Networks and Systems

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Lecture-40
Discrete-Time Systems (3)
Models of Discrete-Time systems.
Realization.
Introduction to Z-transforms

In this lecture, we will first talk about the models of a discrete time system. You recall that, in the continuous case of continuous time system taking the familiar example of an RLC network. The governing differentially, the governing equations for an RLC network and dynamic conditions are differential equations. So, the circuits are represented by differential equations. Now, suppose you are given a differential equation and you are asked to find out an RLC network, for which this particular differential equation pertains then it is not all that easy it can be done, but the problem is slightly complicated.

On the other hand, if you are given a differential equation and you are asked to find out a circuit representation for that, then it is possible to find out a simple circuit using a model which includes integrators, summing amplifiers and coefficients setting potentiometers just like in your analog computer implementation or simulation of a dynamic system. So, in the modeling of a continuous time system a very useful way in which it can be done is using integrators summing amplifiers and potential dividers and so, on and so, forth. On the other hand, if you are talking about a discrete time system.

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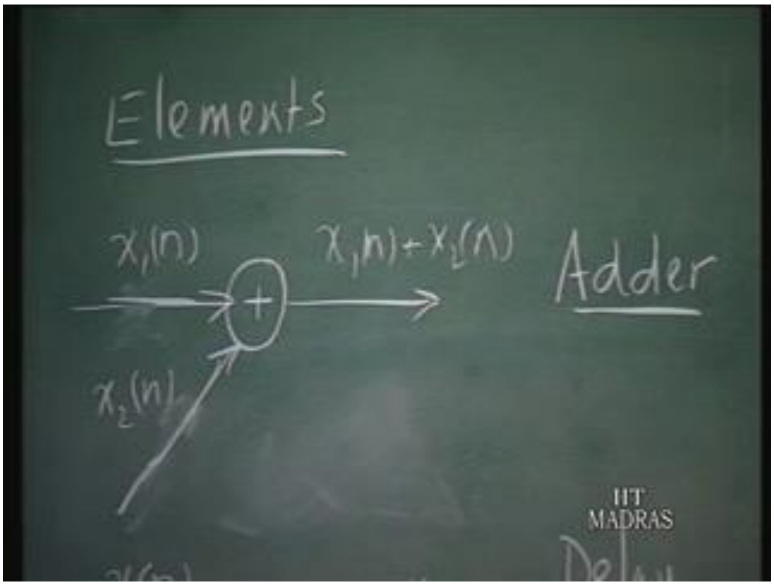

So, we should have a similar type of models such circuit elements, which together constitute a model of a discrete time system. Here the elements that constitute such models are of 3 types 1 is an adder an adder suppose see what we are represented is a 2 input adder, if you have 2 signals x1 n and x2 n are coming in at any sampling instant n the output will be the sum of these 2 inputs So, the x the output will be x1 n plus x2 n this can be extended to an adder which have several inputs for example, if you have k inputs x1 n x2 n up to x k n the output will be the sum of k inputs that is 1 element.

The second element is the delay element, if you are having a sequence of values fed into the delay element at every given instant the output will be, the value of the input at 1 previous sampling instant; that means, the whole sequence of inputs are reproduced here with a delay of 1 unit. Therefore, if xn is the input at that particular instant of time the output will be x n minus 1this is the delay element. So, this produces 1 unit delay if you want 2 unit delays 2 delays you can put 2 such delay elements in cascade. So, this is quite obvious.

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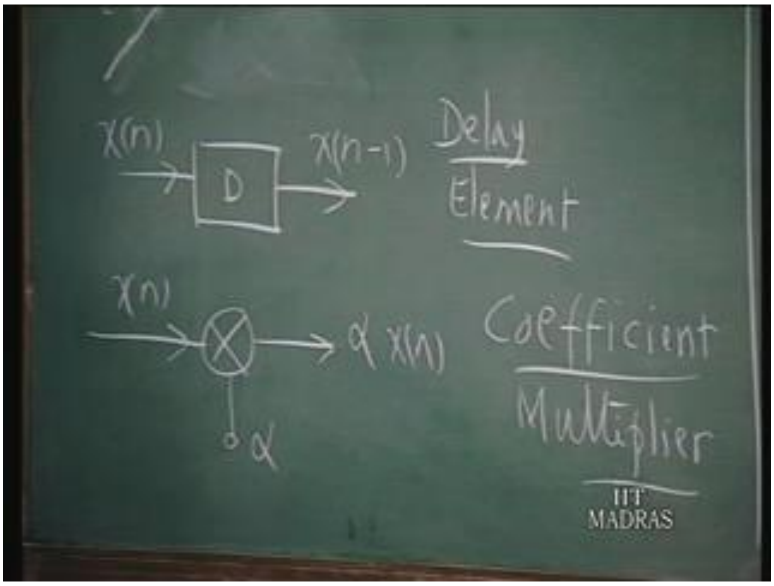

A third element is what is called a coefficient multiplier, in which the signal input xn is the input and this is multiplied by a certain constant that you put this is a coefficient. So, alpha time’s x n will be the output. So, this is what is called a coefficient multiplier once we have these 3 elements, we can model any linear discrete time system or difference equations suitably. So, let us look at a couple of examples to see how you can find out the difference equation corresponding to a system which includes these various kinds of elements.

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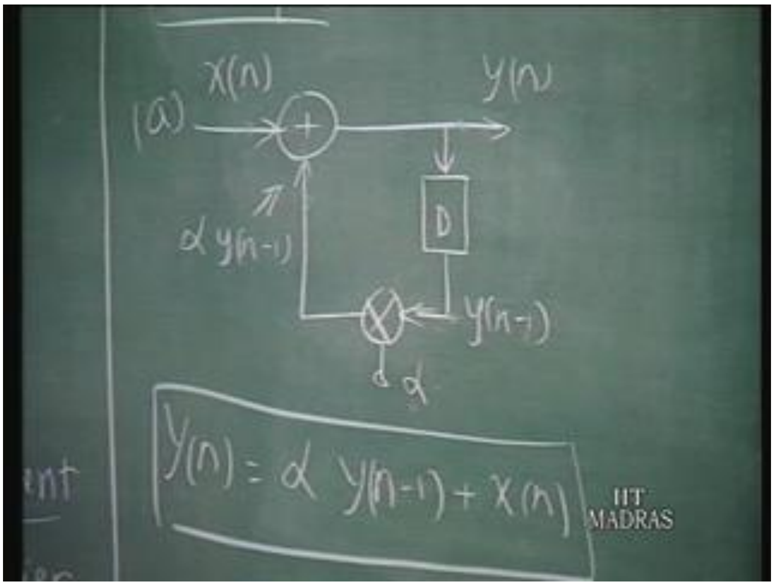

First example: let us take this example where we have an adder with an input xn coming here and another input coming here and the output of the adder is what we call the yn the output, yn is delayed by 1 unit. Therefore if this is yn this signal here will be yn minus 1 and that is multiplied by, alpha a coefficient multiplier. So, the output of the coefficient multiplier is going back to this adder. So, the sum of these 2 signals will be y of n. So, what do we have now y of n is therefore, the sum of this plus, this is xn and this is yn minus 1 this is multiplied by alpha therefore, this signal here will be alpha times yn minus 1.So, yn will be xn plus alpha times yn minus 1. So, yn will be alpha times yn minus 1 plus xn. So, this is the difference equation to which this particular sys system pertains, this is the model of these difference equations this is the first order system.

