1 that has to be added. We also; that means, what we are trying to do is find Z transform of each 1 of this terms and then form the equation.

So, Z transform of x n let this be X of z So, in Z transform domain we have; 1 minus Y z minus half you transform to other side of z minus 1 Y of z equals half of X of z plus the other term which is remaining here, when you met this Z transform of half Y n minus 1 you also have a minus 1 here and that is multiplied by half therefore, minus half. This will be the equation corresponding to the difference equation Z transform domain, this is purely algebraic equation.

Note that the difference equation is converted into denold brake equations, just like in the pendiguise time case, a difference equation is converted into algebraic equation in the transform domain. Now, for to solve for this we need also have some information about x n. So, let this also the additional data is given here.

(Refer Slide Time: 06:37)

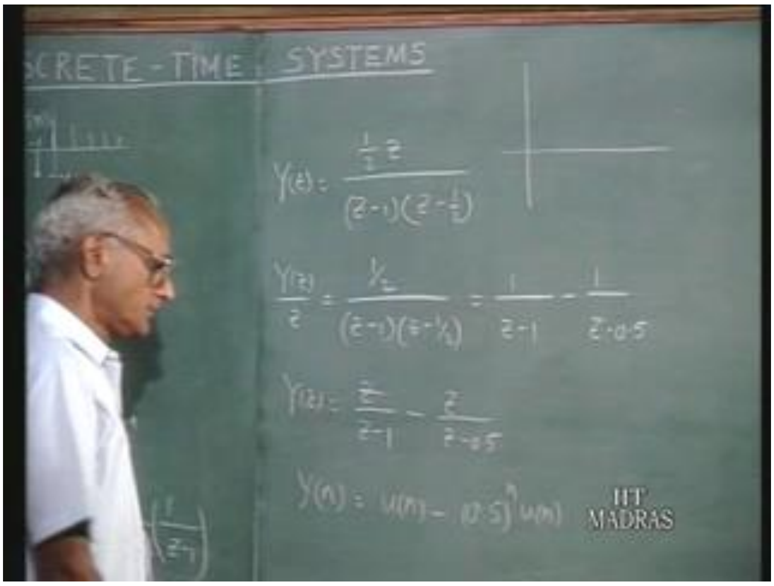


Let your X n be u n.

(Refer Slide Time: 06:47)

So, we want to find out the output corresponding to a unit step input therefore, this will be half of z upon z minus 1, that is, the Z transform of unit step minus 1. Therefore, that will turn out to be half of 1 upon z minus 1.

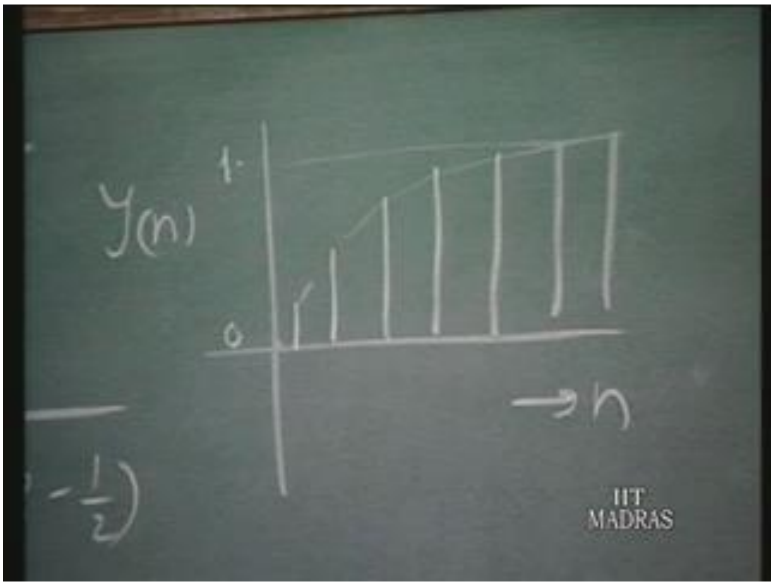
(Refer Slide Time: 07:21)



So, if you solve for Y of z form this algebraic equation you can show that Y of z turns out to be, I will not go to all the intermediate X n half z over z minus 1 times z minus half. That is the Z transform of output quantity. So, to make the partial fraction expansion, I will form Y z upon z to start with therefore, this is half of z minus 1 multiplied by z minus half and that terms have to be 1 over z minus 1 minus of 1 over z minus 0.5. Consequently Y z will be z upon z minus 1 minus z upon z minus 0.5.

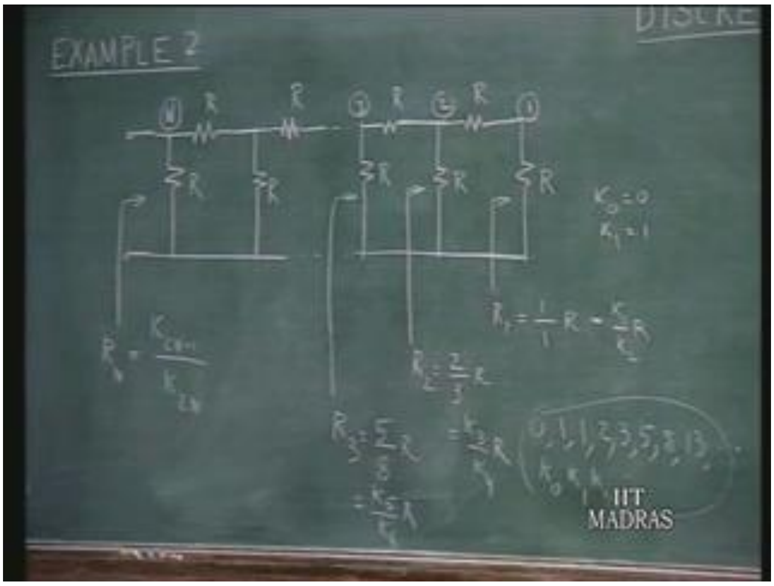
So, each of this is the Z transform of well known discrete time function. Therefore Y of n can be written as; this is u n of course the unit step, this is 0.5 raised power of n u n. So, that is the solution for y of n. How does this function look like? You have a unit step, from that you have 0.5 times n which gradually so, when n equals 0 this is 1 and this is also 1 to start with that 0 0 1 minus 1 0. And ultimately for large values of n this becomes and significantly small and assume tactically it raised to value 1.

(Refer Slide Time: 09:06)



Therefore you will have something like this. So, you have samples where the reaching the value 1. So, that is your solution for y n. The envelope for this curve would be at the form 1 minus 0.5 n. So, that is the solution for your in this particular problem, where the first order difference equation. Now, let us take a second example.

(Refer Slide Time: 09:40)



Let us now consider as a second example, a ladder network which has a value of all resistors equal to R 1s in this configurations. Let us say at this is 1 end of the ladder. So, the effective resistor is seeing here I will call that R 1. So, look in to it right, what is the effective resistance still at this point is R 1. Now, the second step, say let us say this is node 1, this is node 2, this is node 3 and there are all nodes let us say N nodes, this are the node names.

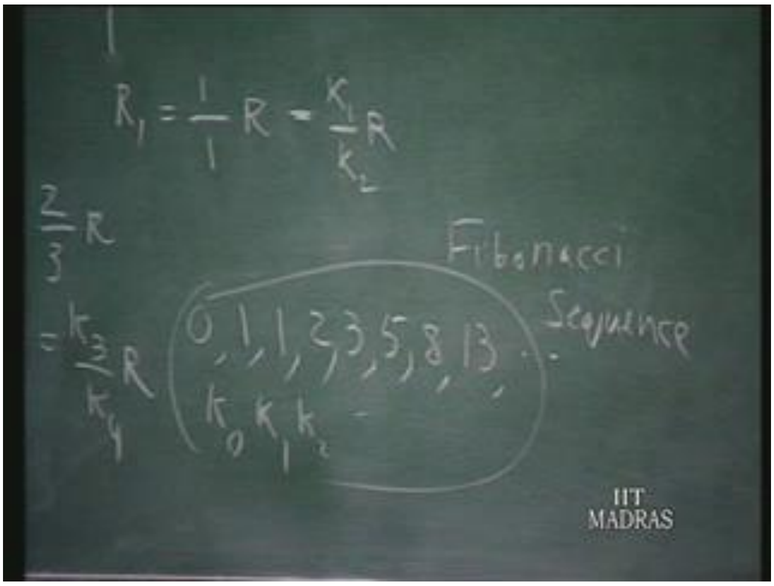
So, the effective resistance seen to at node 2 for the circuit to the right of it is R 2. So, let us say similarly this is R 3. And finally, the resistance seeing here that is the R N. Now, it is clear there as for R 1 is concerned, this is equal to R itself. R 2 is the effective resistance seen here is 2 R in parallel with R so, 2 R times R divided by 3 R R 2 by 3 R 2 upon 3 R. R 3 likewise is; this is what you see here is 2 by 3 R plus R in series; that means, 5 by 3, 5 upon 3 R, 5 upon 3 R in parallel with R; that means, 5 by 3 R times R divided by 1 plus 5 by 3 times R. So, it becomes 5 upon 8 R.

