Networks and Systems
Prof V.G K.Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 04
Introductory Concepts – 4

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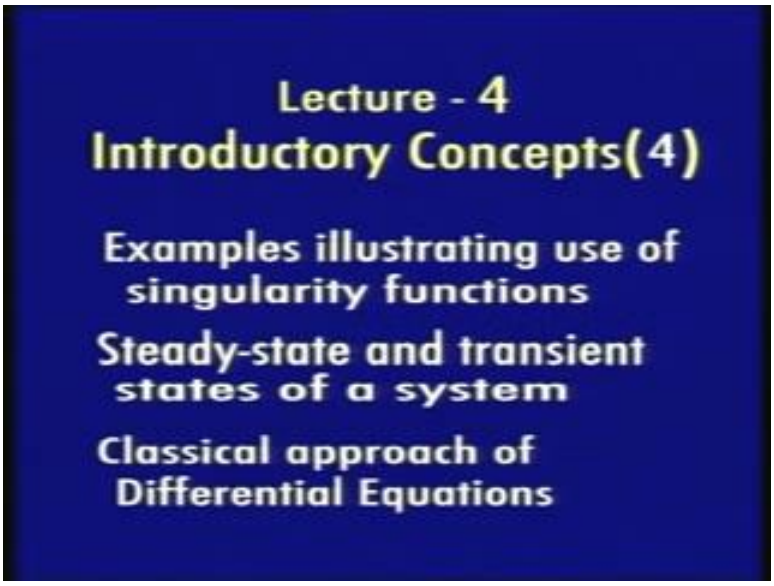

As a continuation of our discussion of the meaning and application of singularity functions let me, take an example suppose you are given this function f of t which has a discontinuity here. And also, discontinuous derivative and so on. F of t is described like this so, what is if f of t is like this. What is the meaning of f of t u t?

So, this function is being multiplied by u of t. That means, the negative values of the function for negative values of t are cut off and for positive values of t it is multiplied by 1. Therefore, f of t u t would be the reproduction of the same curve for positive values of time for negative values of time it is 0. So, this is exactly what will be f of t u t.if this is f of t this is f of t u t where the section for the negative values of time is cut off.

Now, suppose on the other hand if this is f of t i want to know, what is f of t minus 1u t. If this is f of t, f of t is a curve which is delayed by 1second. Therefore, it will have this curve this is minus 1and this break point occurs at 3 units and this is f of t minus 1. And f of t minus 1multiplied by u of t f of t minus 1has values only, for positive t multiplied by u of t will not disturb this.

Therefore, this will be f of t minus 1u of t. On the other hand, if i multiply f of t by u of t minus 1. What will be the result? If i multiply f t by u of t minus 1, i have u of t minus 1is a step starting at plus 1. So, at this point the step starts at and then, you are multiplying that this curve by u of t minus 1 therefore, you have at 1 the curve will be reproduced from t equals 2 onwards only.

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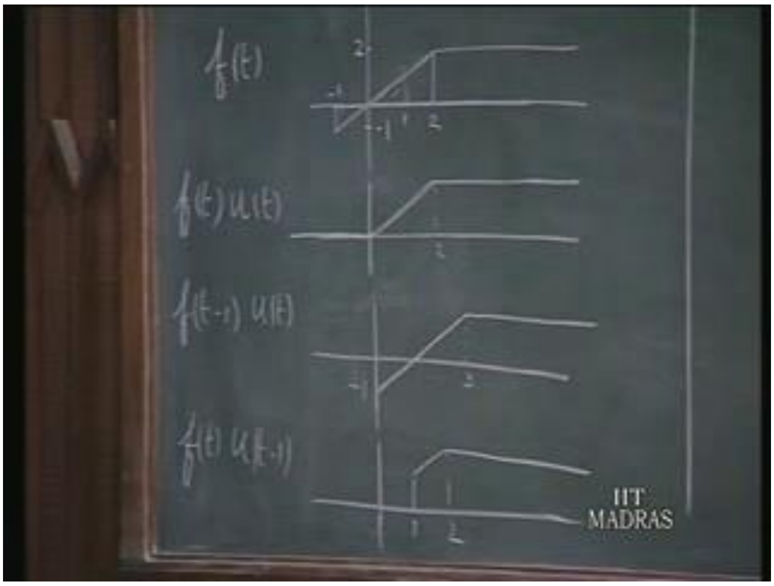

That means, only this portion of the curve gets cut off truncated. So, that is you see the notation f of t f of t u t f of t minus 1u t f of t u t minus 1. So, this is how you can understand the meanings of the various functions of time whenever, u of t minus 1 means if it starts from t equals 1 onwards, and you should be able to get have the ability to recognize the waveforms which when, the f of t is multiplied by step functions with delayed step functions and. so on and so forth.

Let me, take now we have seen 3 functions now U of t the ramp function r t and the delta function. How are they inter related? You see that, if i differentiate u of t i get delta t; delta t is d by dt of u t. Because, when you differentiate this derivative is 0 here and here at this point there’s sudden jump of 1unit and the area under this delta curve is 1unit. That means, when you integrate through the delta you get a rise of 1unit.

