**Networks and Systems**
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**Lecture –41**
**Discrete – Time Systems (4)**
**Properties of Z-Transforms**

In the last lecture, you are introduced to the concept of Z transform of discrete time system. And we also saw Z transforms are some basic discrete time function. Let us see, let us recall what we did.

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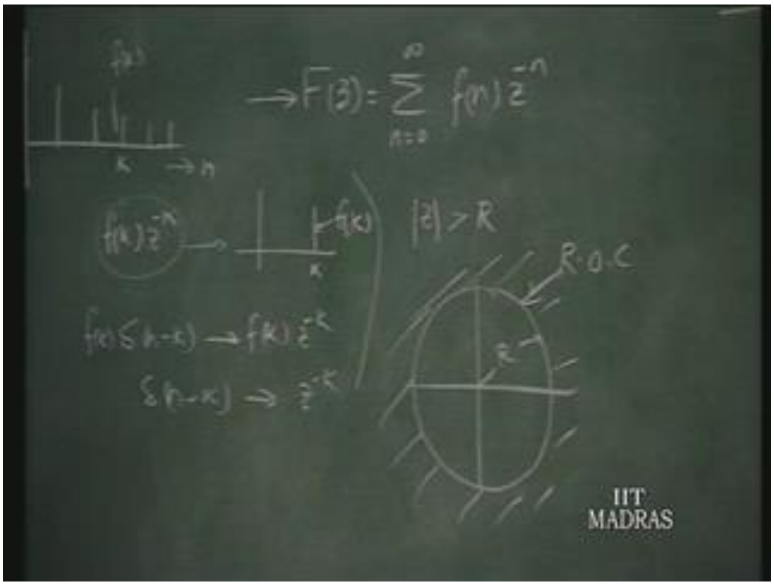


Given a sequence of values f n, we set that Z transform of that F of z is given by f n Z power minus n, n going from 0 to infinity, at this being the 1 sided Z transform of the discrete time signal give further. What this being severely is that the K'th sample which is f K is given a weight Z to the power of minus K. So, f k is met f k multiplied by Z of minus K. And the summation of all such terms gives you F of Z . Conversely if I have f K minus z K alone standing by itself; that is the Z transform of a simple single value f of K standing at the k'th insets. That means: delta m minus K f K has the Z transform f K z minus K. Or delta m minus K has Z transform z minus K, where K is a constant.

We also saw that, this kind of transformation the whole serious f n Z power minus n, converges for values of Z greater than as it constants R, where R defines regional of convergence. So, if this a radius R, so this region of convergence which we abbreviated as ROC region of convergence is indicated ROC. That means: the region of convergence is the region outside a circle of radius R, where the value of R depends upon the particular function f of n.

We have seen that in few special cases for example, u n has the Z transform z upon z minus 1 and the region of convergence for this Z is greater than 1. You if you see the Z transform of u n, it has got a 0 at the origin and a pole at 1. So, the region of convergence is 1 outside this pole. So, it turns out that, the region of convergence is the region outside a circle; the circle must include all the poles on the particular Z transform.

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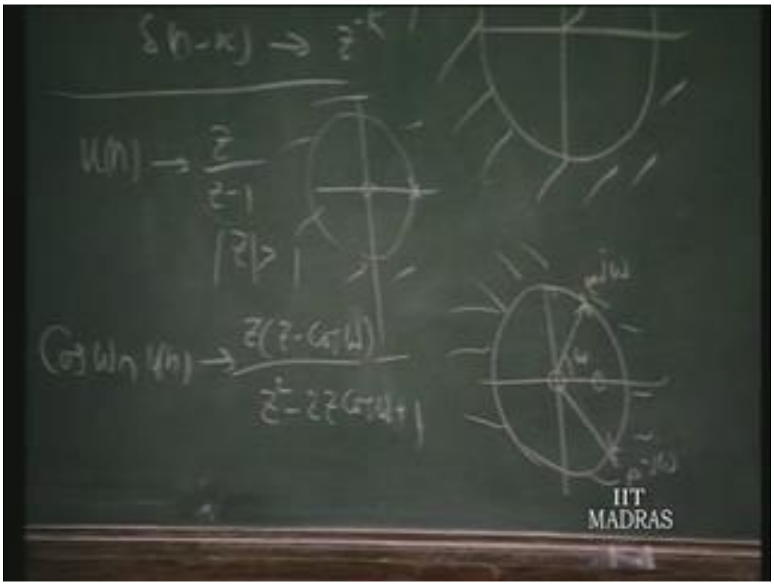


Similarly, if you take cos omega n u n, you find the Z transform of that is z times z minus cos alpha divided by z squared minus 2 z cos alpha plus 1. Now, if you look at the poles and zeros at this particular function, it is it will be like this; there is a 0 at the origin z times z minus cos alpha, there is a pole at there is a 0 at cos alpha, at this denominator gives rise to 2 factors which give the poles at this 2 points. So, this angle being this is cos omega it should be this cos omega, you know this should be cos omega, this should be cos omega, this should be cos omega plus 1. So, the angle of this is omega. So, this is e to the power of j omega, this e to power of minus j omega. So, these at the poles.

Again the region of convergence is outside the poles; that means, the pole which is outer post pole, the pole which is the largest distance of the origin defines the value of R and therefore, the region of convergence such that, it does not include every 1 pole of the Z transform. So, this is how it goes.

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Now, we were also discussed in the properties of the Z transforms. And we mentioned first property linearity which we have already discussed. The second property we mentioned was delay in time.

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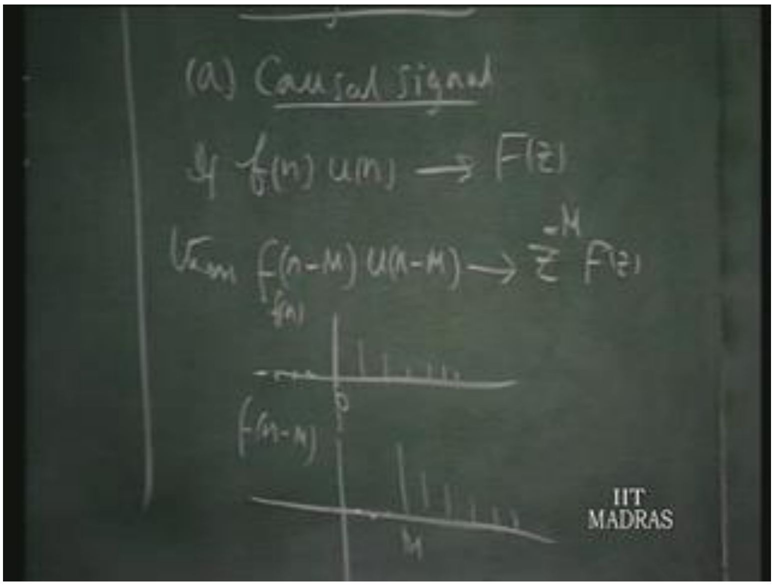


So, let us take first of all a causal signal. So, if f of n u n has the Z transform F of z by set 10, if this function is delayed by a M units f n minus M, u n minus M is has the Z transform Z power minus M F z. Why it happens is because, if you have u n you have samples like this, f n minus M u n minus M you have the same sequences samples, but delay by M units. So, this is 0 this will be M. So, what it means is there is a particular sample f K, it is give at the way z minus K in forming F of Z.

Now the same f K will come here means that later. Therefore it is there wait up Z power minus K minus M. So, every sample here which was multiplied earlier by a certain power of z, now, it is multiplied by certain power of z plus multiplied by z minus M further. Therefore the whole serious now will now be multiplied by z power M. And in the process then you delayed this no mew samples are introduced because, this has the causal function therefore, earlier we assumed this as zeros for negative values are f n. So, when you delayed this function, these zeros this non j values will occur here therefore, there new samples are introduced as a result of this shift.

So, consequently the causal function if it is delayed by M units, you have the Z transform z power minus M F of z.

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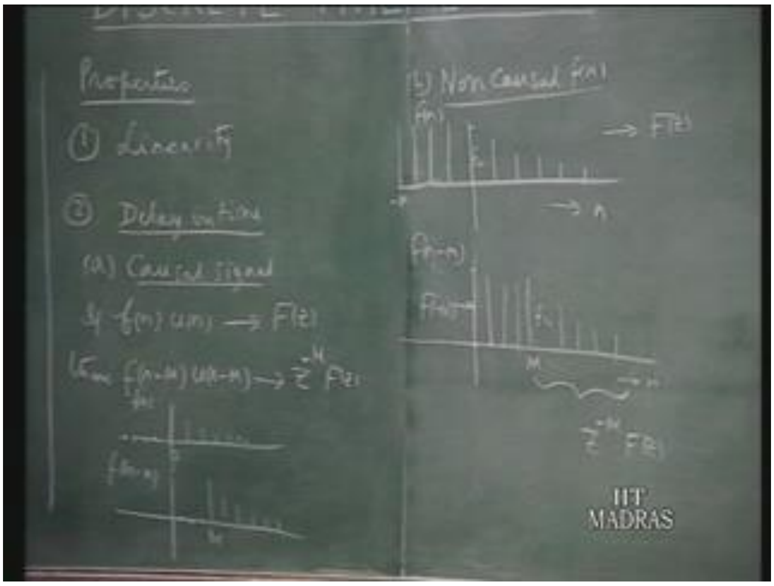


What happens if f of n has some values for negative values of n; so non causal function. So, let us plot this; you have f of n you also have some values let us say. Then, it is delayed by M units therefore, f of n minus M, this original value f 0 gets shifted here at you have this values. But these also come; this whatever you are occurring a negative tends to n will also appear here. So, this is … So, when you are forming the Z transform of this, you not only take that to account the values of this set of samples, but also you have new samples into new here which are not present in the forming F of z from f of M.