So, you observe and interesting feature of the values of the effective resistance as you progress towards the N’th node. I can call this R times 1 by 1 or 1 upon 1. So, I can write this its maths effect 1 upon 1 times R. I do this to see a certain pattern in this. The sum of the 2 numbers which is equal to this, 2 plus 3 equals 5 and the last 2 numbers 5 plus 3 equals to 8. So, as you progress you have a sequence of numbers, where the sequence goes like this 1 1 2 3 then 5 then 8 then next number would be 13 of course and so on and so forth, where each number is the sum at the receiving 2 numbers.

So, you need not to keep this particular form, we can also say the previous number is 0 so, 0 plus 1 equals 1. So, such a sequence of a number you generated and the effective resistance is given in terms of the ration of 2 appropriate successive integers. Now, going in this fashion; the effective resistance seeing here as you can see when you are talking about R 3 is 5 upon 8.

So, if I can call this numbers as K 1 by K 2 times R, where I can define K 0 as 0, K 1 equals 1, this numbers I am writing this as K 0 K 1 K 2 etcetera, this will be K 3 upon K 4 times R and this will be K 5 divided by K 6 times R. And naturally this would be written as K 2 N minus 1 divided by K 2 N because, at the third node you have K 5 over K 6, this is K 2 N minus 1 upon K 2 N. So, this is the sequence of numbers.

(Refer Slide Time 14:30 min)

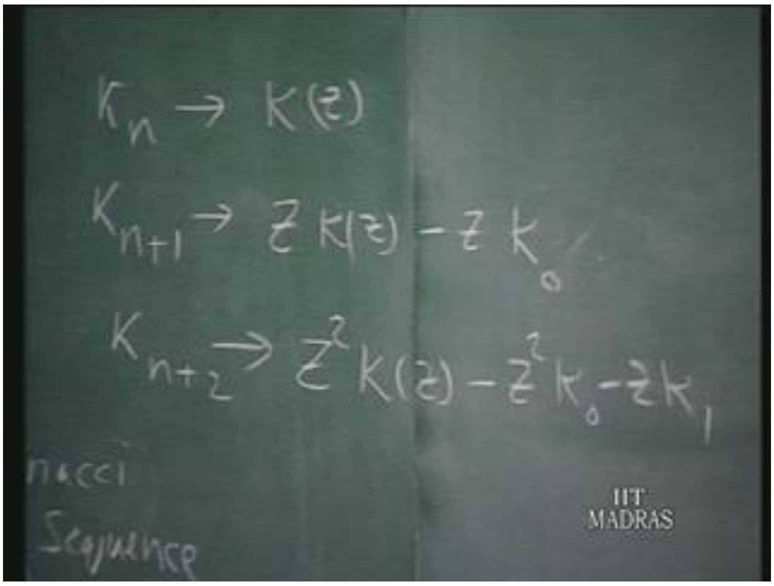


And by the way, such sequence of numbers, where each number is the sum of the proof receding number is known as Fibonacci sequence or Fibonacci numbers Fibonacci sequence of numbers. And interestingly in ladder network, with all equal resistances then the effective resistances at any particular node can react in terms of the Fibonacci sequence numbers. But, we would like to find out a general expression for K n; that is the problem that we have 1 hand.

(Refer Slide Time 15:15 min)

So, what we observe therefore is that, we have K n plus 2 equals K n plus 1 plus K n. So, this is the difference equation pertaining to this Fibonacci numbers; K n plus 1 plus K n. So, this is the difference equation and we would like to solve this using the Z transform approach. This is a second order difference equation therefore, we need to have 2 initial conditions and we use this as a initial conditions. Let us see now.

(Refer Slide Time: 15:32)



So, if these are the numbers we will K n, the Z transform of that we put as K z. Then K n plus 1; that means, you are advancing the signal it is K z times K z, but informing z times

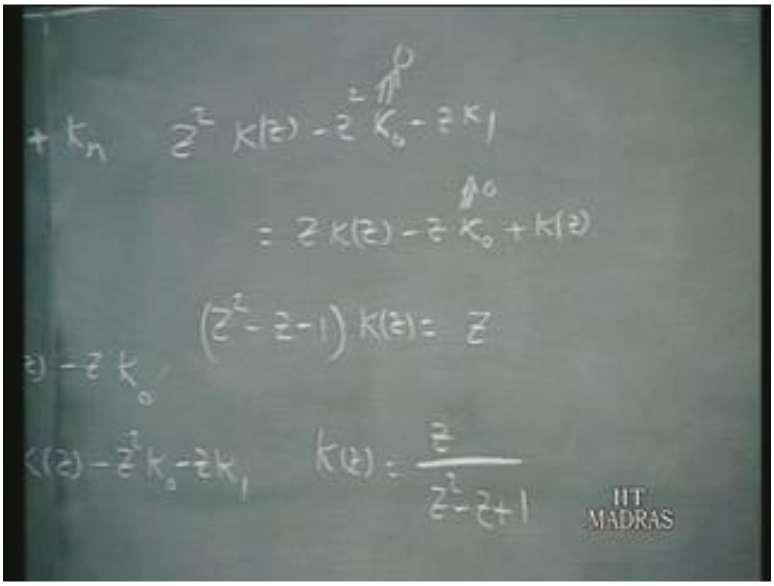
K z, 1 of this K 0 would have gone to the for negative direction and that is last, therefore you must write this for, you must write this minus z times K 0. That is what you are having.

For K n plus 2, again you are pushing into negative direction therefore, Z squared K z, but the samples K 0 and K 1 because, I write at the scale 0 here I will write simply K 0. So, K 0 and K 1 have been transferred in the for negative in the negative part of the x axis therefore, they have lost and therefore, that has to be written. So, z squared K 0 minus z of K 1. So, that is how the Z transforms of K n and K n plus 1 and K n plus 2 look like.

Observe here in this particular example, the independent variable is no longer times. So, it may not be a proper thing to call this discrete time systems, but the same methodology will be valid. The independent variable now is a position the inducts of the node 1 2 3 N. So, except for that minor difference, the entire the techniques that we have at disposer can be used for this. It is not a discrete time system, but the independent variable had discrete values. So, similar case discrete time system.

So, once we have this we will substitute this in the Z transform this equation.

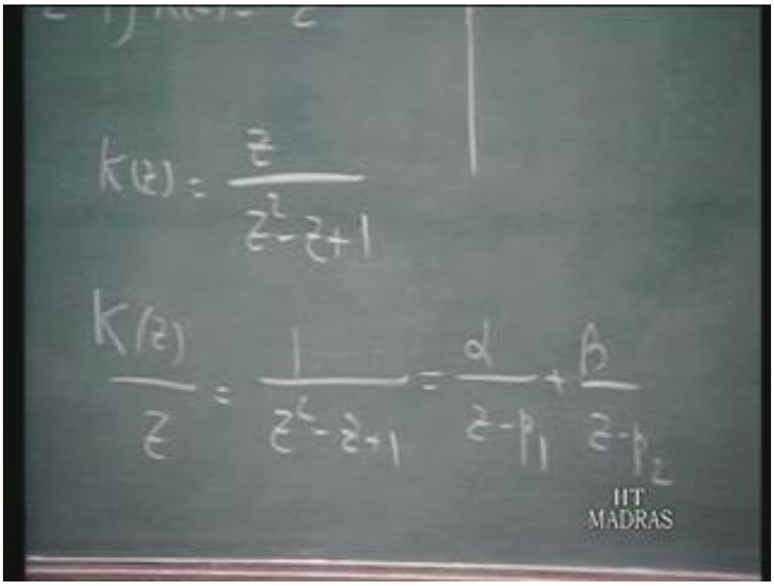
(Refer Slide Time: 17:28)



Therefore you have Z squared K z minus z squared K 0 minus z K 1 equals; that is the Z transform of this. The Z transform of this Z K z minus z K 0 plus this Z transform of this which is place it. So, collective terms which involves K z so, you have z squared minus z minus 1 multiplying K z equals K 0 is 0. So, we do not have to worry about that this is 0. And z K 1 K 1 equals 1 therefore, that is so you have K z equals z upon z squared minus z plus 1.So, that is the Z transform of the N’th Fibonacci number K n.