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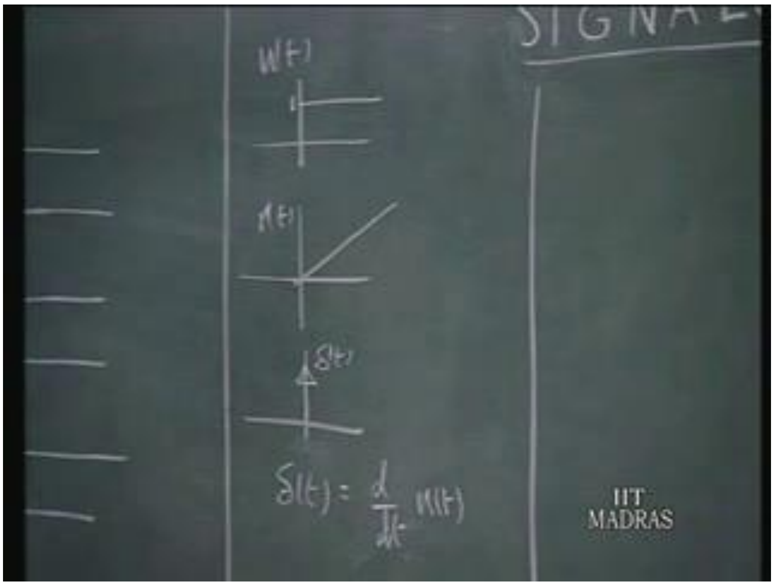

Therefore, u of t is the integral of delta t and delta t is the derivative of u of t. So, u of t can be written as from minus infinity to t delta t dt. So, that is how these 2 are related. In a similar fashion suppose, you have r t you take the derivative you get this step function. If you integrate u t you get the ramp function. So, d by dt of r t gives an unit step function and integral minus infinity to t of u of t dt gives a ramp function.

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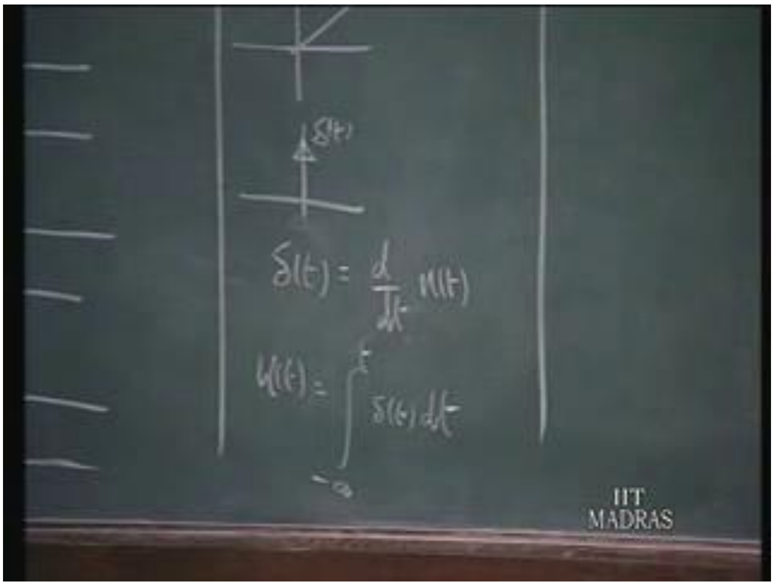

That means, starting with the ramp you take the derivative you get this, you take the derivative you get this or starting from the impulse you take the integral you get u of t, you take the integral of that you take r.

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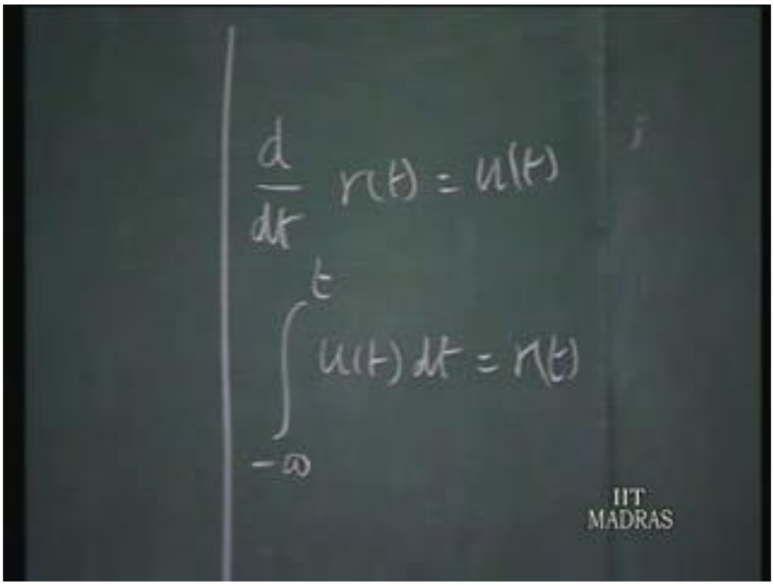

So, these are 3 are interrelated in this fashion. So, using this let me take an example suppose, i have a 4 volts d c source and connect this to a 2 ferret capacitor at t equals 0. I would like to find an expression for current in the circuit. You can see that initially, the capacitor voltage is 0. Let us, say vc 0 vc 0 minus before this vc is closed is equal to 0. If the capacitor is uncharged as soon as, you close the switch the capacitor by Kirchhoff's Voltage Law must acquire a voltage of 4 volts.

So, no way you can avoid it even though, the capacitor voltage is the same or continuous in the normal course of things, but here according to the rules of the game we you close the switch this 4 volts must appear across the capacitor. So, the capacitor voltage before t equals 0 is 0.Immediately after that it must jump to 4 volts. That means, 8 coulombs of charge must be dumped on the capacitor in 0 time.

So, the charge in the capacitor t equals 0 minus is 0. At t equals 0 plus mean, it is 8 coulombs. So, 8 coulombs must be dumped on the capacitor in no time at all in the interval 0 minus to 0 plus. That means, the current must be infinite, but the area under the curve of the current wave form which is really the charge must be 8 units. So, you can obviously, see that the current in the circuit is described by a delta function.