By the arguments that we have already advanced in the case of causal signals; if this Z transform of F of z, this set of samples multiplied with the appropriate power absurd will certainly use of z power minus M F of z. But in addition you have these samples, there must also be given attention to informing the Z transform of f n minus M. So, let us see what this samples what are the addition contributions of this samples.

The sample that is coming here what is originally at minus M, so this will be f of minus M. F of minus M is now standing that the origin.

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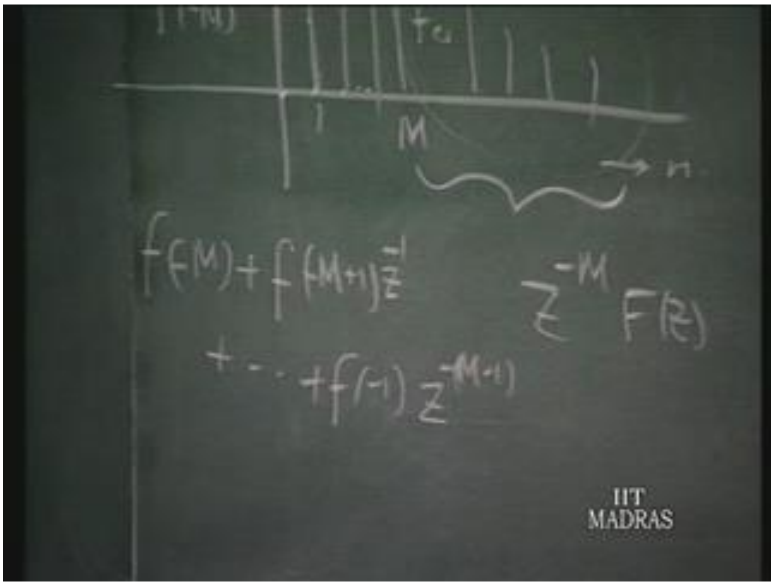


So, when you are taking into Z transform of that you have F of minus M multiplied z power minus 0 that is of course, 1. At this sample is f of minus M plus 1 and that will have a weightage z minus 1 because, it is it is occupied now the position 1 in the series. Like that it goes on. And the last sample that is you need to introduce is f minus 1. And that of course, as a power of z power minus M minus 1. So, when you have a non causal signal, the views at the transform will now include not only this quantity that is also the additional quantities here.

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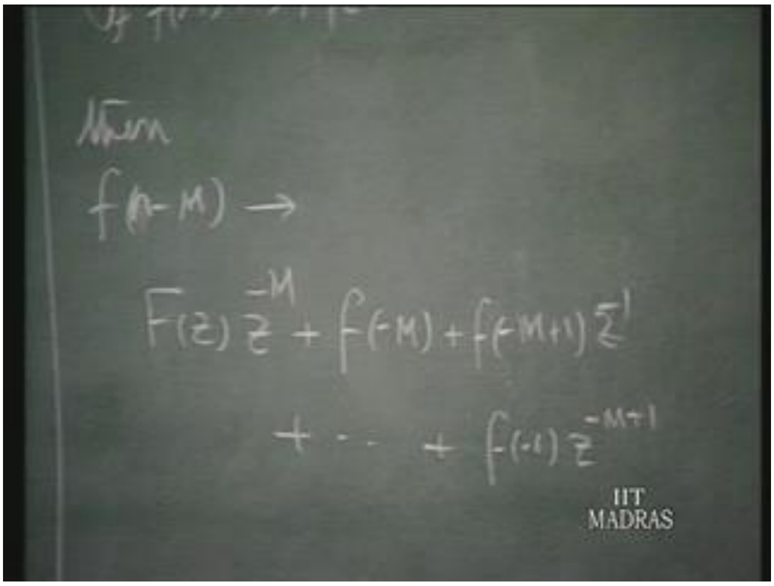


So; that means, we can say if f of n has the Z transform F of z, then f of n minus M will have the Z transform F of z z power minus M plus f of minus M plus f of minus M plus 1 z power minus 1 like that f of minus 1 z power minus M; that is the last 1 minus M plus 1. So, this is an important because, sometime when you take difference equations, you were described certain variables which are initial conditions are prescribed at minus 1 minus 2 and so on and so forth. That means: 1 of the variables is certainly values of negative values of n and therefore, when you delay suppose y is set to have value given values at minus 1 and minus 2, where you find M minus 1 M minus 2 you are delaying that; those values coming to prominence here in y M minus 1 y M minus 2 and you must take their values into account in forming the appropriate Z transform.

So, this is important that we should keep this in mind. For non causal signals it is of course, straight multiplication set of minus M for causal signals, for non causal signals it must take this values at the samples also into account. Now, let us consider what happens the signal is advanced.

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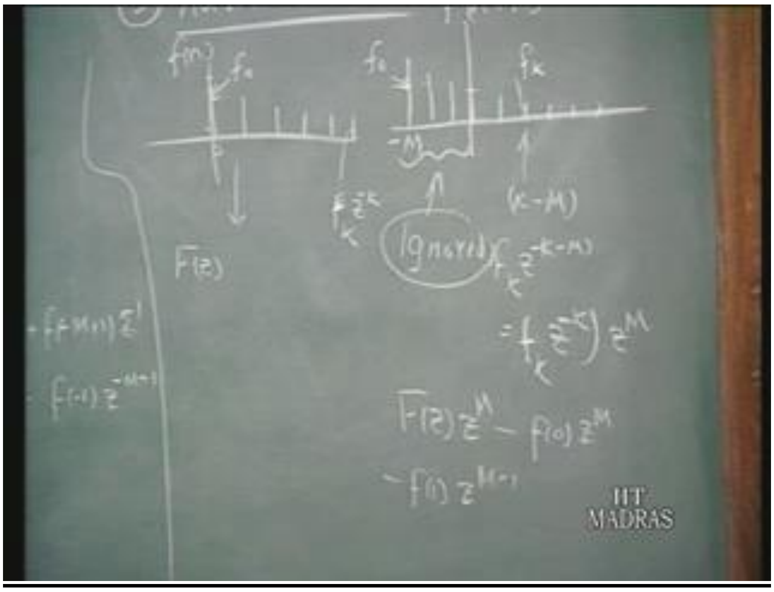


Instead of delay we advance the signal. So, if I have f n like this, if you advance it, it will be like this. So, what was f 0 here occurs at this point at instant M, this is f of n plus M. So, if you advance M sampling instants, the signal occur the sample occurring at 0 n equals 0 now occurs at n equals minus M. So, the Z transform of that is F of z, where a K'th sample here f of K has been multiplied by z minus K in forming this. Now, the K'th sample here the f of K occurs at not at K, but K minus M. Consequently the contribution of f K in this Z transform of the advance signal would be f of K z of minus K minus M or f of K z power minus K multiplied by z power M.

So, if this is F of z, 1 would expect that the Z transform of that would be F of z times z power M. But we must it into account the fact that, when you are bothering of Z transform of this we take into account only these signals all the signals are ignored. But in this expression, the contribution due to this samples also are introduced. Therefore from this expression, we must subtract the contribution of this sample which are ignored in forming the Z transform of n plus M.

So, f 0 would have contributed at Z power M because, after f 0 was standing by itself f 0, but when you are multiplied by z power M f 0 times z power M would have been the contribution to this. So, you must have minus f 0 z power M. The next sample which would have been f M, f 1, in this F of z would have multiplied by z power minus M minus 1, but you are multiplied by z power M therefore, z power M minus 1 is a contribution to f 1. Like that it goes on. The last contribution was due to this, this is now f M.