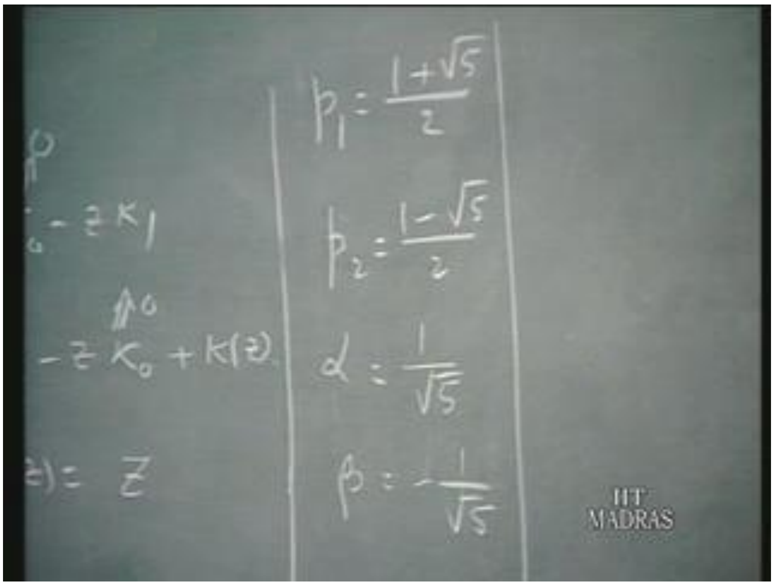
So, we must find the inverse Z transform to find out what K n would be. So, for this purpose we must make the partial fraction expansion again and find out the poles corresponding to this.

(Refer Slide Time: 18:46)



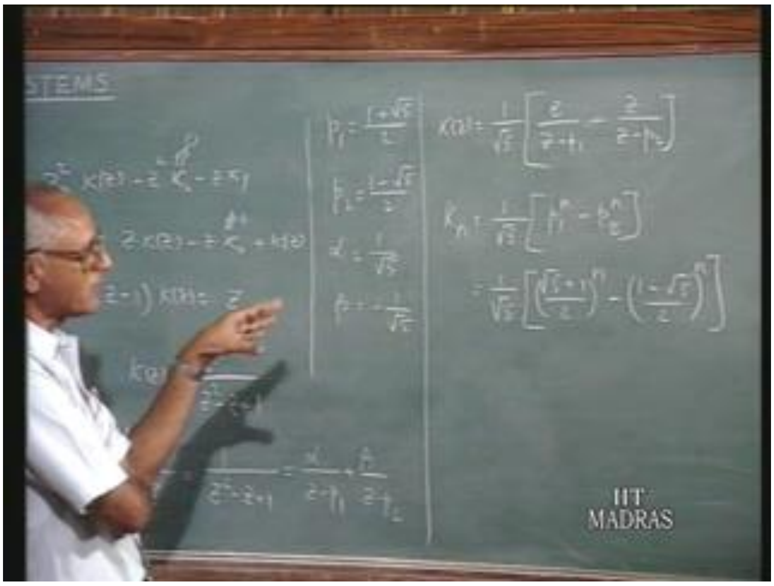
So, to do the partial fraction expansion of this, we will first of all divide right through by z according to our practice. So, z squared minus z plus 1. That we say this is equal to alpha over z minus p 1 plus beta over z minus p 2, where p 1 and p 2 are the poles of this reactional function.

(Refer Slide Time: 19:15)



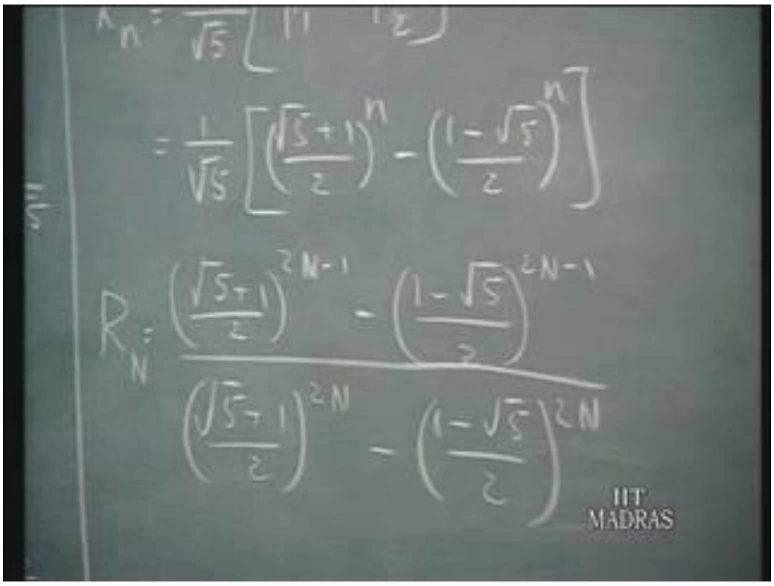
You can work this out and can show that p 1 equals 1 plus root 5 upon 2, p 2 equals 1 minus root 5 upon 2 and further more alpha equals 1 upon root 5 and beta equals minus 1 upon root 5.

(Refer Slide Time: 19:40)



So, the result is that we can write K of z as 1 upon root 5 times z upon z minus p 1 plus minus z upon z minus p 2 because, alpha and beta are negative therefore, this minus sign comes to the picture. Finding out the inverse Z transform of this; you the K n the N’th number in the Fibonacci sequence, will turn out to be 1 over root 5 p 1 raised to the power of n minus p 2 raised to the power of n. Of course this is valid for n greater than or equal to 0. And this is 1 over root 5 root 5 plus 1 by 2 raised to the power of n minus 1 minus root 5 by 2 raised to the power of n. And once you have the general expression, this is the N'th Fibonacci number, even though you seem to have lot of square root sign come coming here, it will turn out to be when you work it out turn out to be a lean picture.

(Refer Slide Time: 21:06)



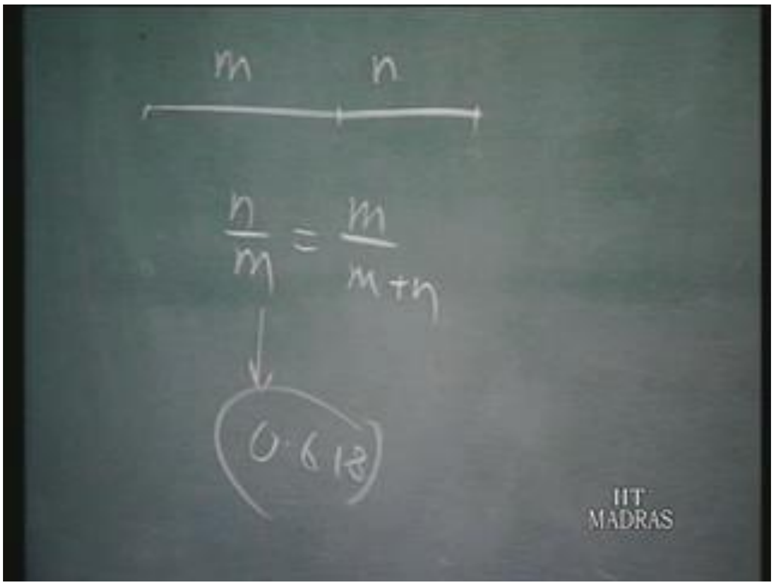
And the R N that we are looking for, the effective resistance seen at this end at the N'th node, which is K 2 N minus 1 by K 2 N, it now can be written as divided by root 5 plus 1 by 2 raised to the power of 2 N minus 1 minus root 5 by 2 raised to the power of 2 N. That is being expression for R N. So, we can find out the effective resistance for general value of N using this particular expression.