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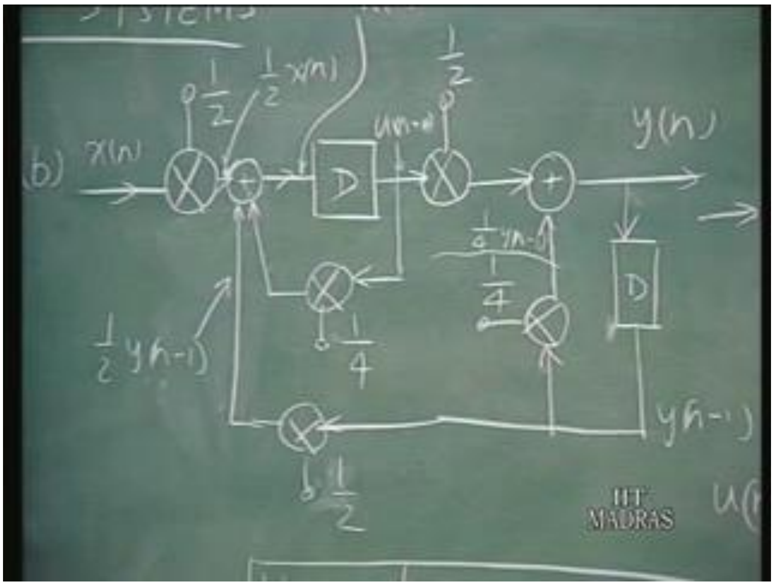

Let us take a more complicated example, where we have 2 delay elements involved this is another system. Let us you take it down, a input x n is given to a summer this is not certainly there is a small error here let me put this here. So, input xn is given to a coefficient multiplier alpha therefore, the signal here will be half times xn and that signal plus 2 other signals are given to this adder and the output of the adder is given to a delay element and that delay element is given to it again. The output of the delay element given to a coefficient multiplier and that is 1 input this adder and the output of this adder is yn yn is delayed by a delay element here.

Therefore, this signal here is yn minus 1 and that is fed to the coefficient multiplier here one-fourth therefore, the signal here will be one-fourth of y n minus 1 that is the signal that is coming here. And now this yn minus 1 is also fed to another coefficient multiplier therefore, the signal here will be half of y n minus 1; now, another signal is coming after this delay element; suppose we call this signal here as un. So, this will be un minus 1. So, this signal here will be un minus 1 because if this is un this will be un minus 1. So, un minus 1 is multiplied by one-fourth and put this to the adder.

So, un will therefore, be xn half of times half times xn plus half times yn minus 1 plus one-fourth of un minus 1 therefore, un will be half of xn plus half of yn minus 1 plus 1 fourth of un minus 1.

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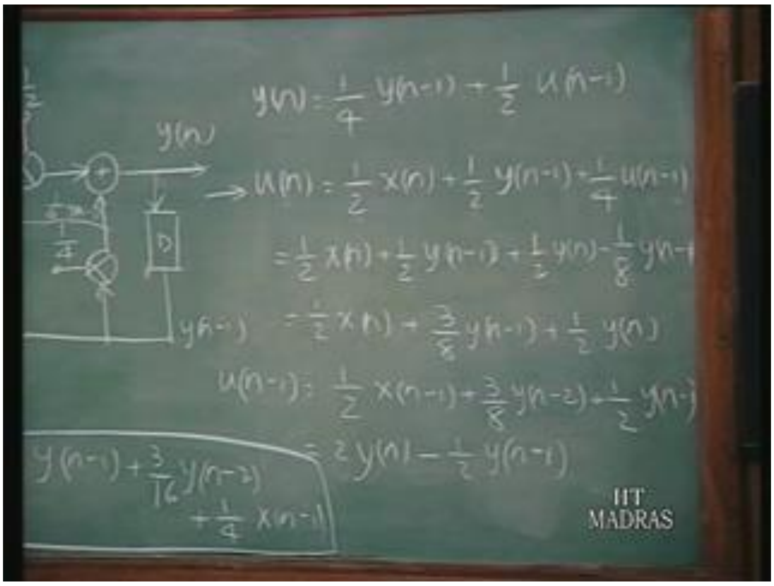

So, this equation represents the characteristics of this adder un will be half of xn plus of yn minus y of n minus 1 plus one-fourth of un minus 1; that is what, we are having. Similarly, your output of this adder yn will be the sum of this if this un minus 1, this signal will be half of un minus 1 plus one- fourth yn minus 1.Therefore, yn will be half of un minus 1 plus one- fourth yn minus 1 and that is this equation, yn equals one-fourth yn minus 1 plus half of un minus 1. So, you have these 2 equations and when to find out the difference equation connecting the output yn and the input xn, we have to eliminate this intermediate variable un.

This can be done in several ways and 1 way in which it can be done is we take this equation un equals half xn plus yn minus 1 plus one-fourth un minus 1 and since you already have a expression for un minus 1 in terms of yn and yn minus 1; we will substitute that here. So, half xn plus half yn minus 1 for one-ourth y un minus 1 you substitute half of yn minus 1 8 yn minus 1 and simplifying that, you get half of xn plus 3 eighth yn minus 1 plus half of yn; now, still we have to get rid of un. So, if this un is this much, un minus 1 will be decrement the independent variable by 1 step.

If un is this much un minus 1 will be half of xn minus 1 plus 3 8 of yn minus 2 plus half of yn minus 1; because n is replaced by n minus 1, but we have an expression for un minus 1 from the first equation; un minus 1 from the first equation is after all 2 times yn minus half of yn minus 1 from the first equation. So, un minus 1 is 2 times yn minus half of yn minus 1. So, if you equate this 2; you have a single equation joining yn and xn in the difference equation form.

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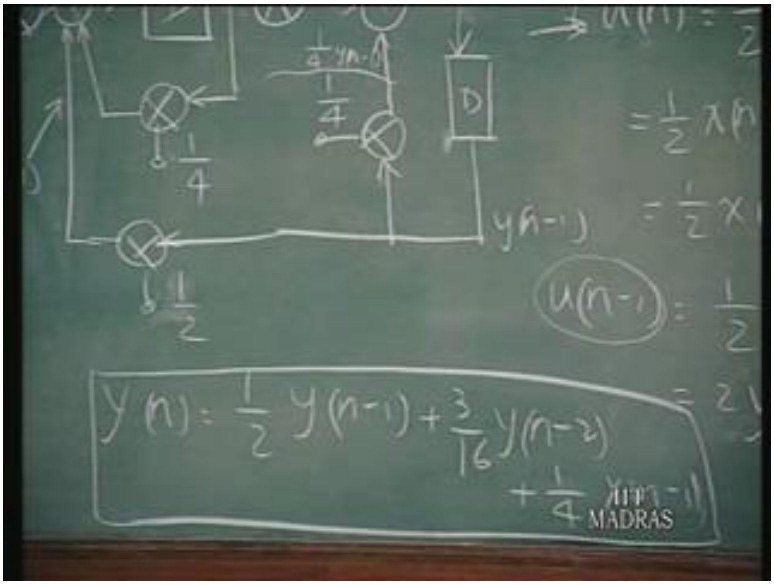

So, equating this 2 we finally, arrive at an expression the difference equation for the system yn equals half of yn minus 1 plus three-sixteenth yn minus 2 plus one-fourth xn minus 1 So, this is the second order difference equation joining the output yn to the input xn in this particular system. So, these 2 examples show you; how you can calculate find out the difference equation pertaining to a model of a discrete time system, which consist essentially of adders, delay elements and coefficient multipliers. The delay elements are somewhat corresponding to the integrators in your analog computer simulation and of course, the others are summing amplifiers and coefficient multipliers or quite similar to the adders and the coefficient multipliers.