So, the current in the circuit i of t can be written as 8 delta t. So, because the area under the curve is 8 units that is it represents the charge that has been put on the capacitor in infinitesimal interval between 0 minus to 0 plus. So, i of t is 8 delta t. You can arrive at this result in a more formal way of course, we have physically argued that this should be the answer, but you can say that the capacitor voltage vc what is the expression for the capacitor voltage?

For time t less than 0 it is 0 for time t greater than 0, it is 4 volts once you close the switch. Therefore, vc can be written as 4 u of t. That is the voltage wave form across the capacitor.it will be like this. We know that, the current in the capacitor is c d vc dot i equals c d vc dt. That is the basic equation in terminal relation for a capacitor. Now, in this case the capacitor is 2 Ferrets.

Therefore, 2 times 4 times d by dt into u t. That is the expression for the current and we have just now observed d by dt of u t is delta t. Therefore, this is 8 delta t so, if you look at the waveform of the current in the capacitor it is a delta of 8 units magnitude sitting at the origin that is the expression for the current in the capacitor. Let me, take a second example: we have a voltage source vs connected to a resistor capacitor combination.

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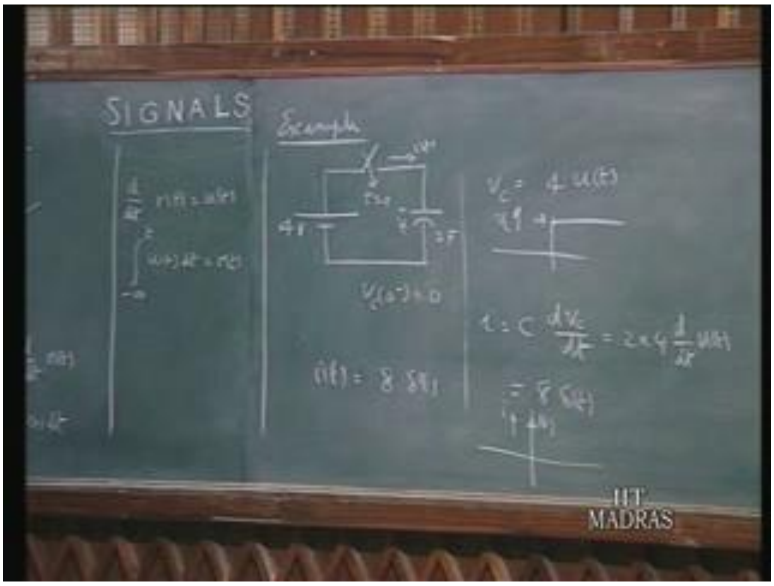

The capacitor is 2 Ferrets and the current i here. So, let the circuit we know the waveform of the Vc and we are asked to find out vs. So let, Vc be in this circuit be having a waveform like this. So, this is a triangular pulse t this is 2 Volts. So, we are given this waveform the function for the expression or the waveform for v c and you are asked to find out Vs. How do we go about it? Now, let us first find an expression for Vc. The plan of attack is once you find an expression for Vc c d Vc dt will give me the expression for the current. You have the voltage across the resistance plus, the voltage across the capacitance and that will give me the value Vs.

That’s how i proceed so, how do we find the expression for Vc? It is 0 up to t equals 0 and then, starts with a ramp which is has a slope of 2 units 2 Volts in 1second. Therefore, 2 r t would be an expression like this. At this point first of all the ramp the rate of growth must be arrested at this point which was earlier increasing. Therefore, you must introduce a negative ramp of 2 units of magnitude starting at t equals.

Now, if only you had only these 2 ramps then, you will have a growing ramp like this. At this time this growth is arrested by a negative ramp and the curve would have continued like that. If you add these 2 you would observe that, the resultant of these 2 would be something like this. But then, we would like to bring this down to 0. So, you must introduce a negative step of unit magnitude starting at t equals 1 in order to pull this down to 0. Which means, you must introduce in further a 2 ut minus 1.

So, the sum of these 3 quantities will describe this triangular pulse. That is your v f c Vc. Therefore, the current in this is c dVc dt and c is 2 Ferrets. So, 2 times the derivative of these expressions 2r t the derivative of that would be 2ut minus 2r t minus 1. The derivative of this 2times u of t minus 1The derivative of the ramp function is the step function. And then, minus 2times the derivative of a step function is an impulse function. So, delta t minus 1, that is your I and Vs equals Vc plus r times I which is equal to 1.

Therefore, Vc plus i the sum of these 2 Vc plus i will give me the value of the Vs. Therefore, you have combining these 2 4 u t a 4 u t here plus 2r t this 1 minus 4 u t minus 1 form this minus 2r t minus 1 from this 1 1 minute minus 4 r t minus 4 u t minus 1 There’s another 2u t minus 1 therefore, i must write here 6 u t minus 1 minus 2r t minus 1 and then, in addition you have minus 4 delta t minus 1.

That is the expression for this 4u t plus 2r t minus 6u t minus 1 minus 2r t minus 1 minus 4 delta t minus 1 that’s your Vs. If you plot this, Vs would have a waveform like this to start with 4u t. So, it starts with 4 till 0 or negative values of time and at that point of time it has a ramp with a slope of 2units. Now, it is going on increasing right up to t minus 1 right up to 1 so up to 1 this will build up to a value of 6 Volts. up to 1. At this point of time you have a negative going ramp of 2 units. Therefore, it means that whatever was increasing earlier is now pulled down by the same slope that means, if you added this function it should have continued like this. But you also, have a step of minus 6 so, instead of remaining constant here this would continue.