Now, let us see what happens for a long ladder, when the N becomes very large.

(Refer Slide Time: 21:52)

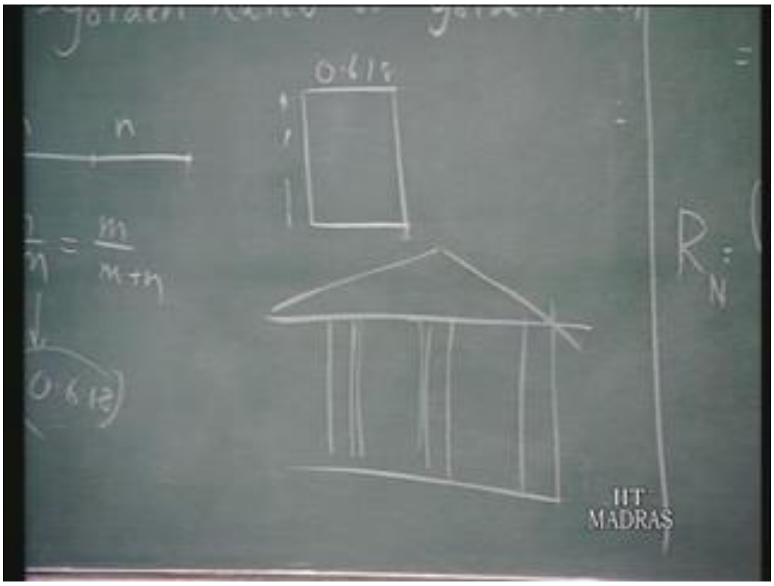
So, limit as N tends to infinity of R N; that means, a very large ladder network; 1 minus root 5 by 2 is a number smaller than unity in the magnitude. Therefore, for large values this cross out. So, essentially it is the ratio of these 2 quantities therefore, this becomes this quantity divide by this raised to the power of 2 N; that means, 1 over root 5 plus 1 divided by 2 or 2 over root 5 plus 1. This can alternatively be written as root 5 minus 1 divided by 2 and this value is 0.618. This is a interesting number. This is sometimes called the golden ratio or golden mean.

(Refer Slide Time: 23:01)



1 can look at this way. Suppose or a straight line and I want to divide this straight line into 2 parts such that, the ratio of the smaller section to the larger section, that is, n over m is also equal to the length of the larger section the whole length that is m over m plus n. So, you want to divide a straight line in this fashion; that the ratio the smaller length to a larger section is the same as the ratio of the larger section to the whole length, then it turns out the n by m once again turns out to be 0.618. And the classical painters and architects apparently believed that, this is the ratio which has the most exclusing proposals.

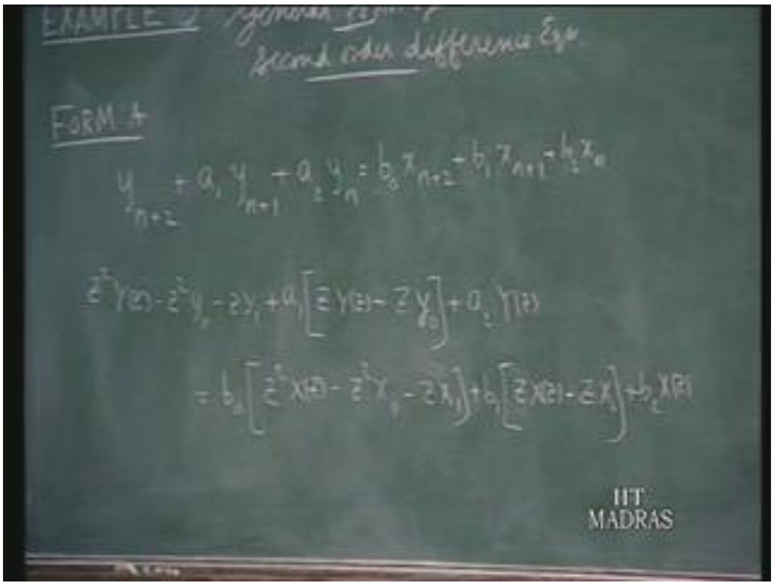
(Refer Slide Time: 23:59)



So, in the classical paintings you find that, this particular ratio is used to form the frame per painting; that means, this is 0.618, this is 1. Architects also used to have this kind of configuration this kind of ratio for doorways and so on and so forth. So, if you have some columns like this in architect and so on, the opening will have the wearing this station. So, it terms this has got this matter of interesting details, for us to know that this particular ratio what is called golden ration and suppose to present rectangle of the most pleasing proposal. We will leave it at that.

Let us now consider it is a third example; the application Z transform technique the solution of a general second order difference equation.

(Refer Slide Time: 24:50)



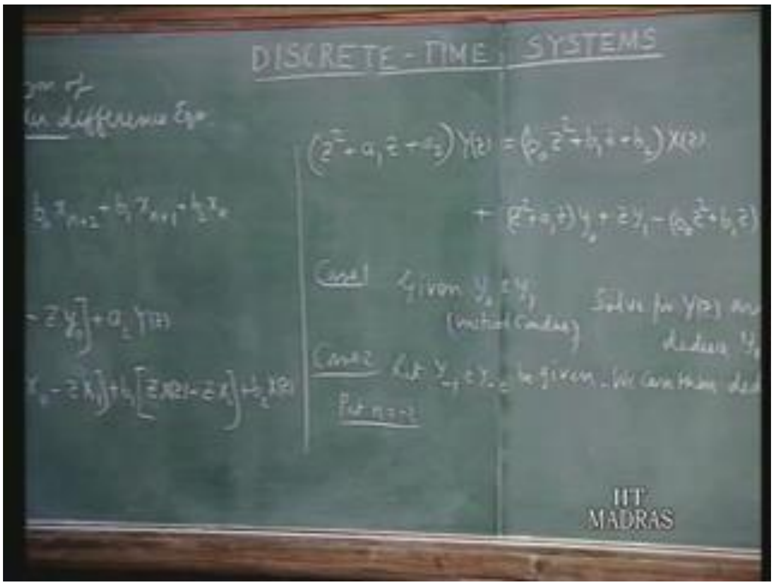
Let us consider 1 form; form A let us say, where the independent variables starts with n and goes upto n plus 2. So, we can write y n plus 2 plus a 1 y n plus 1 plus a 2 y n equals b not x n plus b 1 x n minus 1 plus b, since we start with n plus 2 here we may as well have b not x n plus 2 plus b 1 x n plus 1 plus b 2 x n. That is the general form of the second order difference equation. You might ask why did I put a not, even if you had a not we can revive write through by a not and end up with this form. So, this is the most general form, we do not have to put a not, specifically a not can always regard as 1.

Now, let us Z make the Z transform of each 1 of this terms, we have y n plus 2, we have z squared y z for this z squared y z. Suppose Z transform of y n is Y z, therefore, this is Z squared Y z. Since you are advancing this, you have lost 1 sample corresponding to y 0, this term corresponding to y not and also y 1 so, z y 1. This is something which have been talking about plus the Z transform of this a 1 times z Y z and the sample corresponding to y 0 here have been lost because, we have pushing this in the advance in this; that means, you are pushing that particular sample at n equals minus 1 position therefore, z y 0 plus a 2 Y z, that is, this 1.

This is equal to likewise we carry out the same operation here, b not times z squared X z minus z squared x 0 minus z squared minus z times x 1 plus b 1 times X z times X z minus z times x 0 plus b 2 times X of z. So that is how it goes.

Now, collecting the terms corresponding to X of Z and Y of Z and terms corresponding to y not y 1 x not and x 1 separately.