Now, the converse problem is what is more cater importance to us given I will illustrate this by means of an example: Just now we will take that, second order case again for purpose of illustration and see how we can arrive at the model corresponding to a given difference equation.

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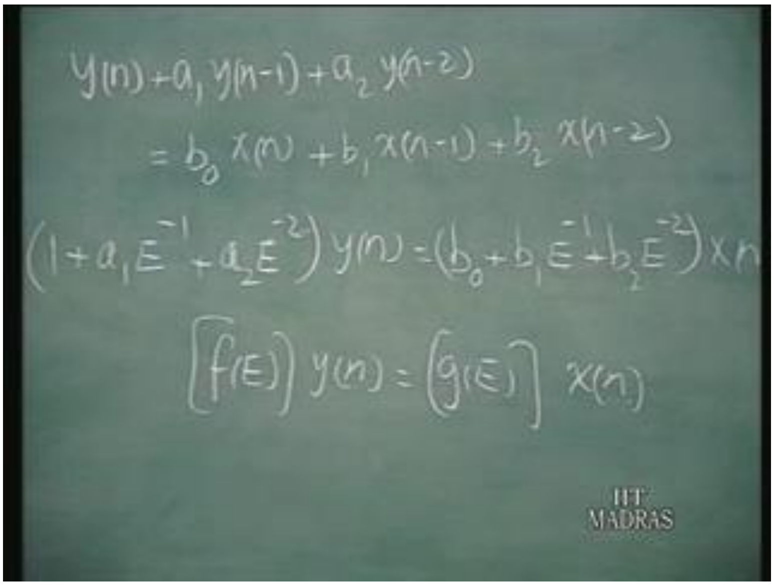

Suppose we have a second order difference equation yn plus a 1 yn minus 1 plus a 2 yn minus 2 equals b not xn plus b 1 xn minus 1 plus b 2 xn minus 2. Now, we can put this in the operator form, 1 plus a 1 e power minus 1 just as E advances; E of yn becomes yn plus 1; E raised to the power of minus 1 yn can be set to indicate yn minus 1. So, 1 plus a 1 E power minus 1 plus a 2 E power minus 2 times operating on yn, would be b not plus b 1 E power minus 1 plus b 2 E power minus 2 times xn. So, this operator polynomial can be written as fE yn equals gE operating on xn.

So, this is the second order difference equation in terms of the various coefficients in terms of the operator coefficients we put it in this fashion. So, we would now like to arrive at a discrete time system model in terms of the various coefficients a1, a2 b not b1 and b2 and arrive at this model.

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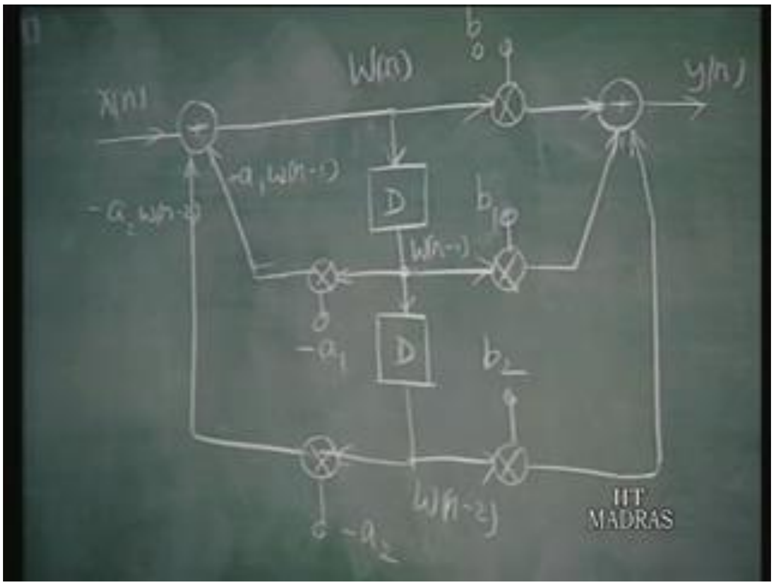

So, for this purpose let us look at this particular circuit discrete time system model, where we have a assembly of delay elements, adder units and coefficient multipliers. You have a summer here which act at 3 inputs and you have an intermediate variable here; suppose, I call this intermediate this signal here as wn and this is multiplied by b not this. So, if this wn this will become wn minus 1 this is the delay element wn minus 1 this is further delayed by another 1 unit; therefore this signal here wn minus 2.

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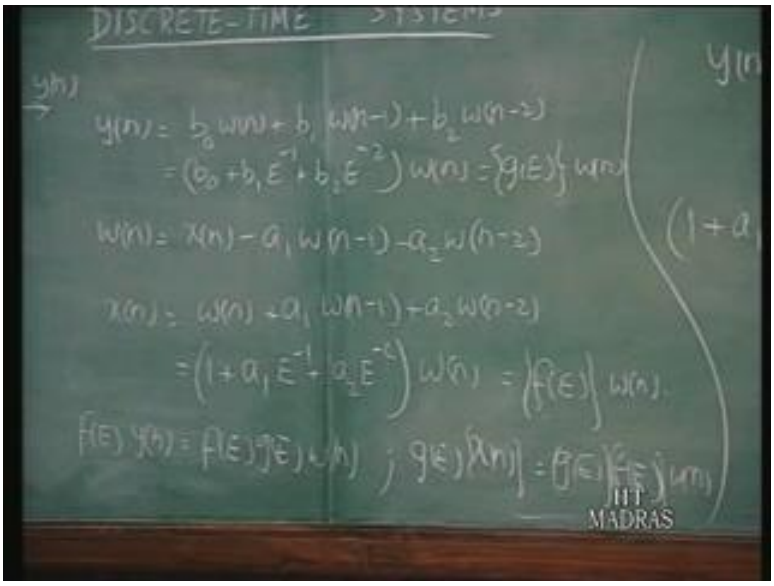

So, you have yn as b not wn plus b1 time's wn minus 1 plus b 2 times wn minus 2; all these outputs of these coefficient multipliers are fed to yn. Therefore; obviously, yn will be b not wn plus b1 times wn minus 1 plus b2 times wn minus 2; that is 1 equation pertaining to this adder. Now, let us look at this situation here the output of this adder will be xn minus a 1 wn minus 1 minus a2 wn minus 2 because the output of this is minus a1 times wn minus 1 and the output of this coefficient multiplier is minus a2 times wn minus 2.

So, the sum of these 3 signals is wn therefore, I can write this as wn equals xn minus a1 times wn minus 1 minus a 2 times w of n minus 2 or I can put this in an alternative fashion, xn as wn plus a1 times w of n minus 1 plus a2 times w of n minus 2. Now, let us see how this particular set up corresponds to the difference equation with which we started. Now, you recall here that in this difference equation a of E the operator polynomial is 1 plus a1 E power minus 1 plus a2 E power minus 2.