This will come down here by 6 units. Therefore, the sum of these first total 4u t 2r t 6u t minus 1 minus 2r t minus 1 would have been just like this. This would have been the result, but we also have a delta function of minus 4 units here. So, in addition to this we have a delta function minus 4. So, that would be the shape of the Vs curve. So, all these expressions describe this particular waveform.

So, you observe now that whenever, you have waveforms of this type with discontinuities and discontinuous derivatives 1 can use the singularity functions with advantage to describe the corresponding waveforms. In fact, we now in all these discussion we did not have to say this is the for t equals less than 0.

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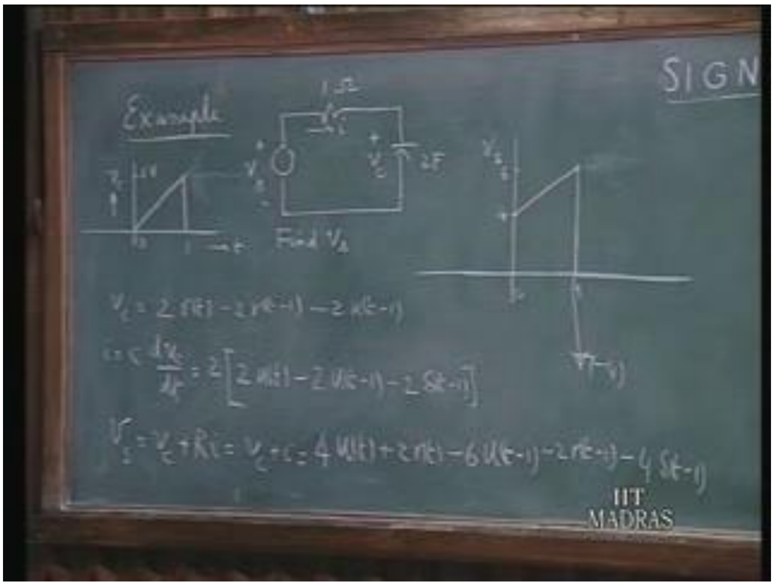

This is the expression for t between 0 and 1, this is the expression for t greater than 1. We have 1 single expression valid for the entire range and this constitutes the central advantage of describing these functions with the help of singularity functions. After having familiarized ourselves with the different types of standard signals 1 has to deal with normally in system analysis.

Let us, now introduce ourselves to some elementary system concepts which will be found useful later on in our in this course. Generally, when we are talking about the analysis of systems we would like to assess the dynamic performance of the systems. And in this context it should be useful for us to remind ourselves of what a steady state of a system means and what the transient state of a system means.

When, the variables of a system or a network are either constant d c values or are regularly repeating periodic functions of time. Then, the system is said to be under steady state conditions. Under sinusoidal circuit theory you deal with systems under steady state conditions, where all the voltages and currents are periodic functions are time which are sinusoidal in character.

Similarly, in a d c circuit you deal with steady state conditions when, all the voltages and currents are constant. So, once again the system is said to be under steady state when the signals characterizing the system are either constants or regular periodic functions of time. The best way to define the transient state of the system is to say, the system is under the transient state when, it is not under steady state. There is no other simpler way of doing that.

So, in general on the account of a switching process you open a switch or you close a switch or some system parameter suddenly changes. Then, the network transits from 1 steady state to another steady state and the intervening period in the transition between 1 steady state to another steady state is referred to a transient state. And it is the study of the transients which forms the central theme of the analysis of networks and systems that we are going to take up.

Strictly speaking most of the transients take infinite time to die out, but in practice as for the engineering approximations are concerned within relatively short time which is 3 or 4 times the time constants involved. We can assume the transients to decay to negligible proportions. Let me, take an example to illustrate this idea suppose, i have a 10 Volts. d c source connected to a r c circuit with an initial capacitor voltage of 4 Volts. and i would like to find out how the capacitor voltage behaves for time t equals 0 onwards, if the switch is closed at t equals 0.

Now, the capacitor voltage Vc to start with is 4 Volts ultimately, it will reach a voltage of 10 Volts. It gets charged to this 10 Volts therefore, the capacitor voltage the steady state in a capacitor is 10. That is the steady state voltage of the capacitor, but at the time of closure of the switch the capacitor voltage is 4 Volts. And there's no reason for that voltage to change because, there is a resistance here and that absorbs any shock that comes in between this is 10 Volts, this is 4 Volts, this 6 Volts. is dropped across the resistance.

So, initially the value of the capacitor voltage must remain as 4 Volts. Therefore, you have a transient term which is minus 6 and from your elementary knowledge of transient analysis this will be t over c r. So, this is an exponential term which decays with time. Therefore, Vc of t is 10 minus 6 e to the power of minus t cr, cr is the time constant of the circuit.

This is the steady state part and this is the transient part. So, in going from 1 steady state to another steady state in going from 4 Volts steady state to a 10 Volts steady state you have a transient arising. So, the capacitor voltage can be this is 10 Volts, this is d c steady state new steady state of course.