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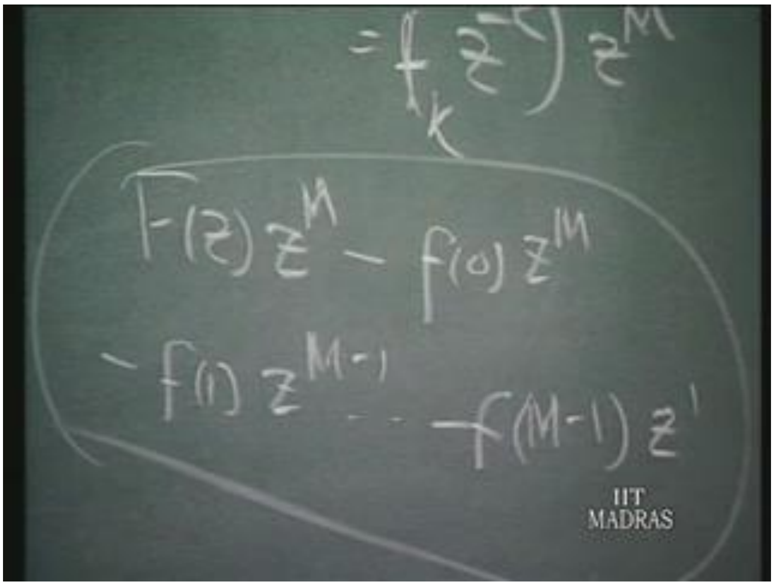


At this sample is f of M minus 1. So, f of M minus 1, in forming this Z transform you would have multiplied by z power 1. So, this then is the reverse in Z transform of an advance signal. F of z times z power M minus the contribution of all these quantities which have been ignored in forming this. So, this is to be taken into account than because, after all you are forming the Z transform or a single sided basis therefore, all the samples which are occurring at negative values and the independent variable are being ignored. And therefore, it must take a suitable correction for this.

In 2 sided Z transform, a sample succeeding from minus infinity to plus infinity are considered therefore, this additional correction will not be there in a 2 sided Z transform. But we are discussing only a single sided Z transform therefore, this correction is necessary whenever the signal is advanced.

Let us consider a few examples.

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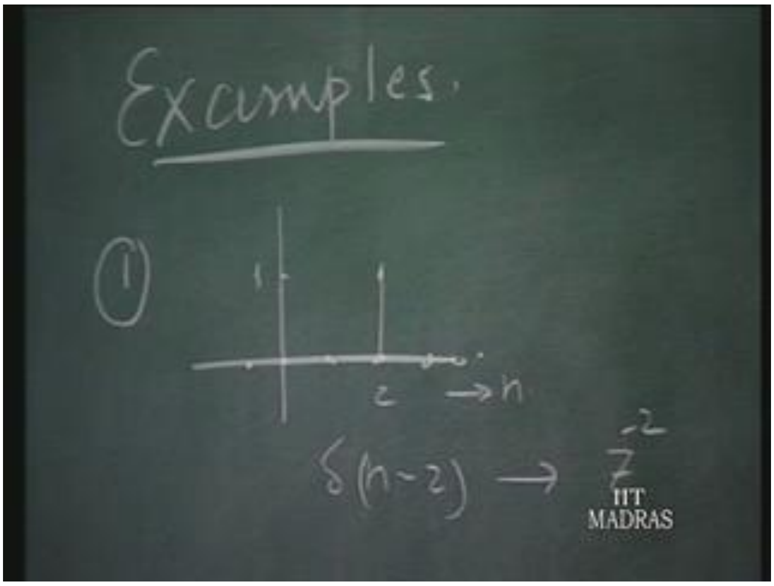


Suppose we have a sample our value 1 at n equal 2 and 0 everywhere else. Now, this is; obviously, delta n minus 2 a unit impulse, if the sample was standing here you would have set it delta n, but it is delayed by it 2 unit delta n minus 2. Since we know delta n has a Z transform 1, delta n minus 2 must have a Z transform z power minus 2 times 1 and that solved at the transform of this.

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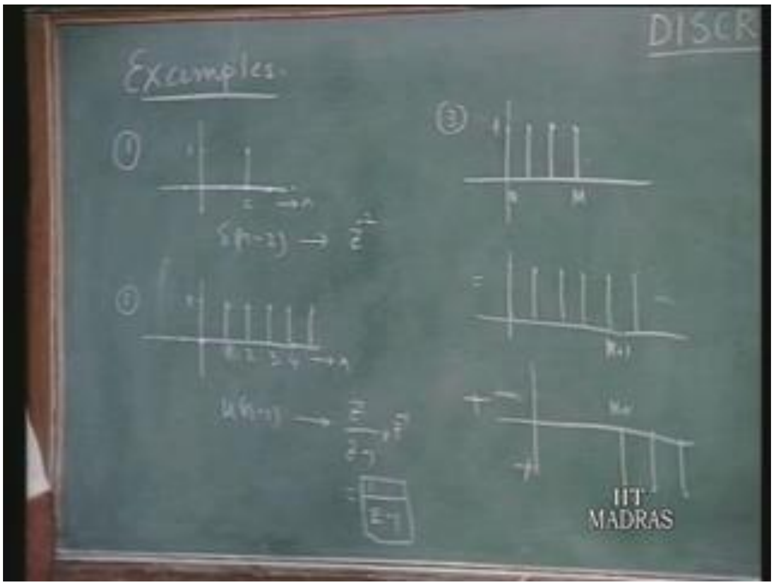


Similarly, suppose we have a function which is like this; 0 everywhere for n less than 1 and for n 1 and beyond it is equal to 1, so, this is also 0. This is; obviously, u of n minus

1 a unit steps starting at n equals 1. Since u n has the Z transform z upon z minus 1, u n minus 1 is z upon z minus 1 multiplied by z minus 1 therefore, this is 1 over z minus 1. This is the Z transform of this.

There is straight third example. Suppose f n consists of unit samples from 0 to say M. Now, this can be written as units samples write through, minus the function which has minus 1 value from this is suppose this is a you want up to M therefore, this is you take M plus 1 onwards; this has minus 1 value. So, if you add this 2 functions, then you get this. So, upto M the value is 1 and beyond that it is 0.

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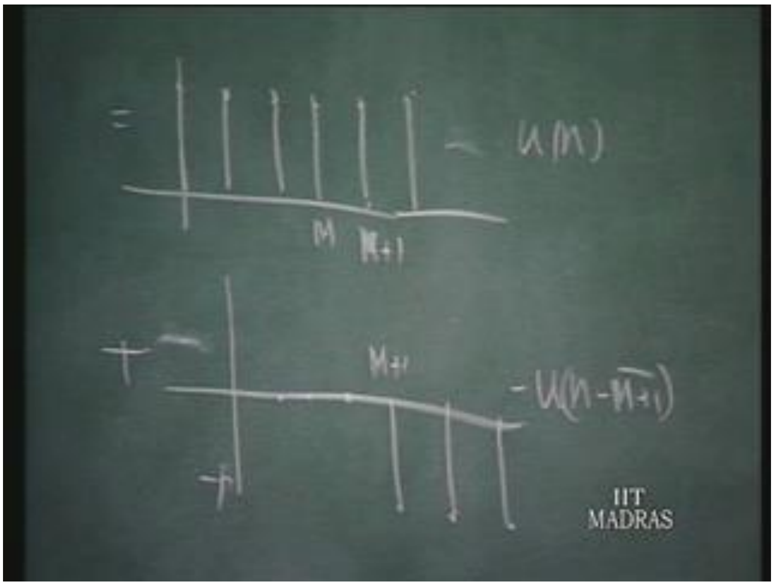


Therefore, you can write this is u of n. This is u of n minus because, is a negative sample this is a minus it is a negative sign here all the samples are negative value. But this step functions starts at M plus 1 you are n minus M plus 1; that is what it is u of n minus M plus 1.

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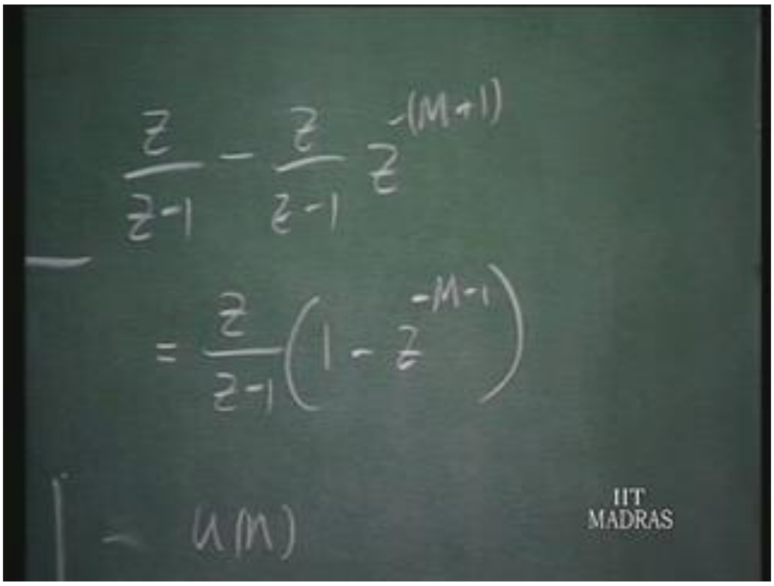


The Z transform of this is z upon z minus 1 minus of the Z transform of this function. The Z transform of this function is the same step function which is delayed by M minus 1 seconds, therefore, z upon z minus 1 multiplied by z power of minus of M plus 1. So, it is delayed by M plus 1 set units therefore, that is the Z transform. Therefore we can write it as z upon z minus 1 times 1 minus z power minus M minus 1.