So, I can in same operator polynomials I can write this as, 1 plus a 1 E power minus 1 plus a2 E power minus 2 operating on wn and 1 plus a1 E power minus 1 plus a2 E power minus 2 we call that fE. Therefore, this will be fE operating on wn. Now, let us see here this can be written as yn can be written as b not plus b1 E power minus 1 plus b2 E power minus 2 operating on wn. And you see that, in the first in the original difference equation b not plus b1 E power minus 1 plus b2 E power minus 2 we call that ge; therefore this is gE operating on un.

So, we have this intermediate variable wn is related to yn and xn by these 2 operator polynomials. So, from these 2 equations we can say from these if I make fE operating on yn equals fE operating on gE operating on wn; that is what, we get from the first equation. Suppose, I make gE operate on xn gE operating on xn equals gE operating on fE operating on wn. So, if you look at these last equations last 2 equation fE gE operating on wn here you have gE fE operating on wn

So, this operator polynomial works same as algebraic expression. So, fE gE is the same as gE fE; that means, this term here and this term here are the same, which means; fE yn equals gE xn therefore, in this particular discrete time system we have the output and the input yn and xn are related in this fashion; fE yn equals gE xn; that is exactly the difference equation that we are trying to model therefore, this particular model represent this difference equation. So, once we have this difference equation we should be able to find out, the discrete time model in this fashion in terms of the coefficients of the second order difference equation we have; a1, a2; b not b1 and b2 we as we know that a not the coefficient of this can always be made equal to 1 and that is what we have done. So, in this the model can be established in this fashion.

Now, we have illustrated this for the case of a second order difference equation and its quite easy to see that, this can be extended to a higher order difference equation also; by extending this in further down by adding more delay elements and adding an array of coefficients of b coefficients and in a coefficients is the can easily be extended to higher order difference equations also.

Now, this type of model evaluation is useful in 2 ways; 1 is for hardware implementation of a particular difference equation. Once you have a difference equation and you would like to bring up a circuit in a laboratory, which simulates the difference equation this is how it can be done. In terms of delay elements, in terms of the coefficients multipliers, in terms of summers; now, irrespective of the nature of the physical variable involved the signals here are usually voltages. So, the voltage represents a particular variable which in turn or corresponding to the difference equation. That is hardware implementation this is useful; a second way in which this can be useful is, because this gives you a convenience in your programming in your arriving at the suitable program for the soft implementation of the difference equation.

So, here you see this is the input and this is the output; the input and the output are related by an intermediate variable wn. So, you can from given input you can generate wn by storing the samples n minus 1 n minus 2. So, from given xn sequence of samples we can generate wn, using this equation and using; that means, you have to store this samples of 1 instant previously and 2 instants previously relating to this intermediate variable.

So, it calculates step by step; once we have this output yn is obtained by the w the intermediate variable wn and the previous samples. So, this makes for convenience in storing the numbers and this is useful for software implementations. Because if you have to do this use this equation, in not only you have to store the values xn minus 1 xn minus 2 xn minus you have n minus 1 output y n minus 1 and output at n minus 2 also. So, this is convenient in some ways of programming for the software implementation. So, this discrete time models that, we have discussed here are useful in 2 ways for hardware implementation and also to give you a guidance a direction in which a programs can be obtained for software implementation.

So, let us now after this discussion of the models of the discrete time system proceed to a discussion of the z transform method, which is the most convenient 1 of solving the difference equations pertaining to the discrete time systems.

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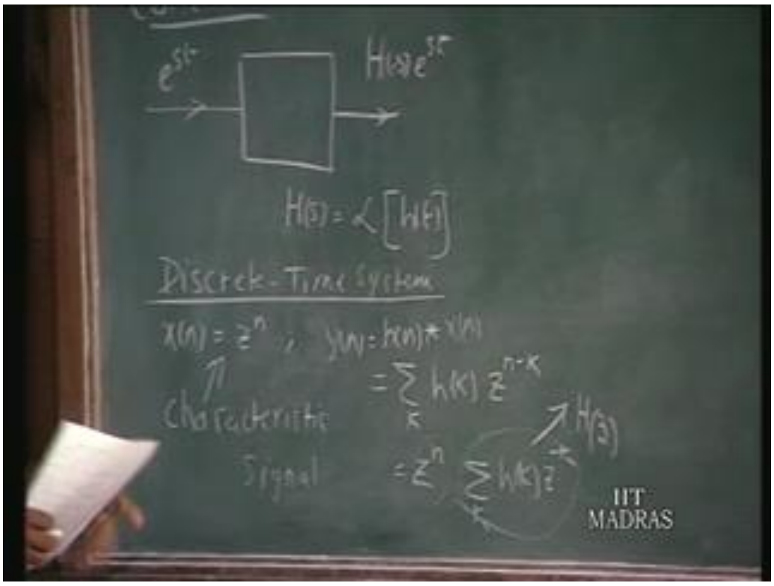

To introduce the z transform let us first of all introduce a possibility argument drawing on our knowledge of continuous time systems. In the case of continuous time systems we said that, if we have a characteristic signal e to the power of st then, the output the forced response is hs times e to the power of st this is a continuous time system, where Hs is the Laplace transform of the impulse response Ht. So, if for a characteristic signal here e to the power of st, you will get the output the characteristic signal multiplied by H of s where H of s is the Laplace transform of the impulse response.

For the discrete time system; suppose, the input xn is Z power n that is the characteristic signal and Hn is your impulse response then, we saw yn equals hn convolved with xn that is what you are having. Therefore, I can write this as hk because signal now is Z to the power of n. So, zn minus k summed on k and that I can write further as, Z power n taking outside summed on k hk Z power minus k. So, we observe that if the input is a characteristic signal characteristic signal the output is the same characteristic signal multiplied by a quantity like this. So, if in the continuous system the characteristics are multiplied by H of s e to the power of st; Zn is multiplied by the impulse response hk samples transformed in some fashion and producing you a function Hz.

So, we can think of this as a transform of impulse response hk; hk multiplied by Z minus k is the summation of all such samples is H of z and since just like H of s is the Laplace transform of H of t; we can consider h of z as a new transform of the impulse response of samples hk. And this provides you a kind of analogy and a motivation for making the z transform.