(Refer Slide Time: 28:17)

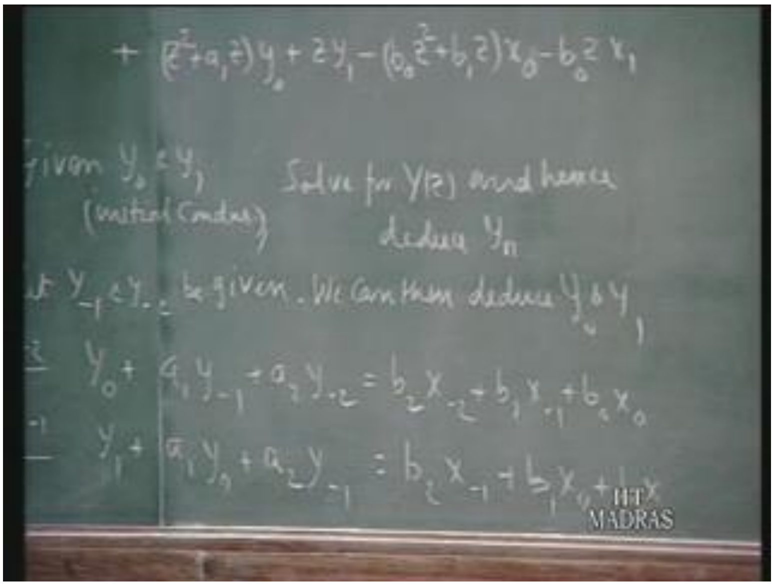


You can show this will be equal to z squared plus a 1 z plus a 2 multiply by Y of z equals b not z squared plus b 1 z plus b 2 times X of z plus z squared plus a 1 z times y not plus z y 1 plus minus b not z squared plus b 1 z times x 1 x not minus b not z times x 1. So, in the transform domain, this is how the algebraic equation look like, connecting X z and Y z and the initial conditions.

So, 1 possibility is; suppose you take case 1: what do you have a second order equation given like this starting with y 1 and going to y n plus 2, a conventional and useful way which is the initial conditions are prescribed are y not and y 1. So, given y not at y 1 are initial conditions. And of course, the x n will be prescribed so you certainly will know what x not and x 1 is. Solve for Y Z and hence deduce y n. So, that is way in which we can attack this problem. Once we have got y not and y 1; these are the initial conditions, this is the second order difference equation, we need to know 2 initial conditions, X of n is given to us so, x not and x 1 is known and also X of Z. So, everything is known as except Y z, you can solve for Y z and hence deduce y n.

Now, suppose we are given not y not and y 1, suppose y minus 1 and y minus 2 are given. Let y minus 1 and y minus 2 be given. Then, we can deduce we can then deduce y 0 and y 1 how because, after all we need to know y 0 and y 1. So, given this conditions we can deduce y 0 and y 1, put n equals minus 2 in the original equation, put n equals minus 2 then, we have in this original equation y n plus 2 a 1 y n minus 1 a 2 y n, in this put n equals minus 2 you get y 0 plus a 1 y minus 1 plus a 2 y minus 2 and so on and so forth.

(Refer Slide Time: 31:47)

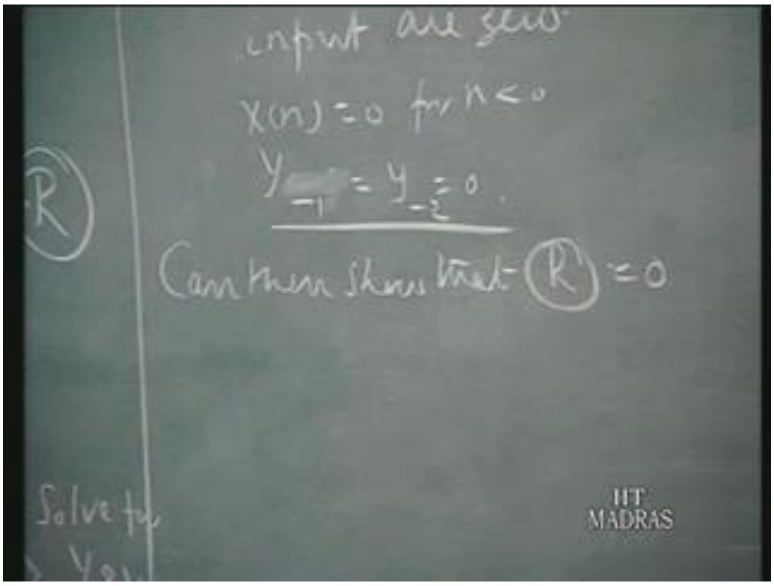


So, put n minus 2 in the original difference equation you get y 0 plus a 1 y minus 1 plus a 2 y minus 2 equals b 2 x minus 2 plus b 1 x minus 1 plus b not x not. Similarly, put n equals minus 1, then you can get a similar way; y 1 plus a 1 y not plus a 2 y minus 1 equals b 2 x minus 1 plus b 1 x not plus b not x 1. Now, normally you are given an input x of n, starting from n equals 0 and we assume that input is causal signal therefore, we can take this to be 0. And now you have you are given y 1 minus 1 and y minus 2, you have got 2 equations; you can solve for y not and y 1. And use this information to find out Y of Z in this equation.

So, it does not mean necessarily therefore, that initial conditions are given to you always y of 0 and y 1. Even if y minus 1 y minus 2, that is, if you are given the input condition initial conditions before the x n case is applied; that is what we are having here. If the initial conditions are as a values of the dependent variable, before the excitation x n is applied, even that information will be useful for us to calculate y 0 and y 1 and then we can solve for this. So, that these for the variation on each possible in that different problems, you may give initial conditions in the form y 0 and y 1 or y minus 1 y minus 2, then also we can solve for this.

But the point to note here is y minus 1 and y minus 2 are the initial conditions before the excitation is applied. So, x n starts form n equals 0; that is the 1 way in which this given and that also we will able arable to solve the problem.

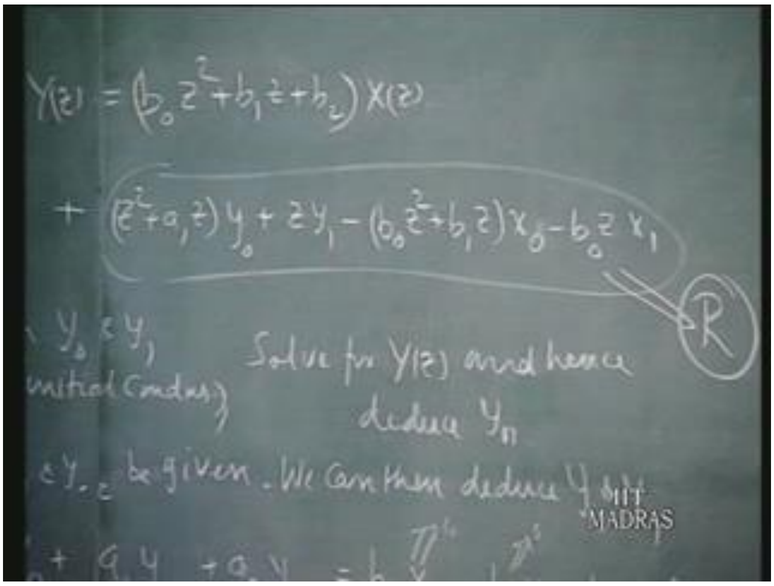
(Refer Slide Time: 34:17)



Now, important case arrives at where, initial conditions prior to application of input or 0. You recall when you talked about transfer functions and so on the continuous time situation, we set the initial conditions are 0 biot the application input. Then there is a proportional to relationship between the output and input in the transform domain. Similarly, here we have a causal signal x n is 0 for n less than 0 and the x n is applied n equals 0 onwards and all the initial conditions are 0 before the input is applied. Therefore y minus 1 equals y minus 1 y minus 2 are also equal to 0.

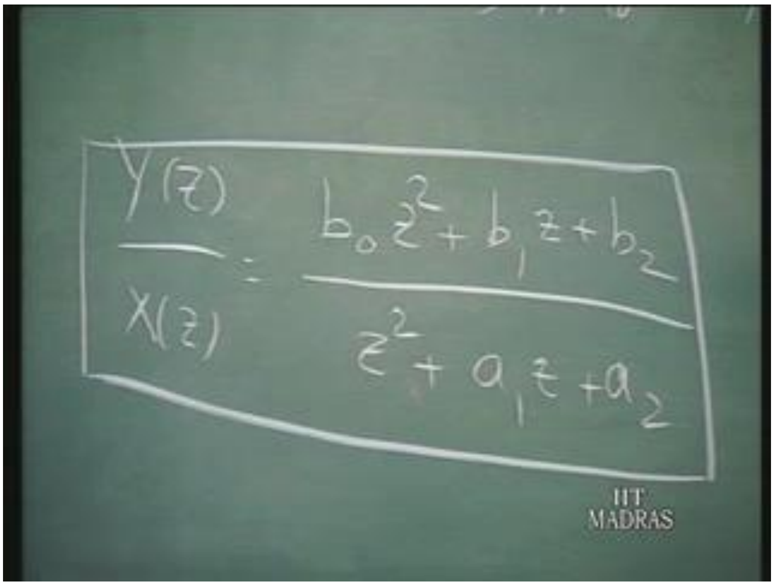
So, if this is the condition, if y minus 1 and y minus 2 are 0 and this is 0. So, y not equals b not x not and y 1 equals b 1 x not x 1. Under these conditions you can show that, this entire thing will vanish, can then show that this entire expression R equals 0.