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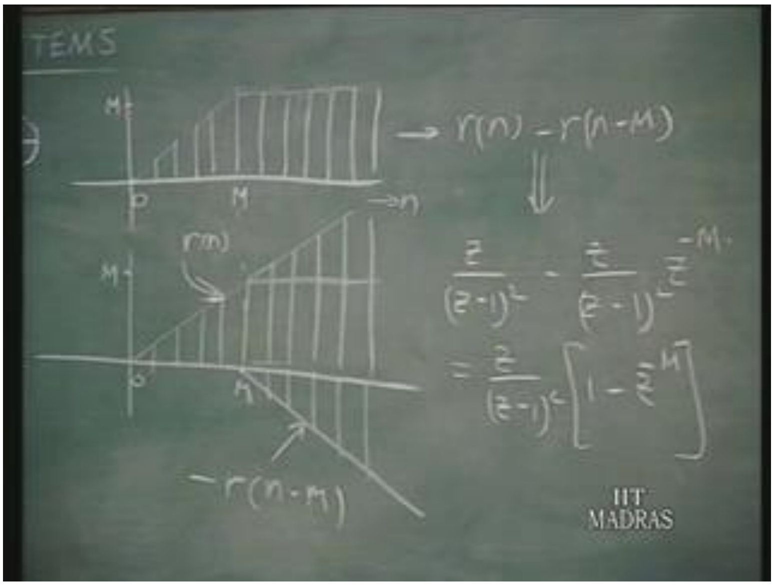


Fourth example: suppose we have a discrete time function which was like this, like this. So, upto M’th sampling is instance very just uniformly the value M and thereafter wards it remains curved it. So, this is your discrete time function as a function of n. Now, to find Z transform of this, we can regard this as the sum of per ramp function not ramp function, which continuously rises. And then at the M'th sampling instant here after all up to 0 to M, the ram function answers the given function of time. But beyond that this graphs could be arrested. So, whatever you are having there must be nullified, the increase must be nullified.

So, if you introduce a negative going ramp here such that, the negative going ramp as negative value so that, whatever increment is cause here its cancelled by this that means: this portion at this portion gets cancelled out. Then what is f here is the going function up to this point and beyond that it retails its value which is equal to M, this is of course, M. Consequently the given function can be thought of at the sum of after all this is r of n, this ramp function. And what is this is also a ramp, but with a negative slope therefore, minus you must have a negative out in front, but this ramp is out delayed, its start at not at 0, but n equals M. Therefore this is minus of r of n minus M.

So, in other words, the given function of time discrete function time is r n minus r of n minus M. So, consequently the Z transform would be z upon z minus 1 whole squared minus of the z transform of this the z minus z upon z minus 1 whole squared, but it is delayed by M sampling instance so, it must be multiplied by z power minus M. So, that is the Z transform of the second ramp. So, this can be put off course, z upon z minus 1 whole squared into 1 minus z power minus M. That is how 1 can absorb, we can use the rule for the Z transforms of delayed time functions, to find out the Z transforms of discrete time function of the type that is touched here.

Let us now consider the next property.

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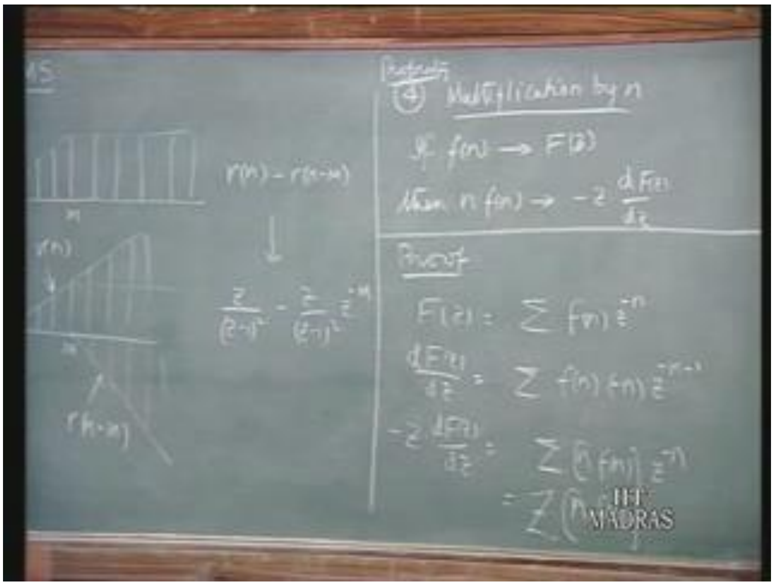

Multiplication by n: If f n has the Z transform F of z, then n f n has the Z transform minus z d F by d z. This is the result. Proof: let us consider F of z; this is after all f n z power minus n. Let us take the derivative of this; d F z over d z, just proving that you want to take the derivative of this series, you can take the derivative on to the summation sign, this will be f of n minus n z power minus n minus 1.

So, consequently if I multiply this minus z, minus z d F z by d z equals; this minus sign cancels with this minus sign and you have n f n and since you are multiplying by z this becomes z minus n. And this is after all the Z transform of n. So, the difference of Z transform of f n is this. So, n f n time Z power minus n is what we are having here that, for Z transform of n f n is quantified; that is what we have got for this is the property 4 which we have just now derived a property number 4.

Let us work out a couple of examples illustrating this property.

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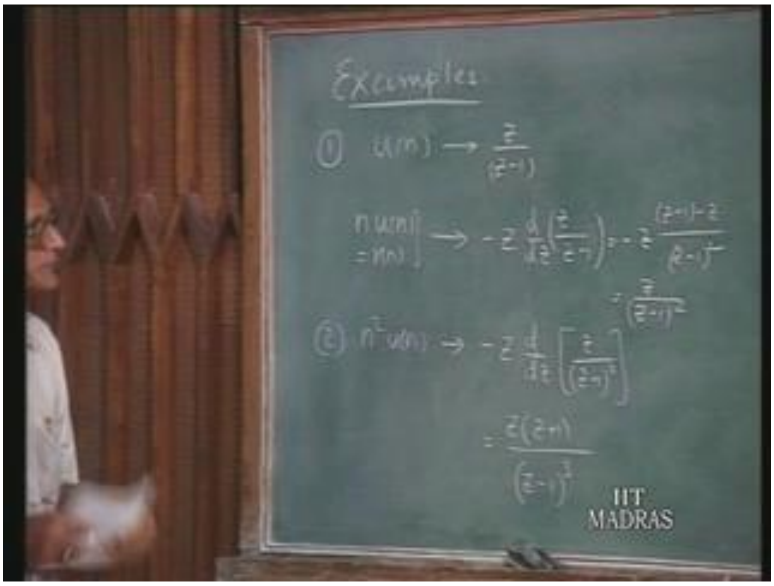


We know that u n has the Z transform z upon z minus 1. Now, what is the Z transform of n u n which is of course, r n? You can find this dead uses from your transform affluent, by using the just property that we have just now discussed; is minus z d by d z of z upon z minus 1. So, that will be equal to minus z times z minus 1 times, the derivative of this final z times, the derivative of this divide by z minus 1 whole squared. Therefore this will be the z upon z minus 1 whole squared there is all that we already know.

Now, suppose I want to find out the Z transform of n squared u n; that means, this particular function is multiplied by n again. Therefore, it Z transform is minus z d by d z of this expression, z upon z minus 1 whole squared. You can work carry this out and show this result equal is z times z plus 1 over z minus 1 whole cubed. So, that is how it works. You do not have to remember this particular Z transform; we can always derive it from the Z transform of n u n.

Let us now consider the next property.

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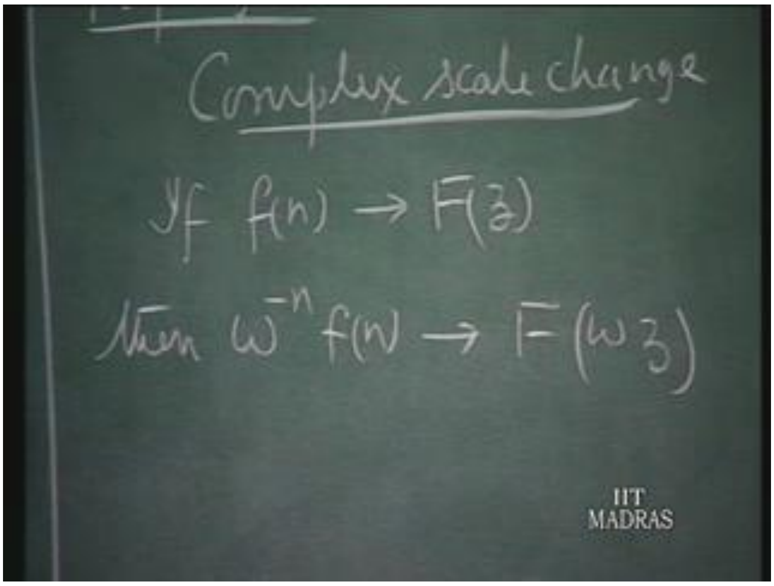


Complex scale change: If f n has the Z transform of F z, then w minus n f n has the Z transform F w z. That means if F w z; that means, it is like a scaling in the complex domain in the Z domain, the absorb is F w z. This result is quite straight forward; you can write down the Z transform for that and get due to this without any difficulties. So, you will not try to construct the proof, this is quite evident. But you will have to work out the examples.