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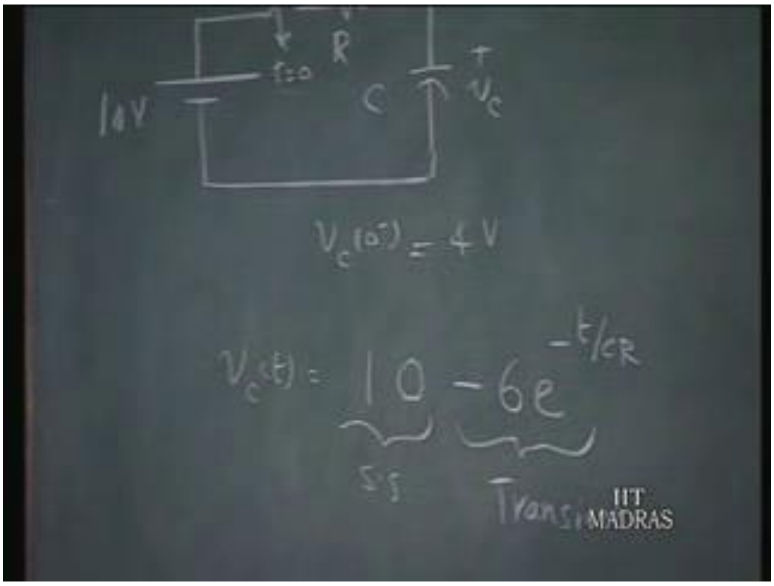

(refer time: 20:12) as 4 Volts and then, you have a transient term which has an initial amplitude minus 6 and decays like this. So, this is the transient component of the capacitor voltage. The sum of these 2 will give rise to a curve like this is the actual capacitor voltage. So, you recognize in this process a steady state value and a transient part and the transient component arises because, the old steady state does not match the new steady state.

So, therefore there is a transient which bridges this gap In order to solve for this transients you have different approaches available. The classical differential equation approach is the most fundamental method. But it is somewhat complicated, but nevertheless since, it is a fundamental approach. Because, most of the equations of the network are written in terms of differential equations.

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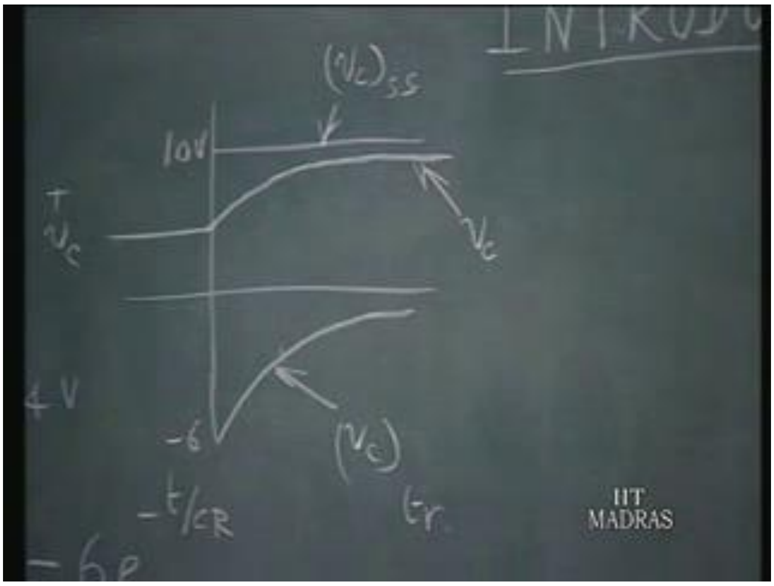

Therefore, it is a fundamental approach and since, you have studied differential equations in your mathematics courses, it would be convenient for us to see how the differential equation approach can be used to study transients in networks of this type. So, we will illustrate we will use this idea through an example and then, later on we will show that some of these associated with the solution of transients through differential equations approach can be overcome using operational methods like, Laplace Transform methods at a later point in the course.

But to get a feel for the advantages of Laplace Transforms we must know, what the difficulties are in the differential equations approach. So, let us take an example. Let us, consider this particular circuit to illustrate the differential equation approach. We have an inductor and capacitor in the circuit and the inductor and capacitor provides the initial conditions that are necessary to solve the problem and normally, the initial conditions are given pertaining to a situation before a closure of a switch.

So, let us say i l 0 minus before the closure of the switch is 0 Amperes and the capacitor voltage before the closure of the switch is also 0. And let us, also say that the source voltage is 12 Volts d c. to solve for this particular solution to solve for the circuit. Let us, take the loop equation approach we have 2 currents i1 and i 2 in the 2 loops. We write down the differential equations pertaining to the 2 loop currents to start with and then, let us say that in this particular problem you are interested in finding out i1.

Find i1of t for t greater than 0 so, this is the solution. So, let me write down the differential equations for the 2 loops. You have for the first loop r1 have 1 plus l times d by dt of i1minus i2 must be equal to Vs and this can be written as, r1 plus ld derivative operator operating on i1. Because, derivative operator d is equivalent to d i1 by dt. D times i1 is d i1 by dt minus l d operating on i2 equals Vs.