(Refer Slide Time: 35:50)



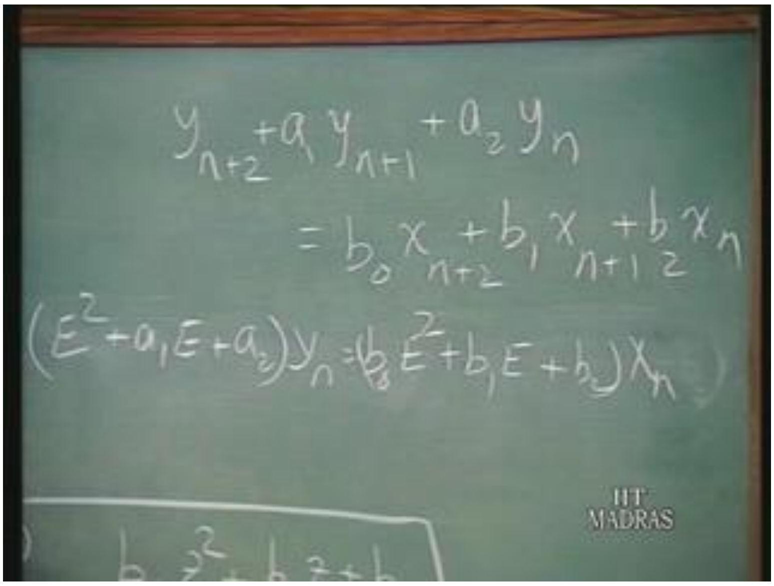
By substituting in this values so, y minus 1 y minus 2 0 y not therefore, its equal to b not x not and y 1 equals b 1 x not plus b not x 1, substitute in those values in to this you can show that this quantity exactly equals to this quantity. And therefore, the entire R is y term.

(Refer Slide Time: 36:15)



We have Y z over X z equals b not z squared plus b 1 z plus b 2 divided by z squared plus a 1 z plus a 2. So, we now have instead of difference equation connecting y n and x n in the transform domain, we have a pure raised your Y z over X z is b not z squared plus b 1 z plus b 2 over z squared plus a 1 z plus a 2. So, provide an initial conditions are 0 prior to the application an input, the transforms of the output and the input in the Z transform domain or related by a pure ratio like this. Now, how do we get this ratio?

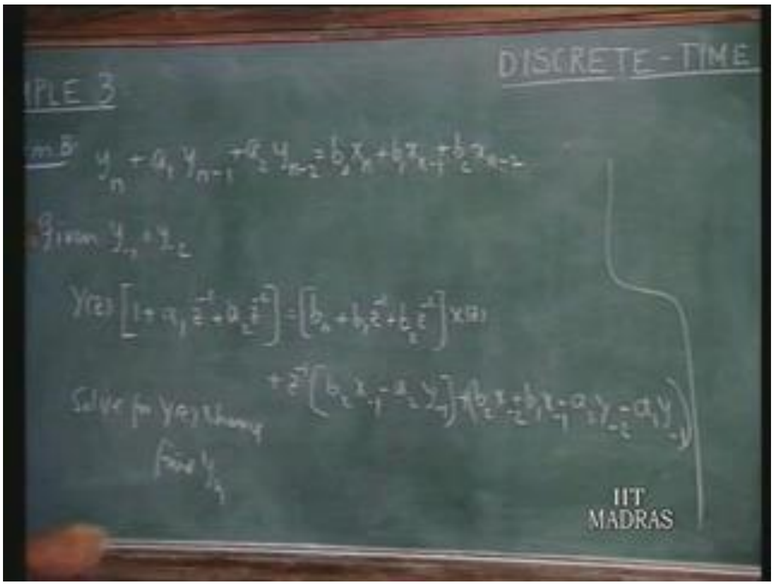
(Refer Slide Time: 37:13)



You recall that this is the equation, which we started out with second order difference equation; y n plus 2 a 1 y n plus 1 plus a 2 y n and so on. So, in operator form, this would be how it will be E squared plus a 1 E plus E 2 a 2 operating y n equals b not E squared plus b 1 E plus b 2 operating on x n. So, in this expression where the substitute the operator E by Z, then the ratio of y 2 x would be simply this, that is, z squared b not z squared plus b 1 z plus b 2 divide by z squared plus a 1 z plus a 2 So; that means, if you have difference equation to start with to find out the ratio of the Z transforms, in is a quite easy; all you have to be substitute the operator E by the variable z. That is that is all it entails.

Now, in this particular form A; we have n ranging from n to n plus 2. Now let us consider the second case, where second order difference equations still, but n ranges from n 2 n minus 2. Let us see what the similar ratio, what type of result we get in the form B.

(Refer Slide Time: 38:30)

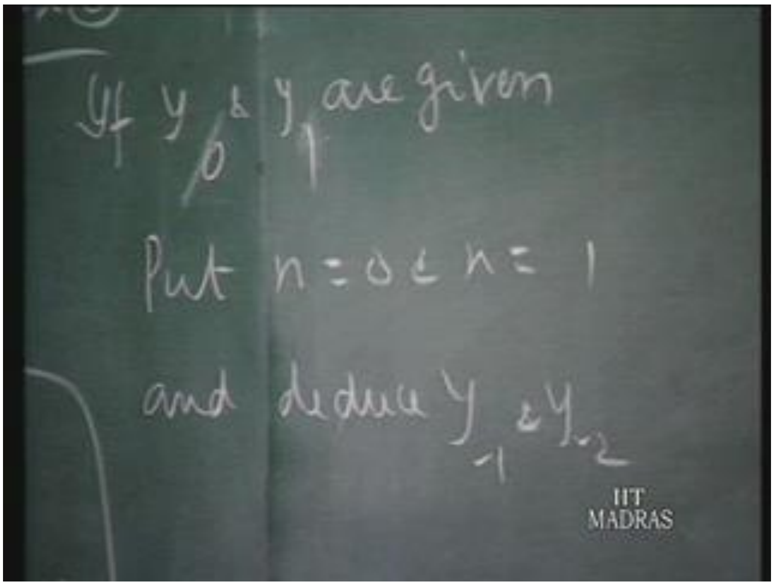


In the previous form if you decrement the independent variable n by 2 units, you get an alternative form which can be put y n plus a 1 y n minus 1 plus a 2 y n minus 2 equals b not x n plus b 1 x n minus 1 plus b 2 x n. Now, on this case, a natural form in which initial conditions are prescribed will be corresponding to y minus 1 and y y minus 1 and y minus 2. That is n equals minus 1 and n equals minus 2.

So, once again trying to take the transform of each term, term by term and then recognizing that now we are delaying and therefore, when you are delaying this function n minus 1, the sample with standing earlier at n equals minus 1 is coming to 0 position and that has to be taken in to account. So, given y minus 1 and y minus 2; let us say this is case 1: given y 1 minus 2 we can and assuming that x minus 1 and x minus 2 are 0, we can find out the equation relating to y and x in the transform domain you can find this out and they can show that Y of z will be times 1 plus a 1 z minus 1 plus a 2 z minus 2 equals b not plus b 1 z minus 1 plus, I should write here x n minus 2 b 2 x n n minus 1 n minus 2 plus b 2 z minus 2 times X of z plus, corresponding to the initial conditions you have; z minus 1 b 2 X minus 1 minus a 2 y minus 1 plus b 2 x minus 2 plus b 1 x minus 1 minus a 2 y minus 2 minus a 1 y minus 1. That is what you are having Z minus 1.