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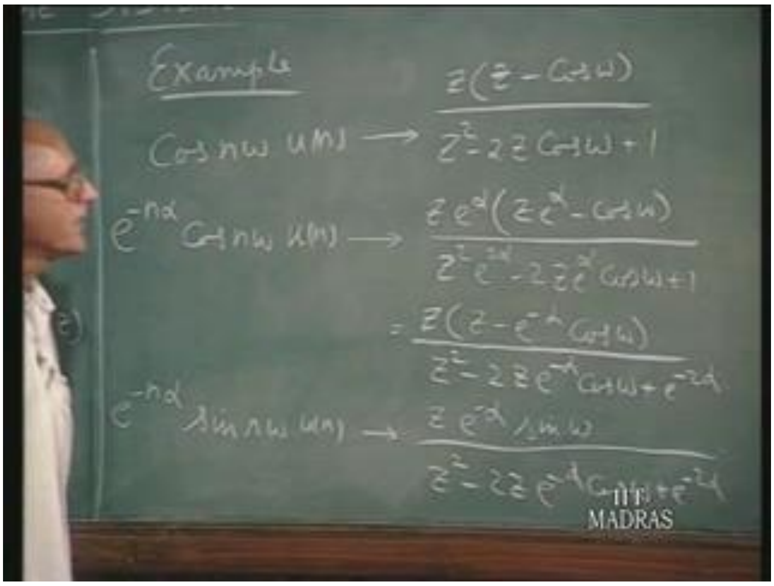

Example: suppose we have cos n omega u n, we know that the Z transform is z time z minus cos omega divided by z squared minus 2 z cos omega plus 1. The second part and transform which should keep in mind we should remember this. Now, suppose instant away side and side function of discrete variable n as we have here. Suppose you have a dumped sinusoids, suppose I take e power minus n alpha cos n omega u n. So, the Z transform of this, since you know the Z transform of this, you are multiplying by e power minus n alpha, if you recall the property that, now we have just now discussed; this w corresponds to e power minus alpha here. So, if you know the Z transform of f M, you can find out the Z transform of the w power minus n f M. So, all we have to multiply this replace z by z times e power alpha.

So, you have Z transform of all this; z e power alpha times z e power alpha minus cos omega divided by z squared e power 2 alpha minus 2 z e power alpha cos omega plus 1. This can be put in a alternative fashion which is often out convenient; z times z minus e power minus alpha cos omega divided by z squared minus 2 z e power minus alpha cos omega plus e power minus 2 alpha. So, this is an alternative expression which is often found convenient.

So, that dumped cosine function we have a Z transforms like this. We can extend this to e power minus n alpha sin sin n omega is similar fashion. Once we know this Z transform of sin n omega, you can find out Z transform of e power minus n alpha sin n omega u n. So, I will give you the result; this will be z times e power minus alpha sin omega divided by the same denominator z squared minus 2 z e power minus alpha cos omega plus e power minus 2 alpha.

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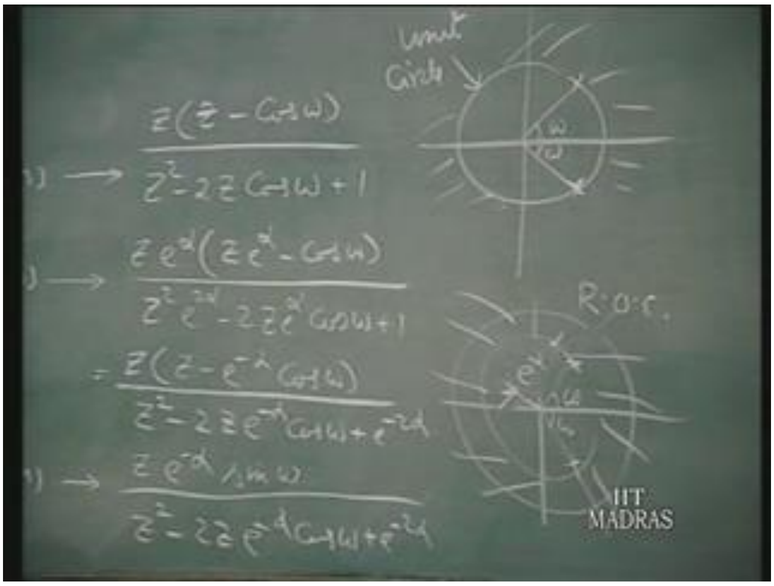


The point note here is that, cos n omega u n has poles in the Z plane at this 2 points on this is unit circle, at an angle plus or minus omega in the x axis. Though at this poles of cos Z transform of cos n omega and deviation of convergence will be outside the unit cells. Now, when you take e power minus n alpha cos n omega u n, the poles now will be of this, this is the unit circle, but the poles will be at a distance e power minus alpha from the origin and at the same angle plus or minus omega. That means: the distance of the poles form the origin is not unity, but e power minus alpha which is of course, less than 1. That means the region of convergence now is enlarged. This is ROC.

The region of convergence there in the first 1 is outside the unit circle, but here outside the circle of radius e power minus alpha. That means: the region of convergence it is enlarged. The reason is cos n omega is something which is amplitude is constant, the e power minus n alpha cos n omega is decaying sinusoid therefore, it is decaying faster therefore, the region of convergence is also increased. That is the purpose of this analysis.

The next property we would like to discuss its 1 relative convolution.

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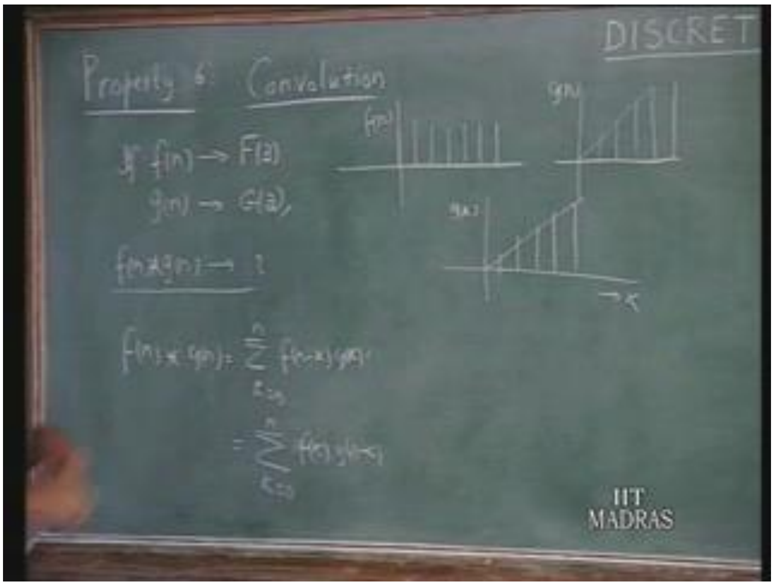


If f n has the Z transform F of z and g n has the Z transform G of z. We would like to know; what is the Z transform of the convolution of f n g n. If you recall, that the convolution of 2 discrete time signals f n g n is defined as f n minus K g K, K from 0 to n per causal signals which we are assuming. This also can be written as K equals 0 to n f of K g of n minus K. This can also be written in this fashion.

A physical significance of this is that, is you are having so, this is f n and let us say this is g n, then what we are doing is in forming this you take g K and multiplied each samples of g K by f of n minus K, this is fn.

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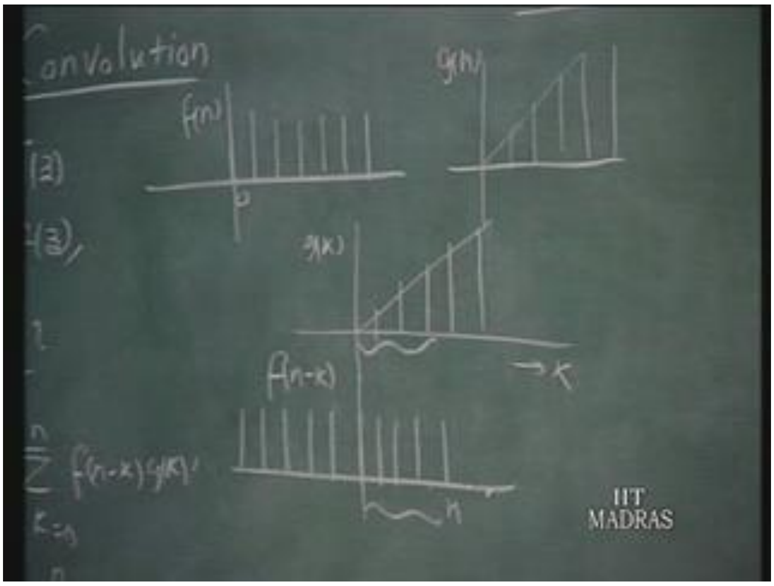


So, f n minus K would be obtained by folding this around the origin then, you will get this and advancing this by n units. So, you will have something like this, this is your f and this 0 samples here, this is f of n minus K, at this now instead of starting at 0 here it starts at here. Now, if a multiply sample g K by the corresponding sample even f n minus K and add up that products from n equal 0 from K equal 0 to M. That means all the samples will end up into the final give the final contribution. These samples are lot, these samples are rot last and therefore, the summation extends from K equals 0 to M.