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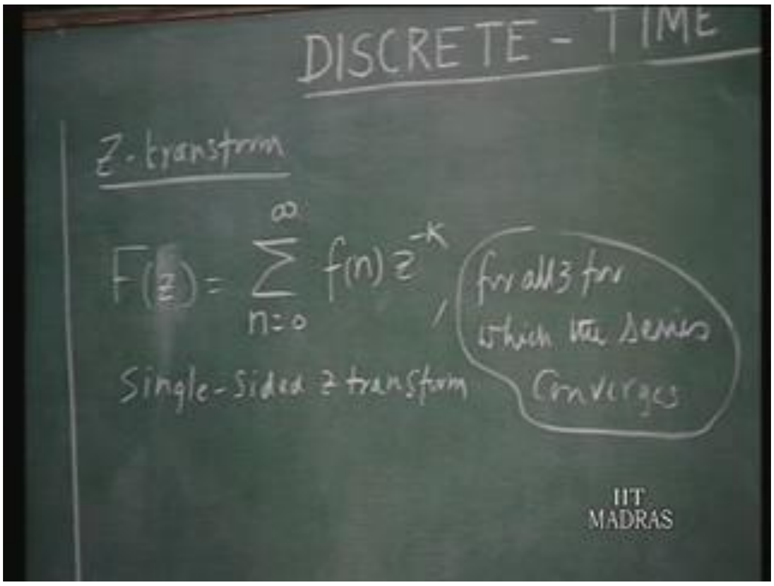

So, formally we can make the Z transform of a function fn will write this as a Fz as summation of the sample fn z minus k; usually we are lower case at this; Fz is fn z minus k. Now, this the summations we extend from 0 to infinity when you extend 0 to infinity this is called single sided z transform. We are usually interested in discrete time functions fn, whose values are important to us for positive values of n non-negative values of n therefore, single sided z transform is most important for us .And this is, this transformation F of z is called the z transform of this we will say for all z for which, the series converges.

So, just as you have the Laplace transform converging for certain values of s here also we have a region of convergence. And the region the complex plane for z in which the series converges this is called the region of convergence. We will see that later, what that region will be for different functions abbreviated usually as R.O.C region of convergence.

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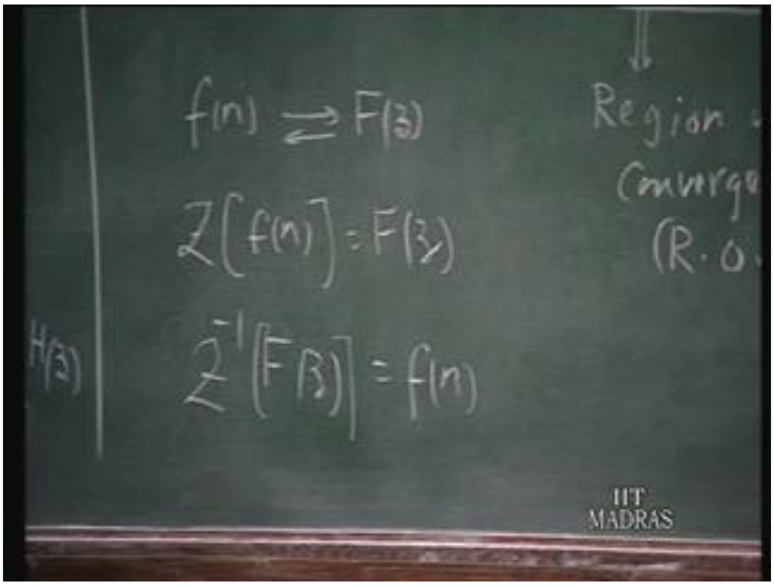

We have a short hand notation say fn and Fz form a z transform pair in discrete time function in this fashion. You can put an arrow like this, you can get recover 1 from the other you can also write Z transform of fn equals f of z and inverse z transform of f of z equals fn. These are all compact ways of expressing the discrete time and the transform pair now depending upon the value of fn there is a certain region of convergence and for single sided z transform like this usually this region of convergence turns out to be a region outside a circle of certain radius.

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So, this is the radius you are having certain say r. So, this is the region of convergence for a single sided z transform. So, the magnitude of z must be greater than some values R; for single sided z transform the region of convergence turns out to be the region outside a circle of certain radius and this R is a function of fn; R depends on f of n depends on the particular function that you are talking about.

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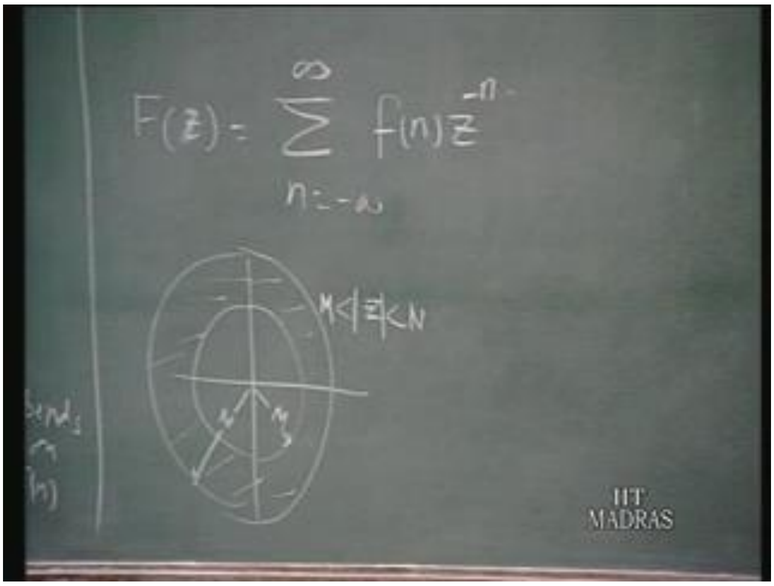

In addition we have a 2 sided z transform which we are not going to talk about much, but I will just mention this is in passing, where you take recognize the sequence of values of fn for both positive and negative n. Then F of z will turn out to be in this case, n from minus infinity to plus infinity of fn z minus n same z of course, we use it turns out that here, the region of convergence would be the angular region between 2 circles. So, if this is M; the inner radius is m the outer radius is n then the region of convergence from that is, this z must be larger than m magnitude and less than n. So, that is a region of convergence for a 2 sided z transform.

But in our work we will not use the 2 sided z transform, because that is not necessary for our purpose it is only to given here to give you some additional information, but 2 sided z transform. You must recognize that the region of convergence is the angular region. and 1 sided Z transform is a special case, where n extends to infinity.

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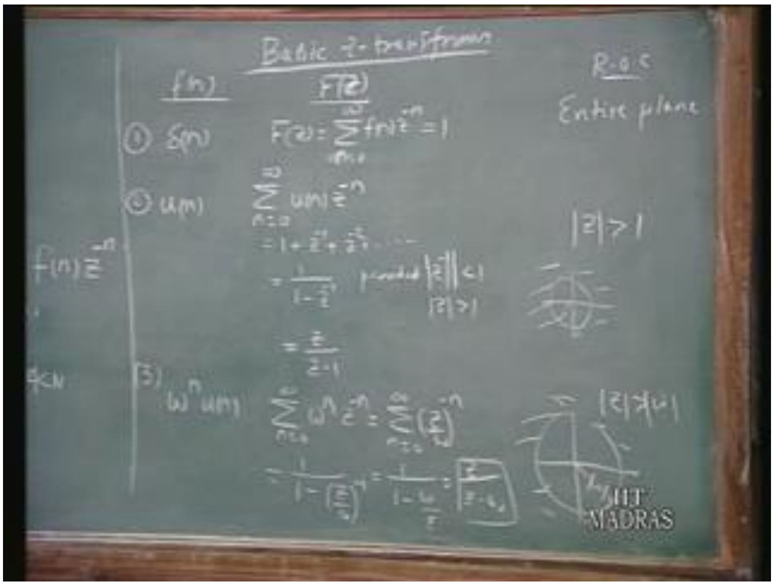

Now, let us look at the Z transforms of some basic functions, basic Z transforms 1 we will write here fn then, let us take delta n in unit impulse. The corresponding f of z of course, is you can write n equals 0 also fn zn; this is the def general definition and our particular input delta n has value only at 1 equals 0 and nowhere else And therefore, in this whole summation you have only 1 term which is non-zero; therefore, at n equals 0 this is 1 and z to the power of minus 0 is also equal to 1 therefore, the value of this is equal to 1.