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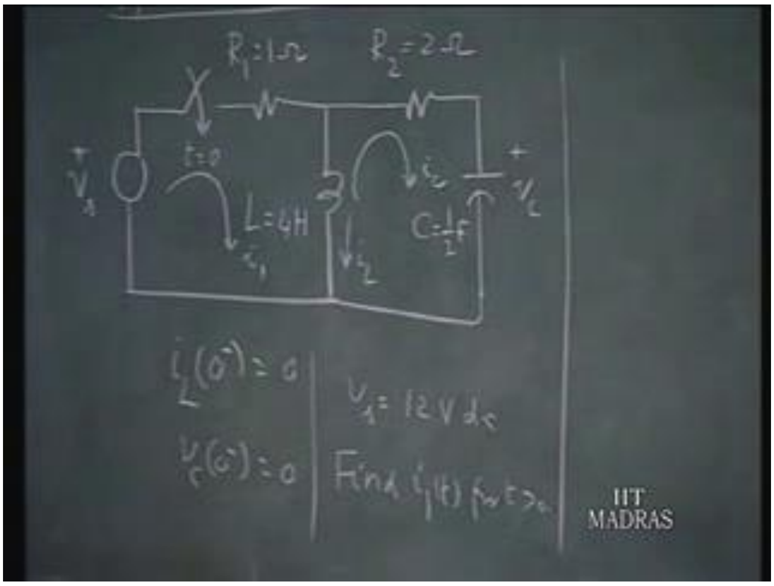

That’s the first equation where, by definition d by definition is d up on dt. Similarly, d square by definition is d the second derivative of the particular quantity. Now, for the second loop suppose, you take the voltage drops in the direction of the loop current i2 We have r2 times i 2 1 over c d that is the integral of the current i2 divided by c. That’s the voltage Vc plus d by dt l times d by dt of i2 minus i1 is the voltage drop across the inductance in this direction.

So, if you write that respective terms you get r2 plus 1 over c d plus l d operating on i2 that is equal to 0. Because, this 1 over c d indicates the integral of i2 dt 1 over c d i2 means 1 over c integral i 2 dt.

That’s the voltage across the capacitor so, that is what we are having. Now, we are interested in solving for i1 Therefore, we have to eliminate i2. If they are purely algebraic equations would have multiplied by suitable factors and we would have eliminated i2. The same thing can be done here also, recognizing these are derivative these are operator functions, but we can manipulate them in the same way as the algebraic factors.

So, you multiply this factor by l d and this factor by r2 plus 1over c d plus l d and eliminate i2 in the process. And substituting numerical values you get for this particular circuit eliminating i2 we have 6d square plus 5d plus 1 operating on i1 equals 2d square plus d plus 1 operating on Vs. This is the final differential equation that you have connecting the source voltage and the response quantity i1 that you are interested in.

In system representation, we can put this as an input quantity Vs and the desired output i1 represented by this fashion. This is characteristic of the type of differential equations we will get in all these problems. So, this can be put in this fashion, an operator function f d operating on i1 equals another operator function g d operated on Vs. whereas, this is the input this is the output. The output and input are relation are corrected by a differential operators of this type f d i1 operating f d operating on i1 will be g d operating on Vs.

So, this is characteristic of all single input, single output systems. For the solution for this in the classical differential equation approach is done is 2 parts. The complimentary function solution this is part of the solution. So, i1 complimentary will consist of 2 terms depending up on the roots of the Auxiliary equations.

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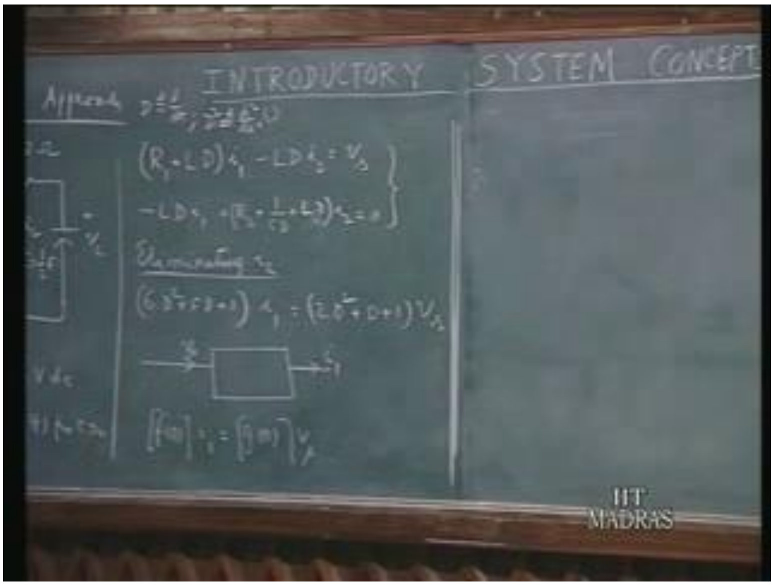

So, the Auxiliary equation we have here is 6m square plus 5m plus 1 equals 0. This is the 6d square plus 5d plus 1 is 0. We substitute use m for d and this is what, we get as Auxiliary equation and the values the solutions for this will be m equals minus half and minus one-third. So, the complementary part of the solution will have an arbitrary constant a multiplied by e to the power of minus t by 2 plus another arbitrary constant b e multiplied by minus t by 3.

So, this is the complementary part of the solution which is familiar to you through your elementary course on solution of differential equations. Now, we have something to say about this now, this complementary solution whatever, you are getting is also termed the ports free solution or the natural response of the system natural response. Because, this portion of the response the characteristic of this response this t by 2 and t by 3 minus t by 3 will remain there no matter what source you take what forcing function you take.

Whatever, forcing function you take that portion will remain the same. Therefore, this is inherent to the system and this is a natural response and this equation is called the characteristic equation. Now, it is called the characteristic equation because, of the fact that no matter what variable you wish to choose to solve for in the circuit whether, you want to solve for i1 or i2 or the capacitor voltage or the inductor voltage. Whatever, you want to do the characteristic equation will remain the same.

This you will get; that means, this f of d you get the same f of d no matter what you take as the response quantity and. So, this is something which is characteristic to the system and it is independent of the forcing function that is why it is called characteristic equation. And the roots of the characteristic equation are the natural frequencies minus half and minus one-third are called are the natural frequencies of the system.