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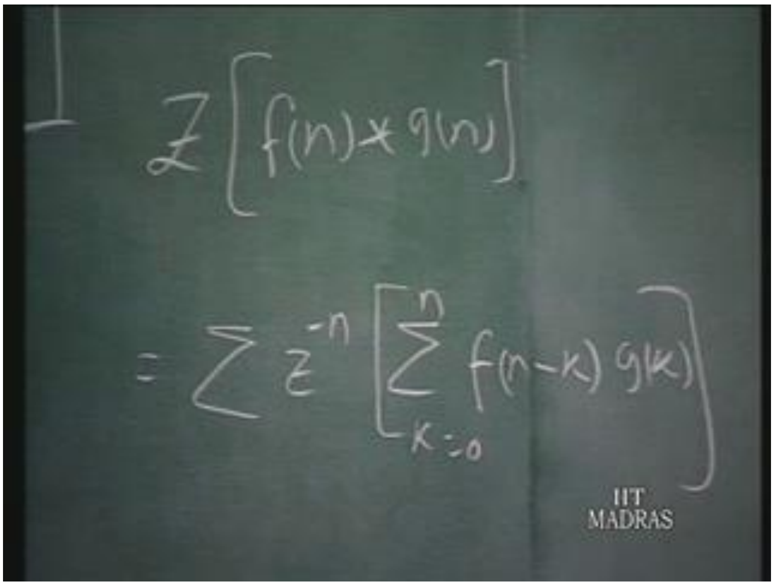

Now, the Z transform of f n convolved with g n is therefore, equal to by the very definitional Z transform; z power minus n times the convolution of this, what is the convolution of that; f of n minus K g K, K from 0 to n. So, this is the convolution summation of f n starts g n and that multiplied by Z power minus n, n from 0 to infinity. Now, when K is now summed up from 0 to n, when K gets passed; n f n minus K, in the argument n minus K becomes negative. Therefore since you are assuming causal signals, your n minus K is 0 for K greater than n. And because of this, I can write this summation z minus n.

Now, the second summation I can say K is equal 0 to infinity without altering anything because, I am only adding zeros because; f n minus K happens to be 0 for K greater than n. So, for instant of K'th in the upper limit and summation to n, I can as well take it in upto infinity. So, this can be written as f n minus K times g K because of this. Now, what I can do is; I can interchange the summation, I will search the start summing up with n first and then with K.

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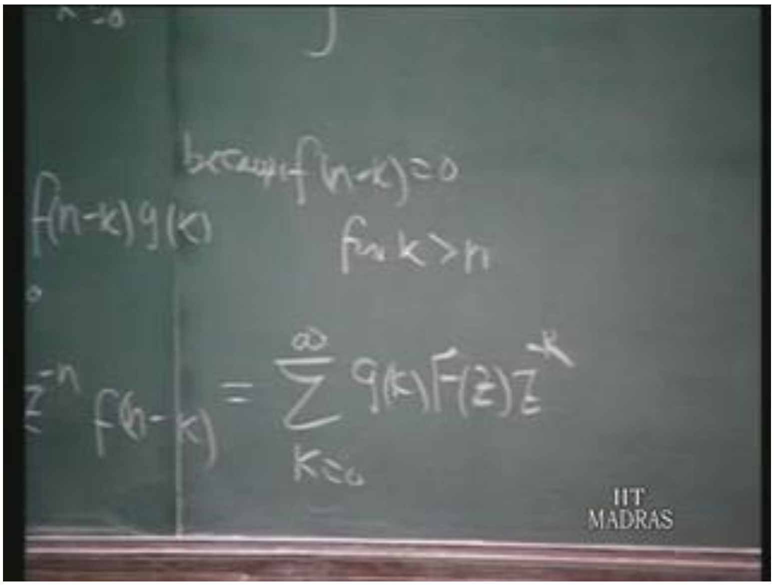


Therefore in the double summation, I will take with the summation with reference to K later and I will first take up the summation with reference to here. Therefore since, I am summing up with reference to n as a first step, all those which you put here; z power minus n f n minus K. And all those which are do not involve are not involved with n can we brought outside, there constants are summation concerned. Now, this as you can see is K equal 0 to infinity g K, this is after all the Z transform of n minus K.

If f of n has Z transform of F of z, f of n minus k delayed function will have F of z times z power minus k. And in this summation now F of z is a constant that can be pulled outside the summation and all you are now having will be F of z multiplied by K equals 0 to infinity F g K z minus k; that is after all the Z transform of g of K. Therefore, I can write this as F of Z times K equals 0 to infinity of g K Z power minus K which is indeed F of z G of z.

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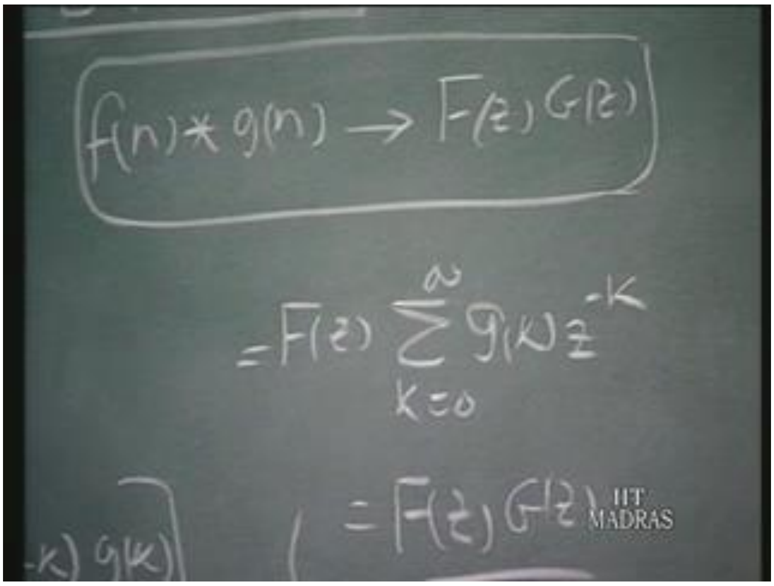


So, up sort of this is that, f n convolved with g n the convolution summation has the Z transform F z times the G z. Convolution in time domain corresponds to multiplication the transform domain, the result which is trickling similar to what we had in the case in the continuous time case, where the Laplace transform of the convolution of the 2 type function f t and g t it is a product of Laplace transforms. Exactly the similar result is available here also.

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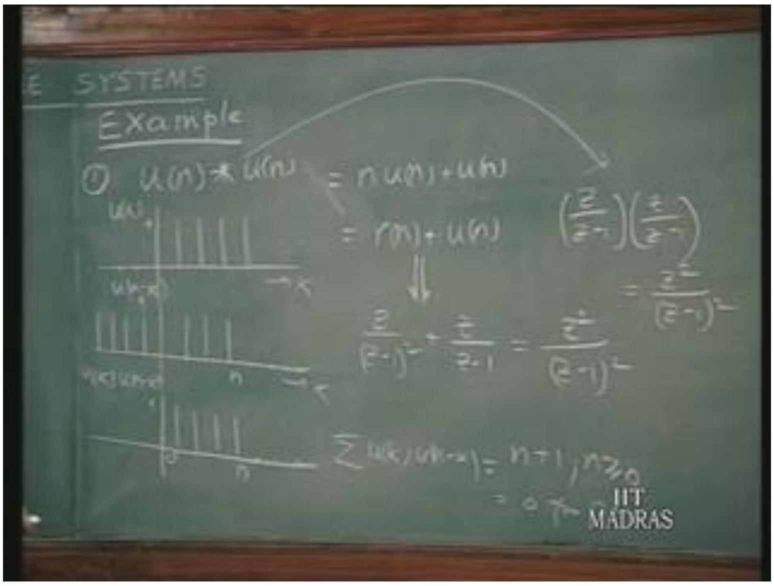


Example: let us take a first example u n convolved with u n. The convolution summation upto step functions. So, to do this we first plot u K verses K. So, that is a set of discrete values of value equal to 1 per positive K. Then you have to multiply this by u n minus K. So, u n minus K will have u n minus K will have values for negative values of K, but u n minus K you shift it by n instance. So, you have like this. And beyond that up to n it is equal to 1, this is K and this is u n minus k. So, beyond here it is 0 because, u n minus K is the same function folded up at the negative direction, but you shifted forward by n instance therefore, this is u n minus K.