So, it is very neat result that the unit impulse delta n has a z transform equal to 1; it is very similar to what we have in the continuous time system, where we have delta t a signal has a Laplace transform equal to 1 almost a similar result you are having here also. And the region of convergence after all whatever z you have this is the result is 2 therefore, no restriction entire plane. So, where z values can take any value does not really matter.

Second; suppose you take the unit step function un then, the z transform would be n from 0 to infinity, un z minus n and n equals 0 this is 1 and this is also 1 n equals 2 this is 1 this is z power minus 1 n equals 2 this I still 1 this is z power minus 2 and so, on and so, forth.

So, this is an infinite series 1 plus z minus 1 z minus 2 extending on up to z minus infinity. So, this can be written as, 1 over 1 minus z power minus 1 provided; z minus 1 is less than 1.The magnitude of that is less than 1 or z magnitude is greater than 1. So, as long as z magnitude is greater than 1 z power minus 1 has the magnitude less than 1 and this is the z transform of that. This can be more compactly put as more conveniently put as z over z minus 1. Now, the region of convergence for this as you can see that, z must be greater than 1 that is the region of convergence.

So, in the unit circle outside the unit circle is the region of convergence for this unit step function. Now, let us take the third case; suppose I take a complex number wn un, then the z transform for that would be wn and n equals 0 to infinity. In that range un is going to be equal to 1 therefore, I drop that out and z power minus n. So, this is equal to n equals 0 to infinity of z upon w raised to the power of minus n. And this has the same form as this series except instead of z; I have z over w therefore, this is 1 over 1 minus z over w to the power of minus 1 or 1 over 1 minus w upon z or z upon z minus w. So, this is the z transform of this and the region of convergence for this once again is z upon w must be greater than 1 or z must be greater than w.

So, here instead of here the unit circle, here the region of convergence is a circle of radius w outside the circle of radius w. So, that is how 1 can calculate the z transforms of basic discrete time signals, we will extend this to few other waveforms in a moment.

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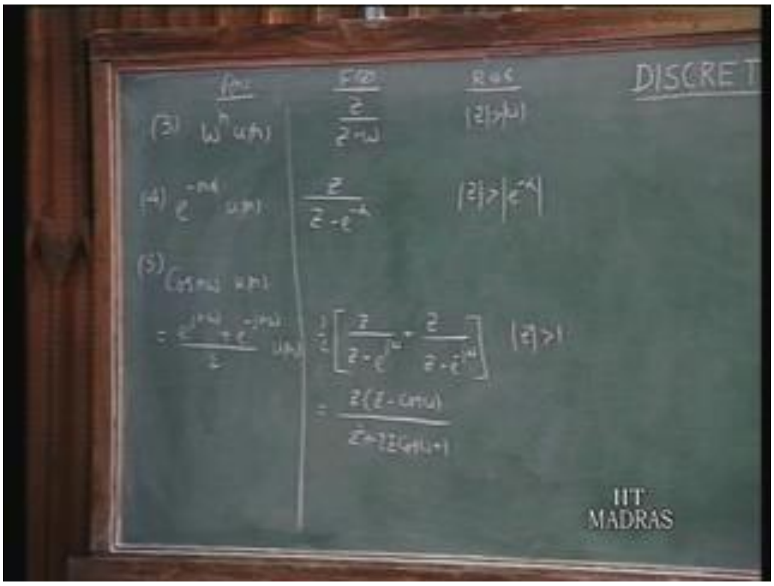

The last waveform discrete time signal for which we found out the z transform was this wn un a z transform z over z minus w region of convergence z magnitude is greater than w. We will extend this now; let us say, we have e to the power of minus n alpha un that is an exponential signal, decaying with a certain constant time constant E to the power of minus n alpha un. Now, we can use the earlier result to get at this after all e to the power of minus alpha is can be identified with w therefore, I can write this as z over z minus e to the power of minus alpha.

So, e to the power of minus alpha can be used in place of z and that is what you are having. So, the region of convergence is e to power of minus alpha if alpha is real of course; this is real quantity if alpha is complex you have to put this. There is no reason for e to the power of minus alpha not being complex because wn is a general complex number. So, we can now extend this to find out the z transform of cos n omega un. Now, cos n omega can be written as e to the power of jn omega plus e to the power of minus jn omega divided by two.

So, we can find out the z transform of e to the power of jn omega un and e to the power of minus jn omega un add them up divide by 2. Because z transform satisfies the property of the linearity, which is quite easy to show; now, compared with the earlier result instead of e to the power of minus alpha you have e to the power of z omega that is the only difference. Otherwise this fits into this or fits in within this therefore, for the first term you will put half here to start with, because of this for the first term e to the power of jn omega un the z transform of that will be z upon z minus instead of e to the power of minus alpha, I have e to the power of z omega that is the only difference e to the power of z omega.

For the second term you have e to the power of minus j omega instead of e to the power of minus alpha. So, z of minus e to the power of minus j omega. So, that is the only difference that you are having here. So, you can have a common denominator z square and minus z times e to the power of z omega plus e to the power of minus j omega; together, they can be written as minus 2z cos omega and the product of these 2 is equal to 1. Therefore, this is this is the denominator z square minus 2z cos omega plus 1 as far the numerator is concerned we have z square term here and a z square term here and z times e to the power of minus j omega z times e to the power of j omega.

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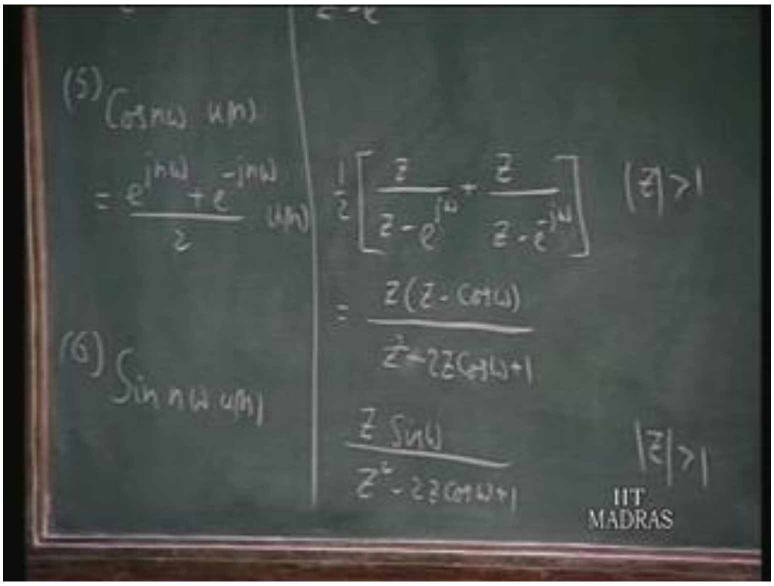

So, together they will give 2z times cos omega with a negative sign and because of this half, that 2 can be cancelled out and you get z times z minus cos omega. So, that is the z transform of cos n omega un and this is very important relation. Now, the region of convergence for that, z must be now magnitude must be greater than the magnitude of e to the power of j omega is of course, known to be 1. Therefore the region of convergence is the region outside the unit circle. Likewise you can establish the z transform of sine n omega un proceeding exactly the similar fashion except now, you have 2; 1 over 2j and a minus sign in front otherwise the treatment is the same. I will give the final result this turns out to be z sine omega divide by the same denominator z square minus 2 z cos omega plus one. So, that is the z transform of sine n omega un.