So, the complex frequencies present in this particular signal are minus half and minus one-third. They happen to be real in this case, but in terms of the general concept of complex frequencies these are the 2 values minus half and minus one-third. They are the complex frequencies for of the system. They are called the natural frequencies because, they are inherent to the system and they arise irrespective of the forcing function that you are having.

So, this is the complementary function solution which is obtained from solving the characteristic equation, Auxiliary equation. We have to of course, solve for a and b because, they are arbitrary constants. They depend up on the initial conditions, but before we proceed further we need to have the particular integral solution. The particular integration solution from your elementary study of differential equations will be obtained as g of d by f of d operating on the forcing function Vs.

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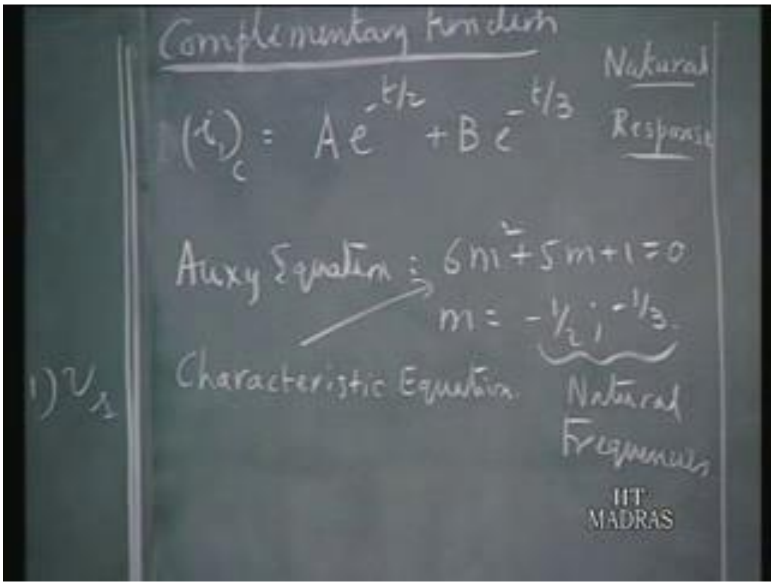

This is an operator function we know that, if Vs is of the form say e to the power of alpha t then, the particular integral solution is obtained as g of alpha divided by f of alpha a e to the power alpha t. That's the operator d is replaced by simply the factor alpha in the exponent that is how it goes. Now, in our case Vs equals 12 Volts d c. Therefore, we can take 12 e to the power of 0 times t alpha is 0.

Therefore, the particular integral solution i1 particular integral solution is g 0 divided by f 0 multiplied by 12. If you substitute d equals 0 in this equation you have this f 0 is 1 and g 0 is 1. Therefore, this is 12 particular integral solution is 12 Amperes. Now, this is also called whatever solution you get here this is also called forced response.

Because, this is the response which depends up on this particular input that you are having. This is the Vs forces this kind of response through the differential equation and if complex frequency is present in the forced response, it will be exactly the complex frequency that was present in v of s. If v of s is a has a single complex frequency alpha then, this also will have the solution will also have the complex frequency alpha.

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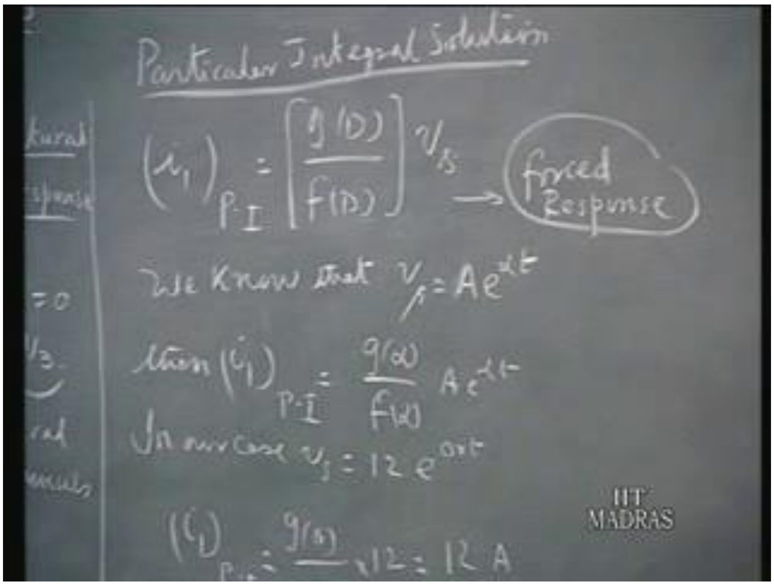

If this has a complex frequency s, this will have complex frequency s. If this has a variety of complex frequencies s1, s2, s3, s4 then, i1 particular will also have all those complex frequencies. So, the force part of the solution has the same complex frequency that was present in the input waveform. Whereas, the complementary solution will have complex frequencies, which are inherent to the system.

They are called the natural frequencies now, we have got the particular integral solution as well as the complementary function solution. So, the total solution is i1 equals a e to the power of minus t up on 2 plus b e to the power e minus t up on 3 plus 12. This is the natural part natural response this is the forced response. Now, that we have got the total solution the form of the total solution we have to evaluate these arbitrary constants a and b for that we need to know the initial conditions, from the initial conditions you should be able to find out a and b.

We need 2 conditions so it is convenient in these situations given by 2 conditions on i1 we need to find out i10 plus and the derivative of i1 evaluated at 0 plus d i1 by dt evaluated at 0 plus. You can think of nay 2 equivalent conditions any 2 conditions dependent conditions, but these come naturally and they are convenient to calculate i1 0 plus and i1prime 0 plus.