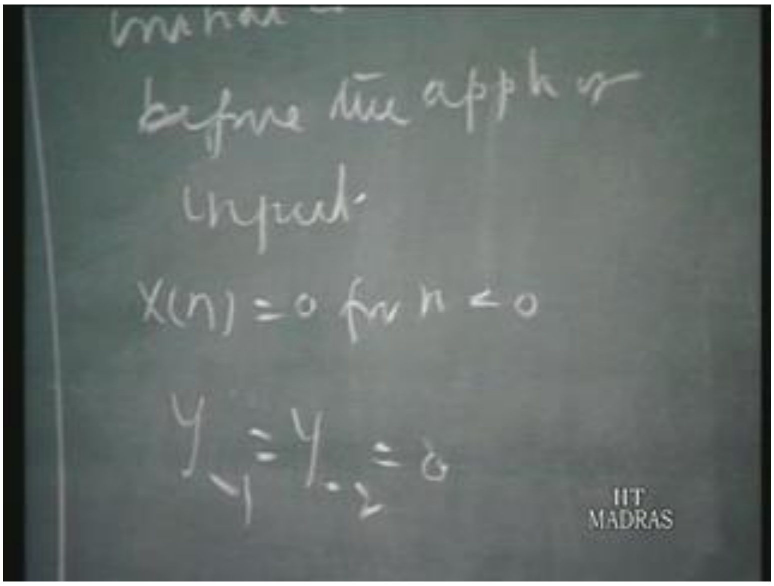
So, y what y minus 1 and y minus 2 are given and there assume that X minus 1 and X minus 2 are 0 for the causal signal or even it specified you know that values. So, given this solve for Y Z and hence find y n. So, the procedure is straight forward.

(Refer Slide Time: 41:27)



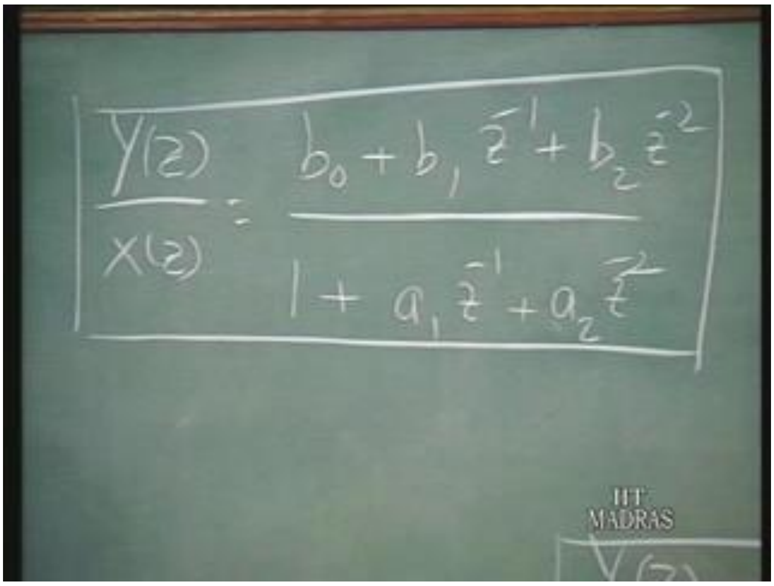
Now, suppose y 1 and y 2 are given. If y 1 and y 2 are given y 0 and y 1 we have y minus 1 y minus 2, alternatively y 0 and y 1 are given, put n equals 0 and n equals 1 in the deduce in the difference equation and deduce y minus 1 and y minus 2, you can do that by putting n equals 0 and n equals minus 1 here and can deduce this values. And once you have got y minus 1 and y minus 2, you can proceed in this fashion.

(Refer Slide Time: 42:18)



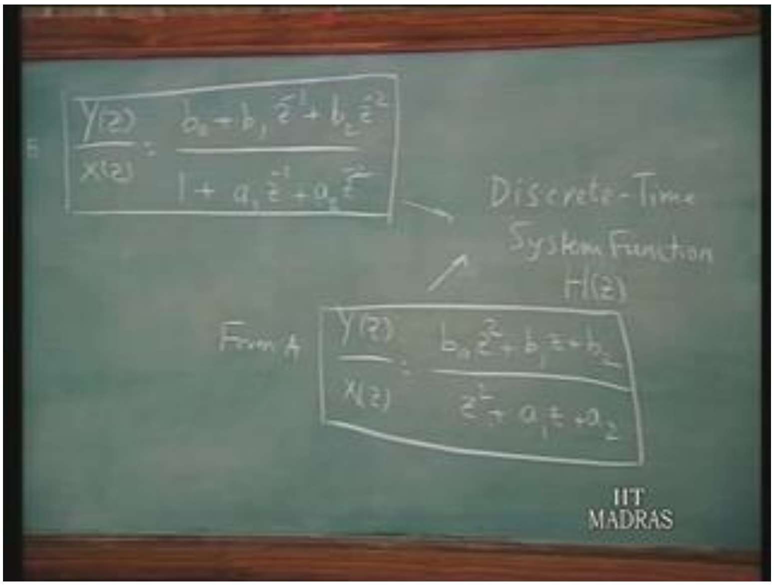
This important thing here as before is; if there are 0 initial conditions before the application of input the causal signal. So, we assume that x n is 0 for n less than 0 and y minus 1 y minus 2 is also 0. Then you look way to here x minus 1 is 0 y minus 1 is 0 x minus 2 x minus 1 y minus 2 y minus 1; all these are 0.

(Refer Slide Time: 42:59)



And you have straight away Y z over X z would be will be Y z over X z will be b not plus b 1 z power minus 1 plus b 2 z power minus 2 divided by 1 plus a 1 z power minus 1 plus a 2 z power minus 2. So, that is the ratio of the output to the input in Z transform domain in form B.

(Refer Slide Time: 43:36)



Notice that, this is the form that we got in form A. This pertains to form A. And what we have obtained here the ration in form B. If you look at these 2 expressions closely, you will observe that both are the same. 1 is can be derived from other by dividing by z squared and multiplying by z squared both the numerator denominator. So, both are essentially the same. That means: whether you take the difference equation ranging from n to n minus 2 or n 2 n plus 2 this is the second order difference equation both are the same. See the difference equations are the same, the ratio of the transform also must be the same and the ratio of the Z transform the output the Z transform for the input is 0 initial conditions is what is called system function, in much the same way as we have in the continuous time case.

But in these situations these 2 are called discrete time system function. So, both are the same; this is called discrete time system function be aggravated this due the symbol Hz for this. Now, the let me once again mention that the discrete time system function is defined as the Z transform all the output to the Z transform of input, which 0 initial conditions biot application of the input. To derive the discrete time system function from the difference equations is almost trivial because, once you setup the difference equation use the operator E and by substituting z for E you simply get the ratio of the 2 polynomials in z and that is the discrete time system function.

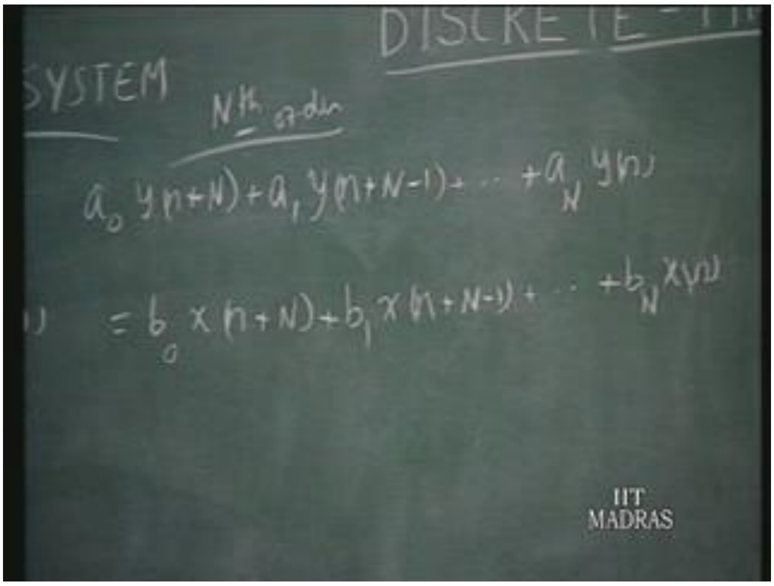
And what you have done for the second order case can be extended to higher order case, for as the same way, no difference at all. So, in the in higher order case the polynomials of the operator F B and G E will be of a higher order therefore, the corresponding numerator and denominator in the system function will also be a higher order. So, after having studied how to find the system function from the second order situation and how the same procedure can be extended to higher order systems also, let us now look at the properties of the system function a general way.

So, what are the properties of H of z; that what will take up later.