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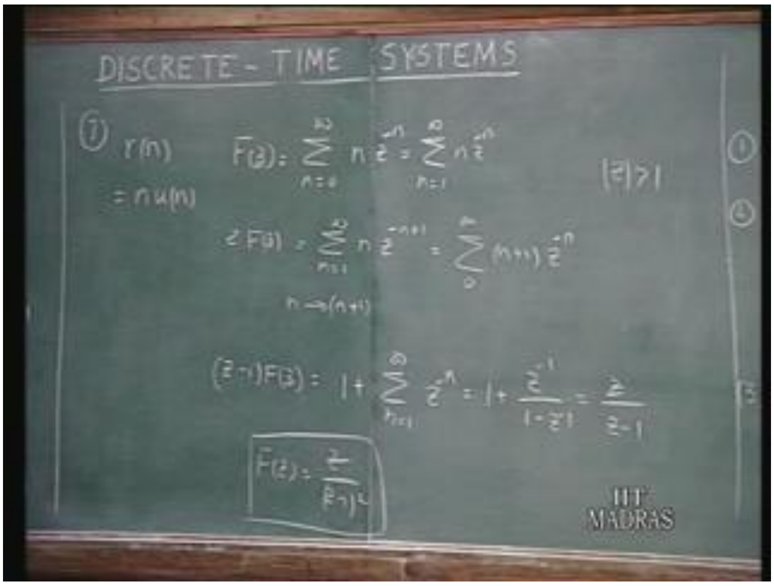

We will have 1 more let us find out the z transform of rn which of course, is n times un we can write the z transform for that, F of z equals n 0 to infinity of n times z minus 1; which can be written because n equals we can write n equals 1 to infinity of n z minus n. To find out the summation of this, we can we will follow a little trick little manipulation here; suppose, I multiply this z by f; 1z then, all I have is n from 1 to infinity of n times z minus n plus 1 that is what you are having.

Now, this is a summation with an index n from 1 to infinity; suppose, I change the because after all it is a dummy index once I have this summation done n disappears and you have purely a function of z that is what you get. So, I would like to replace n by n plus 1; suppose I do that put let n be replaced by n plus 1 then, I have n plus 1 then I have n is replaced by n plus 1 minus n minus n plus 1 therefore, I have z power minus n. And since the old n is replaced by new n plus 1. So, n is equal to 1 becomes this n equals 0 here because new variable here has a value 0 to infinity. So, here z f of z is 0 to infinity of n plus 1 z power minus n.

Now, suppose I subtract this from this I have z minus 1 f of z equals; I subtract this series from this. So, when n equals 0, this is 1 times z power minus 1 will be there and from then onwards, I have n from 1 to infinity n plus 1 minus n that is 1 times z power minus n. And that can be shown to be 1 plus z minus 1 by 1 minus z minus 1 and you can simplify this and show that is z upon z minus 1 and therefore, f of z equals z upon z minus 1 whole square z upon z minus 1 whole square. And the region of convergence for this can be shown to be the same as for the unit step function and the other sinusoidal function and. So, on

So, these are the basic z transforms the importance of the region of convergence comes because when you want to find out the discrete time function corresponding to a particular f of z it is important for us to know what the region of convergence is because it will enable us to expand the f of z properly in the form of a polynomial.

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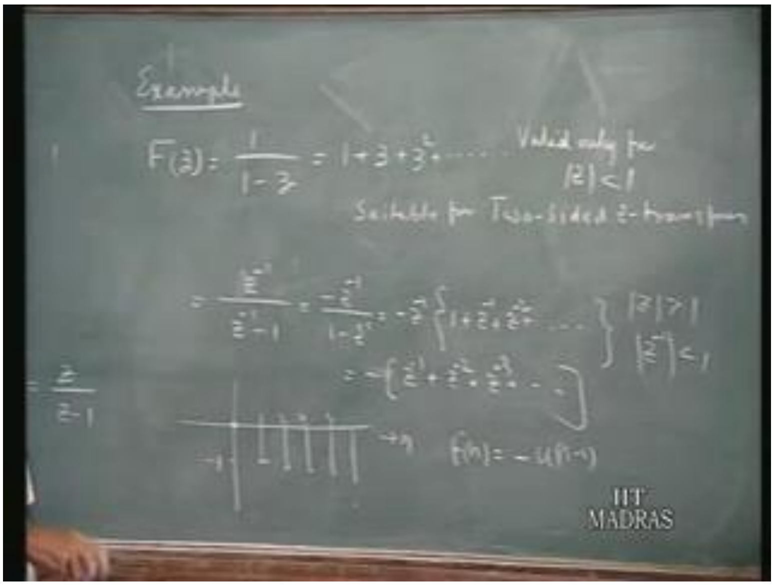

Example: supposes I asked you to find out the discrete time function corresponding to 1 over 1 minus z. So, if you have 1 over 1 minus z I can think of suppose I expand this 1 plus z plus z square like that and going up to infinity. This expansion is valid only for the magnitude of z less than 1 therefore; obviously, this does not pertain to a the z transform of a sing single sided z transform. So, this particular expansion is not valid for a single sided z transform it turns out that this is suitable for valid only for 2 sided z transform suitable for 2 sided z transform.

In fact, for a 2 sided z transform just like z minus 1 indicates a delay, z plus 1 indicates a shift in forward direction and we can interpret this suitably, but suppose we have shifted a 2 sided, 1 sided z transform then the expansion in this form is not valid; therefore, we must write this as suppose I multiply this by z minus one. So, z minus 1 divided by z minus 1 this is what we are having suppose I multiply both numerator and denominator by z minus 1 this is what you will be having. Then this I can write this as minus z minus 1 by 1 minus z minus 1 and now expand this minus z minus 1 one plus z minus 1 plus z minus 2 and so on and so, forth. And this can be written as z minus 1 plus z minus 2 plus z minus 3 etcetera all the term negative sign outside.

Since we know f of z f of n times z minus n; that means, minus 1 is the sample at n equals 1, minus 1 is the sample at n equals 2, minus 1 is the sample at n equals 3; therefore, the time function corresponding to that, will be like that 1, 2, 3, 4 all minus 1 and at n equals 0 there is no constant term that is equal to 0. Therefore, the f of n corresponding to that is minus of u of n minus 1 because this unit step delayed by 1 unit and also has a negative value. So, f of n corresponding to this minus of un minus one. So, the region of convergence now, dictate towards the type of expansion that you should have, because for a given F of z can be expanded in different ways the region of convergence tells you which the proper expansion that is necessary is because only when z is greater than 1 or z minus 1 magnitude is less than 1 is this expansion possible right now we have discussed in this lecture so far; the first of all the we talked about, the modeling of discrete time system using various elements like delay elements adders and coefficient multipliers.