And then you multiplied this 2 functions; that means, u K multiplied by u n minus K will now be; after all this is 0 multiplied by 1 there is nothing therefore, you have a string of ones upto and starting from 0 and going up to 1 n and this is equal to 1. Now, in the convolution summation, you take the u K multiplied u n minus K. And this means that, you must add up the product values from 0 from all the product value that you got and only non zeros values starting from 0 to n and therefore, there are n minus 1 such units we picked up therefore, this is n plus 1. And this is true for positive per n 0 or positive because if n is negative for example, that means, after shifting folding u K to u n minus K if n is negative you shifted in this direction therefore, there will be no overlap between this non 0 samples here and here. That means whatever result you got is applicable only for n greater than or equal to 0.

For n less than 0 this; that means, this sample trained the lower shifted in the negative direction therefore, the product will be identically 0. Therefore this is 0 for n less than 0. So, what it means is; u n convolved with u n in working out in the transform in the time domain, we now observed that this is equal to n plus 1 for n greater than or equal to 0 therefore, you combine this as n u n plus u n, which means: this is r n plus u n.

So, if you find the; what the Z transform of this, after all the Z transform of this would be r n is z upon z minus 1 whole squared and u n; z upon z minus 1 therefore, the product of this the sum of these if you have a common denominator z minus 1 whole squared z times z minus 1 z squared minus z plus z, that is, z squared upon z minus 1 whole squared. So, the working out with purely time domain, discrete time domain, he observed he obtained the Z transform of u n convolved with u n as z squared upon z minus 1 whole squared.

But by applying the convolution principle; that means, just now discussed we can also arrive at this Z transform of the independently. We know the Z transform of u n is z upon z minus 1. Z transform of the second u n also z upon z minus 1. So, the convolution in time domain is equivalent to multiplication the transform domain and this is also equal to therefore, z squared z upon z minus 1 whole squared. So, what you have obtained in the time domain is also the same as what we obtain using the convolution principle which we now just now discussed.

Let us consider a second example.

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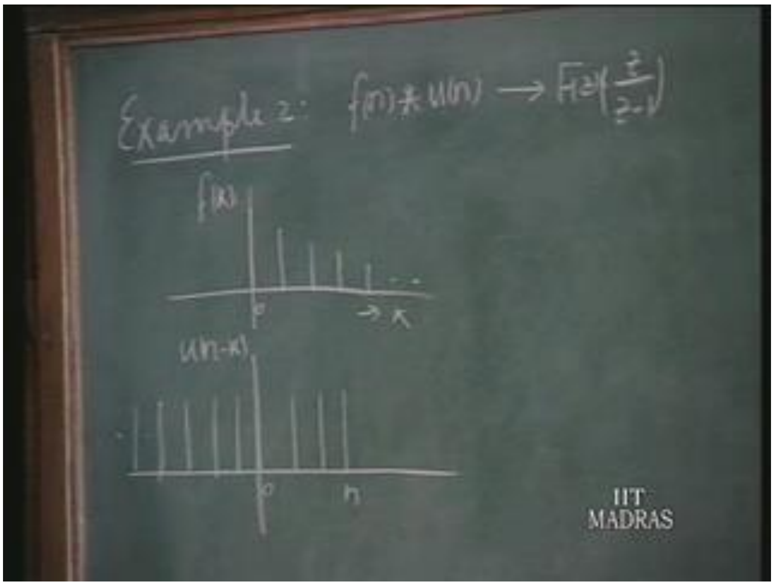


If a general function f of n is convolved with u n, we now the Z transform may will be F of z times z upon z minus 1. Now, let us look at the meaning of the convolution f n star u n. So, if f of n is f of K verses K verses is a sample try like this, u n minus K would be a negative going step staring at n at n going backwards. So, f of K times u n minus K will be the set of samples starting from K equal 0 to n.

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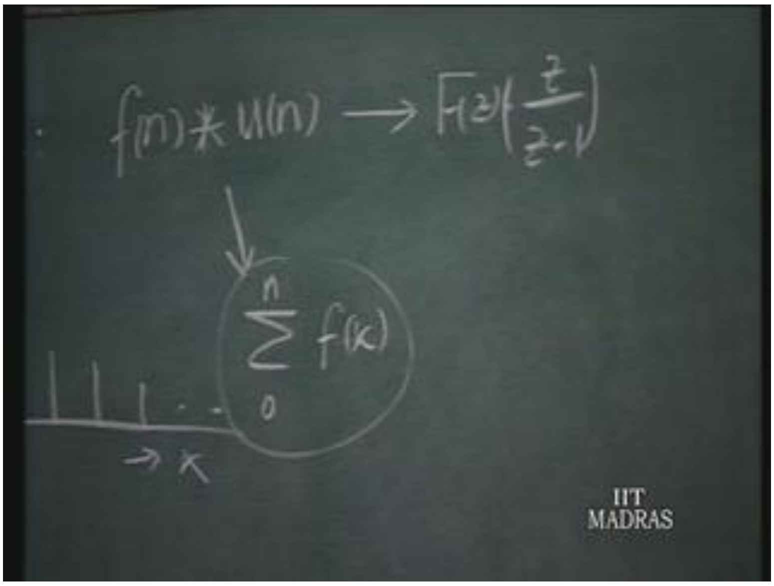


So, the summation of this set of samples is indeed f K from K equals 0 to n because, the product will be 0 outside this range for less than n less than 0 and the K less than 0 and K greater than n. So, in other words the convolution of f of n with u n is the summation of the first n plus 1 samples from 0 to n. So; that means, if you have f of n has the Z transform F of z, the product 0 to n of f of n, f of K that is well be equal to your F of Z times z upon z minus 1.

So, this will be suppose you have got g n defined in this manner 0 to n f of K; that why you have the Z transform F of z times z upon z minus 1. This the result which is 1 again similar to what we had in the continuous time system, where the convolution of unit step with the function f of t is equivalent of is integration of this f of t from 0 to t. That is something which we have already discussed.

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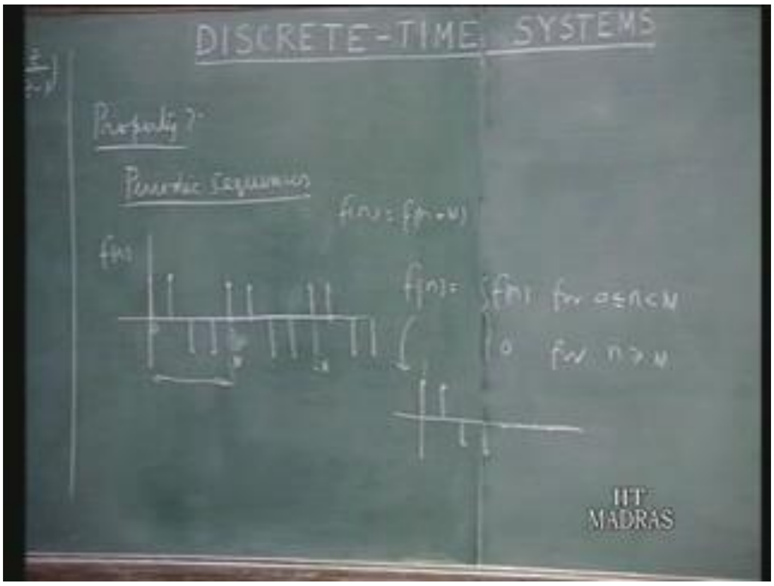


Suppose we have a periodic sequence, you have f of n which repeat itself for heavily n samples instance. So, in other words f of n is equal to f of n plus N for all values of n. Now, let us take the first n minus 1 samples which defined 1 period; that means, if f 1 n is defined to be f of n for 0 greater than n greater than or equal to 0 and less than n. That means first n minus 1 samples and 0 for n greater than or equal to N suppose we define that. And that will be just to the samples during the first 1 period first period and everywhere else 0, this is f 1 of n.

If you know the Z transform of f 1 of n you can find out the Z transform of f of n because, it is a repetition of whole thing again and again.

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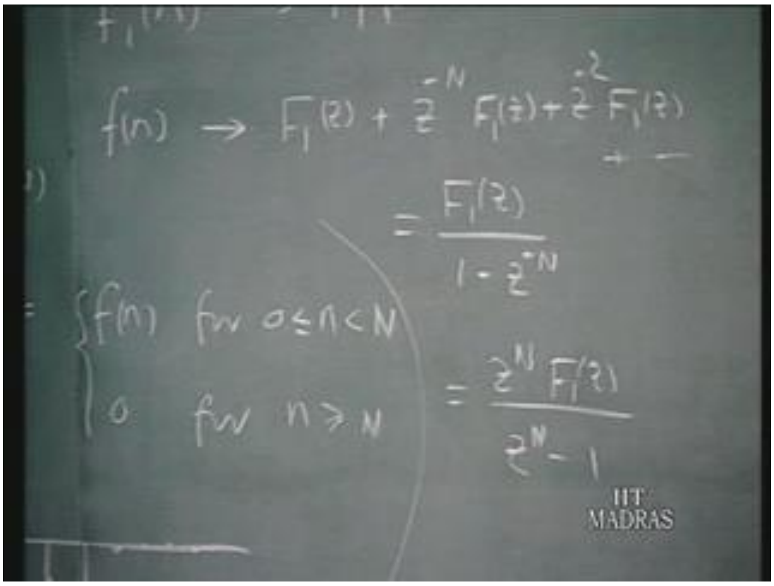


So, if f 1 of n find the Z transform F 1 of z, then f of n is the obtained by having f 1 of n plus f 1 of n minus N f 1 of n minus 2 N and so on and so, forth. Therefore, this is obtained by the basic 1 period has the Z transform. The next cycle of samples for the next period will be z minus n F 1 of z plus z minus 2 F 1 of Z etcetera. Therefore, this will be F 1 of z divided by 1 minus z power minus N. So, that is what we have. It is something very similar to what we had in the case of continuous time systems, where you have e to the power of 1 minus e to the power of minus s t is what we had here, in this case this is how it will be.