(Refer Slide Time: 46:06)

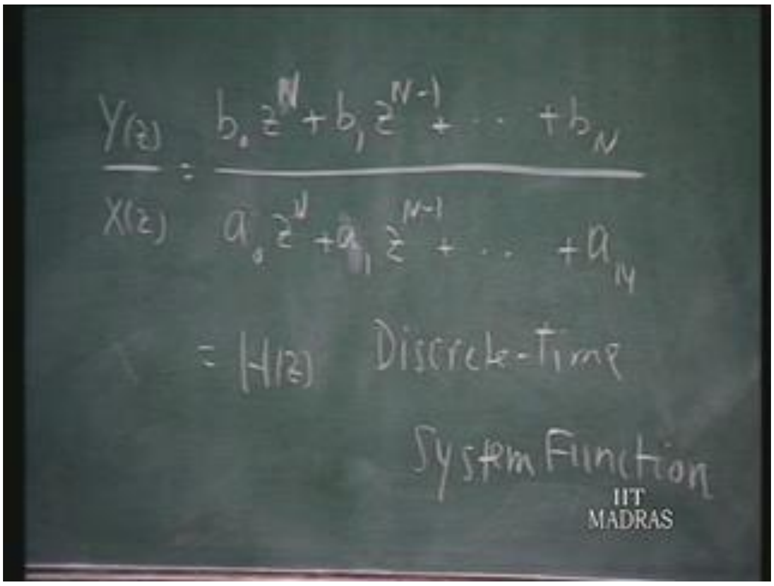
Let us think of this as a single input single output discrete time system and we assume 0 initial conditions, when we mean 0 initial conditions it is implied that the conditions are 0 before the application of the input. That means y minus 1 y minus 2 and all this are 0, provided this is a causal signal.

(Refer Slide Time: 46:33)



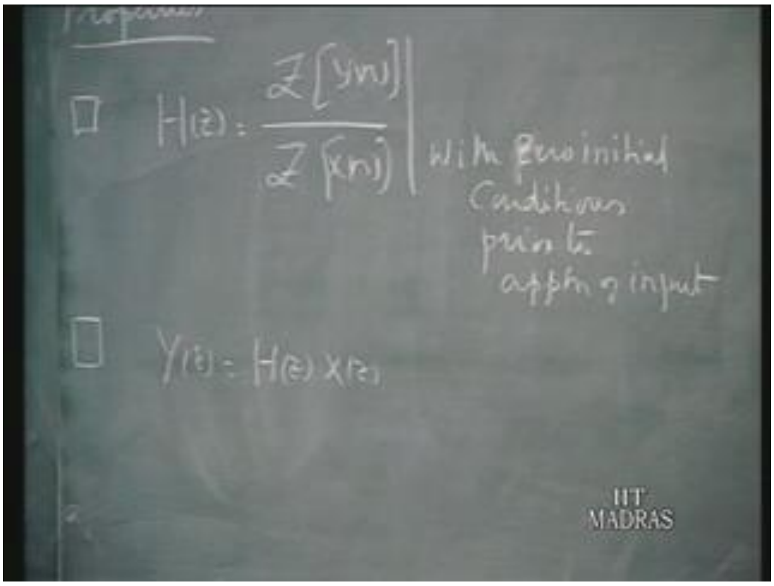
And in general case the y n and x n may be connected by N’th order difference equation where N is involved. So, this is a Nth order differential equation difference equation and this is in this form. From what we have discussed in the second order case can easily saw that, the 0 initial condition and with the N'th order difference equation.

(Refer Slide Time: 47:00)



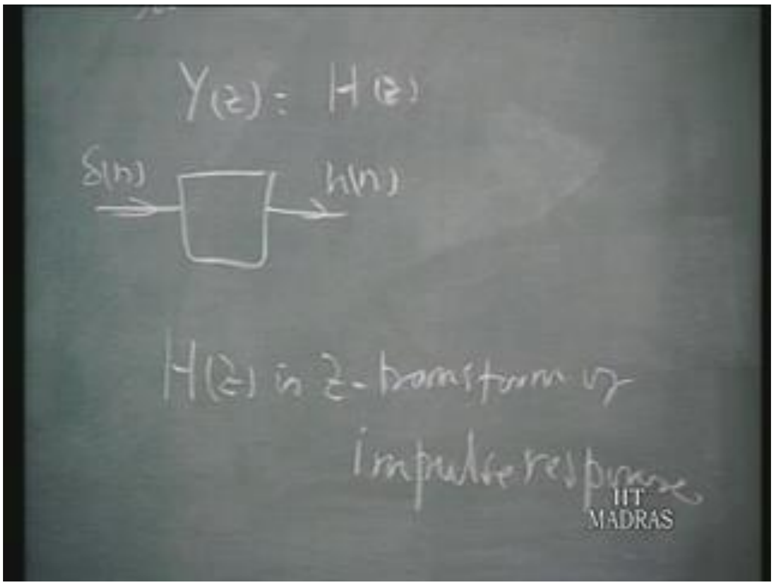
Then, difference equation Y z over X z is simply can be written as b not z power N z power N plus b 1 z N minus 1 down the line to b N divided by coming from here a not z power N plus b 1 Z power N minus 1 and so on, this is a 1 and goes upto a N. So, that is and this is what we called H of z the discrete time system function, which corresponds to what we have done in the continuous time case. That the ratio Z transform such a output to the input to 0 to infinity. Even if you have written down the difference equation in alternative form, starting with a y n a 1 y n minus 1 and like that a N y n minus N and so on, you would end up with the same difference equation same final form H of z.

(Refer Slide Time: 48:18)



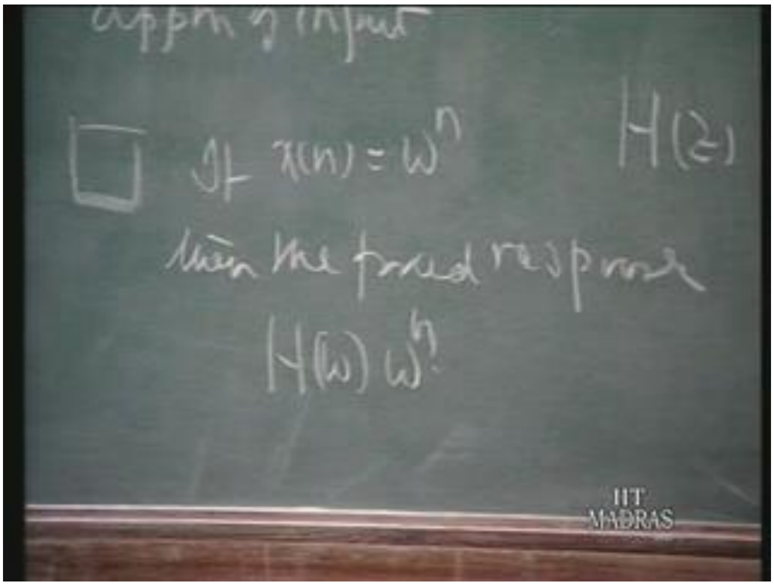
What are the properties of the system function? First: H of z is that we repeat this Z transform of output quantity by the Z transform of the input quantity, with 0 initial condition prior to the application of input, it obvious. That is the definition. Second point of course, with 0 initial condition once again Y z equals H of z times X of z. This is just repeating, but putting in alternative fashion.

(Refer Slide Time: 49:09)



The third point is if x n happens to be a unit impulse, then X z of course, is equal 1 and therefore, Y z is H of z. So, the if delta n produces h n is a impulse response, then that in the impulse response has the Z transform H of z. Therefore H of z is Z transform of impulse response. And therefore, you can see that Y z is equal to H z set z.

(Refer Slide Time: 50:00)



So, in time domain y n equals h n come all with x n, which is something which you already know. Why soon the impulse response, the response vary input can be obtained by a convolution impulse response and the input we get y of n. So, we know find that even here we have the same situation, same except z the Z transform at the impulse response.

The fourth point which like to mention is; if x n happens to be w n, then the forced response will turn out to be H of w times w n because, this is the characteristics schedule and therefore, the output also will have the same type of the behavior, it will be H w time w n, which will see in the next lecture and then proceed from then all. So, the system function plays a very important role in continuous discrete time systems, just as did in the case of continuous time systems. And we will talk more about this later in the next lecture.