So, then we introduce ourselves to the concept of z transform and found out the z transform of various basic discrete time functions. The next topic that we should take up is the property various properties of z transform just as we have taken the various properties of Laplace transforms, in our earlier discussion and 1 important property which we would like to discuss just now and leave it at that the others we will take up in the next lecture.

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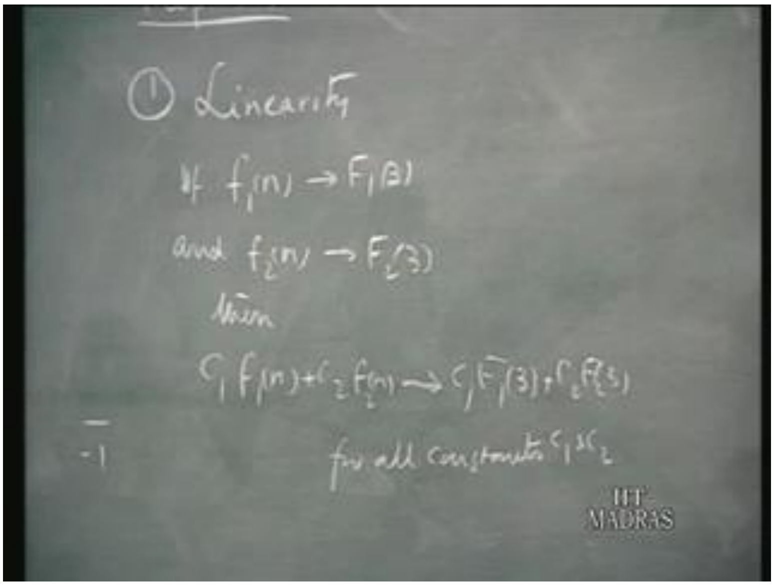

Linearity because this is very simple and a trivial property which we can always prove if f1 n has the Laplace trans has the z transform f1 z and f 2n has the z transform f 2 z then C1 f1n plus C2 f2n has the z transform C1 of f1 z plus C2 times f2 of z for all real constants for all constants C1 and C2 And in fact we have made use of this property in deriving the z transform of cos n omega and sine n omega, where we broke up cos n omega and sine n omega in terms of exponential functions and C1 and C2 happens to be 1 over 2 or 1 over 2 j as the case may be. This is a particular illustration of this; the second property, which I will just briefly mention we will discuss this later in detail later.

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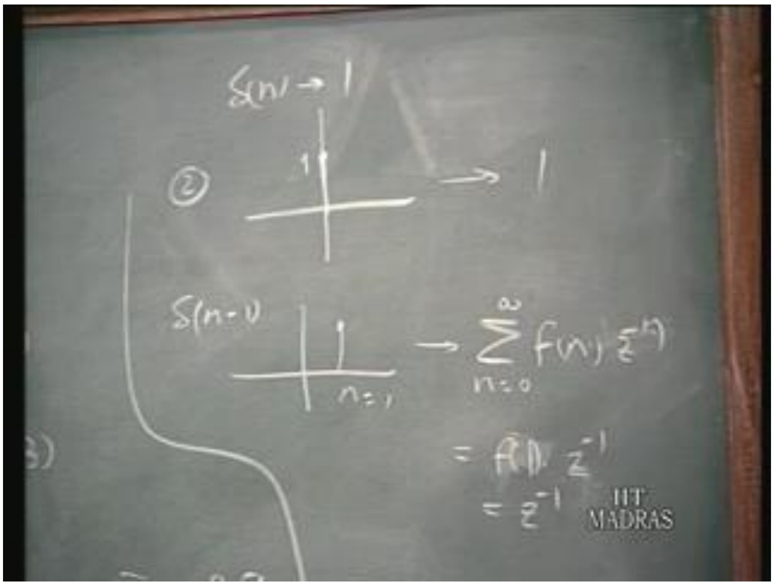

Suppose delta n has the z transform 1 as we have already seen; what is the z transform of a unit sample standing at n equals 1, that is delta n minus 1 its clear that the z transform of this is equal to 1, the z transform of this in the expansion fn’s z minus n gives a sample n equals 1 only no other way no other place. So, from n equals 0 to infinity all the other samples are 0 , this will be equal to f1 times z minus 1; f1 happens to be delta this is delta 1 delta this is 1; f1 happens to be 1 therefore, this is simply z minus one.

That means if you delay this impulse by 1 sampling instant 1 unit the z transform is getting multiplied by z minus one. So, a general rule therefore, is that if you delay a particular fn un suppose you are having and this is F of z if you delay all the samples, the sequence by k units fn minus k un minus k then, this will be z power minus k f of z; that means, every sample here in the original sequence is now occurring k units later.

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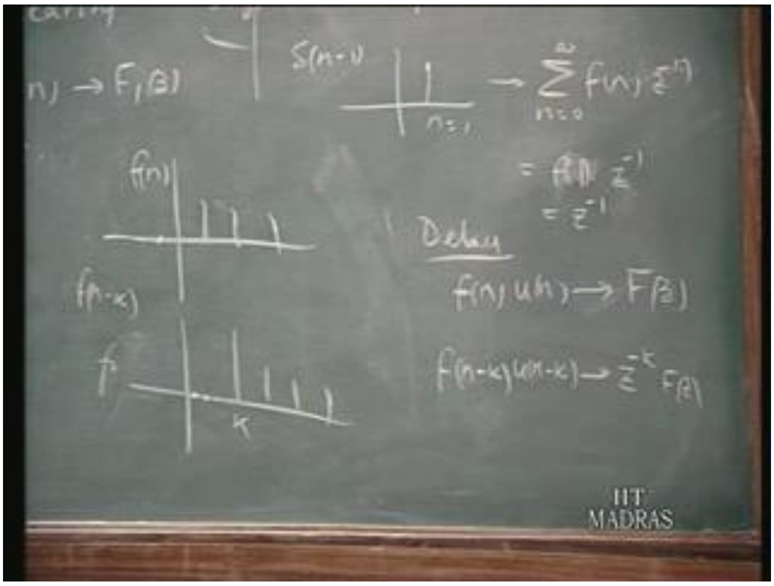

So, if you are having here fn now assuming here fn is a causal signal the same sequence of values occur k instance later. So, a particular sample if it was multiplied by z minus n in the earlier case, it is now became may get multiplied by z power minus of n minus k. So, every coefficient every term here is now multiplied by an additional term z minus k therefore, if fn is the case fn minus k will have this z transform. So, this is an important property delay of a causal signal.

So, if a; that means, when you delay this all this values will become 0; because earlier signal was causal signal therefore, when you delay this these up to k they are all delayed they are all 0. And once you have a casual signal delayed by k units the z transform gets multiplied by z power minus k. Now, what happens if this is not a causal signal suppose there are some sample values here, when you delay this you have some other additional values here what happens this will not be result will not be true; you will have some additional terms and that we will take up in the next lecture.

So, after discussing the after introducing our self to the concept of z transform and finding out the basic, the z transforms of basic functions or discrete functions of time. We have talked about, the linearity property and the property of the z transform when the discrete time function is delayed by k units the consequence in the transform domain is multiplication by z power minus k ; some additional ramifications of this result is what we will take up, first thing in the next lecture and from that point onwards we will proceed further with a discussion of properties of z transforms.

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