This can be further written as z minus N times F 1 of z divided by z minus N minus. The next property that we would like to consider is.

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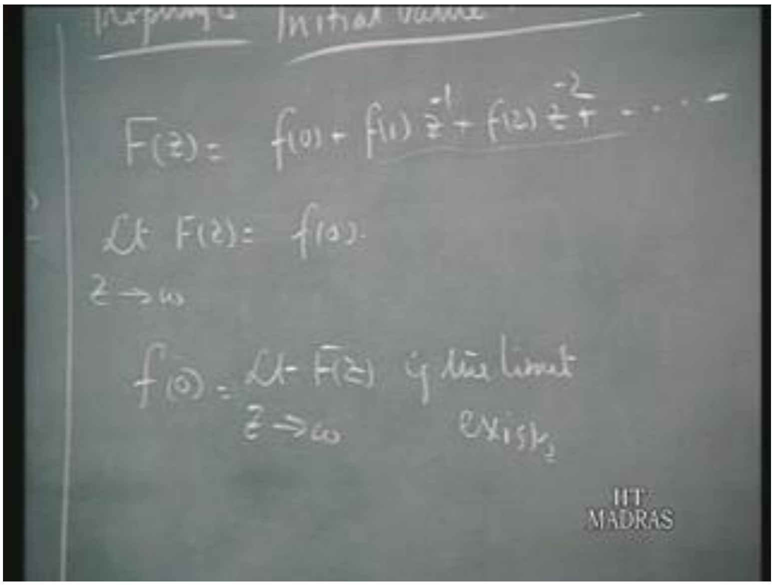


It is the initial value theorem, which is counter path of what we had in the continuous time systems. By definition F of z equals f 0 plus f 1 z minus 1 plus f 2 z minus 2 etcetera. So, if you take the limit at z tends to infinity of F of z, if you take z to very large value and take the limit of z tends to infinity, all this samples become in sequently small and you get f 0.

So, the initial value theorem states; f 0 is limited at z tends to infinity of F of z, in such a limit exists. So, this is very simple role, that is, initial value theorem. And we must also keep in mind that, z is going to infinity; that means, z equals infinity must be the region of convergence. So, if you are having a single sided Z transform, you have this region of convergence therefore, z is going to infinity is allowed. This may or may not be true in the case of 2 sided Z transforms.

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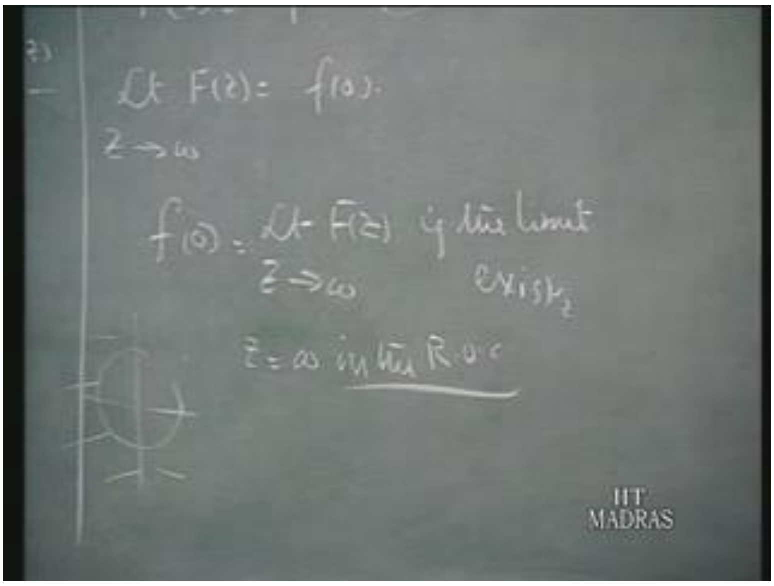


So, the z equals infinity must be in the region of convergence.

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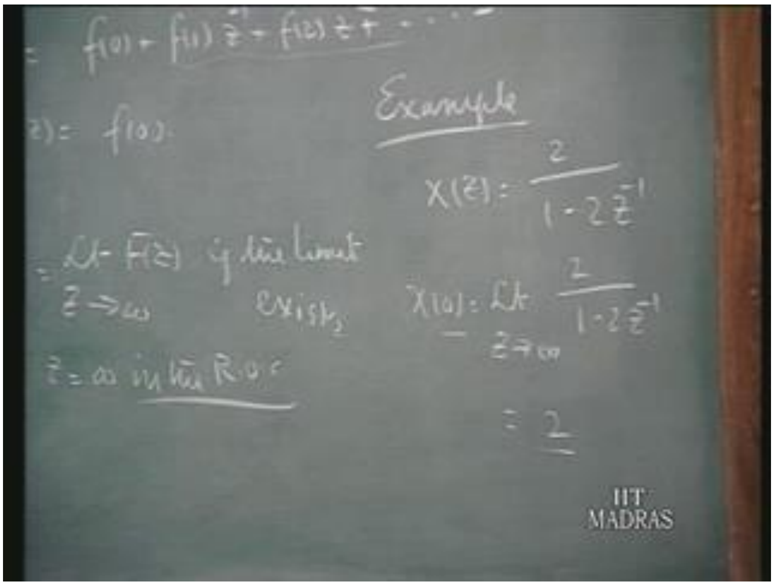


Example: we have suppose X z happens to be 2 upon 1 minus 2 power 2 times z minus 1. This is the Z transform of a certain function X n. Therefore from this you can find out X 0 has limit z goes to infinity of 2 upon 1 minus 2 power 2 times z minus 1. And z goes to infinity at this become 0 therefore, this the second term will become 0. So, 2 divided by 1 which is 2, X 0 equals 2. And when we are talking about the Z transforms discrete time function, there is no nothing like 0 plus 0 or minus is only the values at discrete points therefore X 0 equals 2. So, this is an application of the initial value theorem.

We also have the final value theorem. I will just give you the result and we will discuss this in the next lecture in it is a implication and its proof. But let me just state this property.

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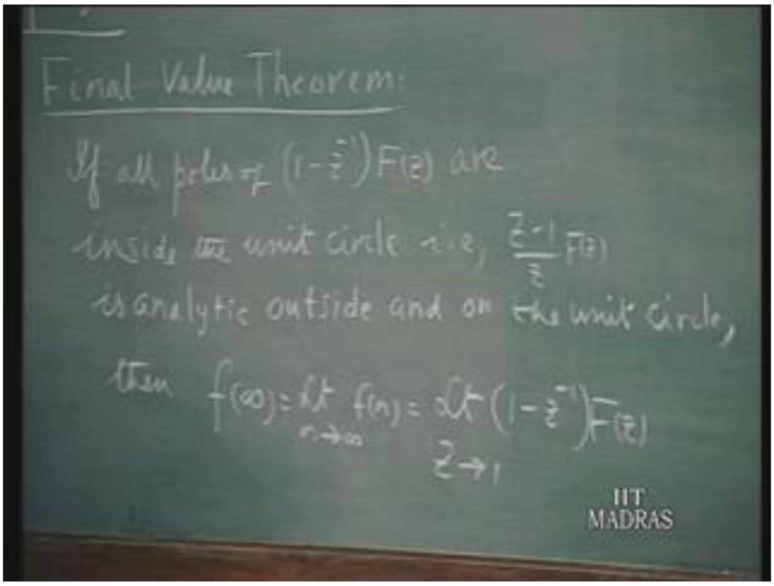


Property 8: final value theorem. The statement of the theorem will go like this; if all poles of 1 minus z power minus 1 F of z are inside the unit circle, that is, z power z minus 1 divided by z times F of z; this function, is analytic outside and on the unit circle. If this condition is fulfilled, then the final value of f of n f infinity which is really limit as n goes to infinity of f of n is given by limit at z goes to 1 of 1 minus z power minus 1 F of z. So, that is the statement on the final value theorem.

The conditions that required are; 1 minus z power minus 1 times F of z should not have any poles in the unit circle, I mean outside the unit circle and even on the unit circle itself. So, provided all the poles of this are inside the unit circle, then the final value of f of n is given by this especially. The proof of that will be constructed in the next lecture. But record we can notice that is; this is the counter part for the final value theorem that we had, in the case in the continuous time situation.