Now, how do we go about it? It is the initial conditions and their reactive elements which provide a link between the situation before the switching operation and after the switching operation. They form the bridge between the conditions in the circuit before the switching takes place and after the switching takes place.

So, it is ultimately related to il 0 and Vc 0.We assume that, Vc 0 plus is the same as Vc 0 minus normally, the capacitor voltage is continuous unless there are impulse currents which we see later. Normally, we can assume this to be true and there's no reason for the capacitor voltage to change suddenly as far as this circuit is concerned and il 0 plus is the same as il 0 minus.

So, we can say that Vc 0 plus is also 0 volts and il0 plus which is really i1 0 plus minus i2 0 plus. That il equals i1 minus i2 this is also 0. So, this is the information that is given to us and from this information we should be able to manufacture the values of i10 plus and i1prime 0 plus. How do we do that? Now, let us see at t equals 0 plus you have this 12 Volt source and you have r1and r2.

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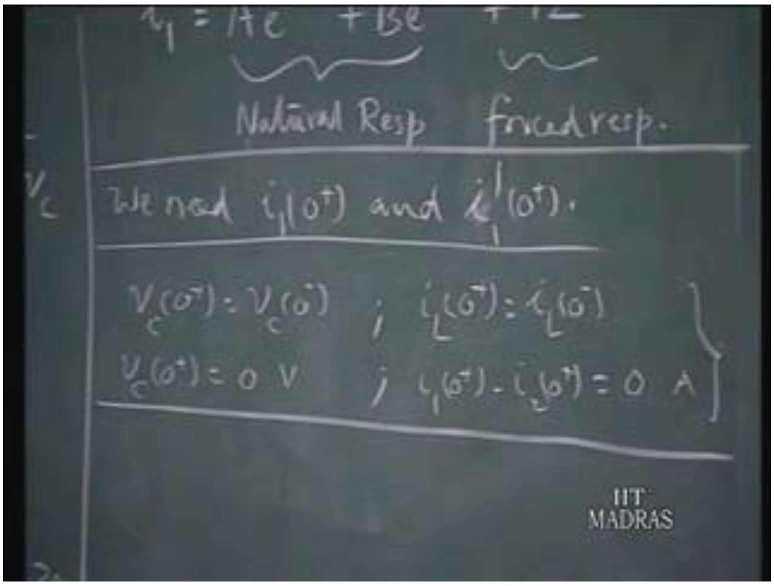

The current in the capacitor inductor is 0. Therefore, that is almost an open circuit and the capacitor is a short circuit because, the voltage across the capacitor is 0. So, you have r1 1ohm r2 2 Ohms. So, the current i1 here is 12 divided by 1 plus 2 3 Ohms. Therefore, i 10 plus is 12 up on 3 that is 4 Amperes. That is the initial value of the current 12 up on 1 plus 2 4 Amperes. Now, how about the derivative of the current at t equals 0 plus.

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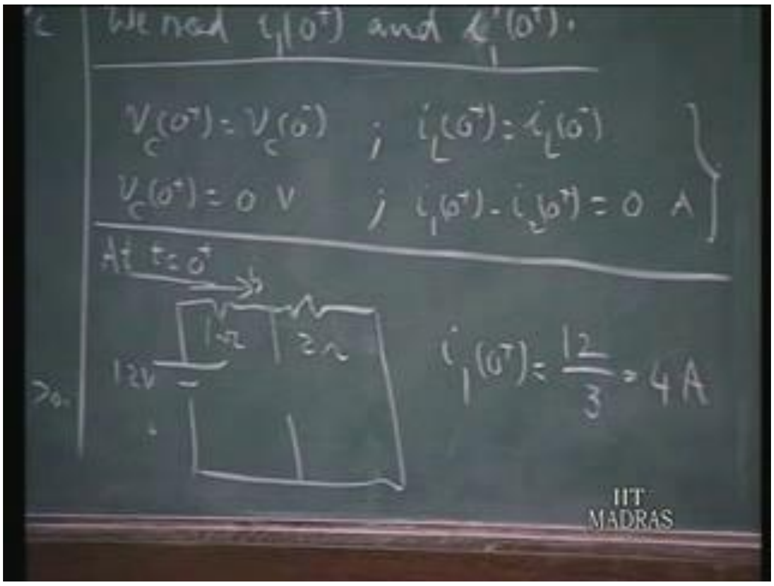

Now, to that we have to some kind of manipulation from this circuit we can see that, Vs equals i1 r1 plus i2 r2 plus Vc. This voltage equals i1 r1plus i2 r2 plus Vc for t greater than 0. Let us, take the derivative of this is of course,12 Volts. All the time this is 12 for t greater than 0. Take the derivative of this, Derivative of this constant is 0.R1is 1 Ohm. Therefore, i1 prime 0 plus 0 does not matter at any time t this is i 1plus and then, this is i 2 plus and i2 prime times r2 which is 2 Ohms plus d vVc dt that is what you are having. I 1 plus i1prime plus 2i 2 prime plus d Vc dt r1 being 1 Ohm and r2 being 2 Ohms right. Now, this is true for all values of time greater than 0 at t equals 0 plus this is 0i1 prime 0 plus 2 times i2 prime 0 plus what is the value of d Vc dt at t equals 0 plus.

The current in the circuit the current in c here at t equals 0 plus is 4 Amperes. This is 4 Amperes so, what does it mean? That C dVc dt that is the current in the circuit at t equals 0 plus equals 4.

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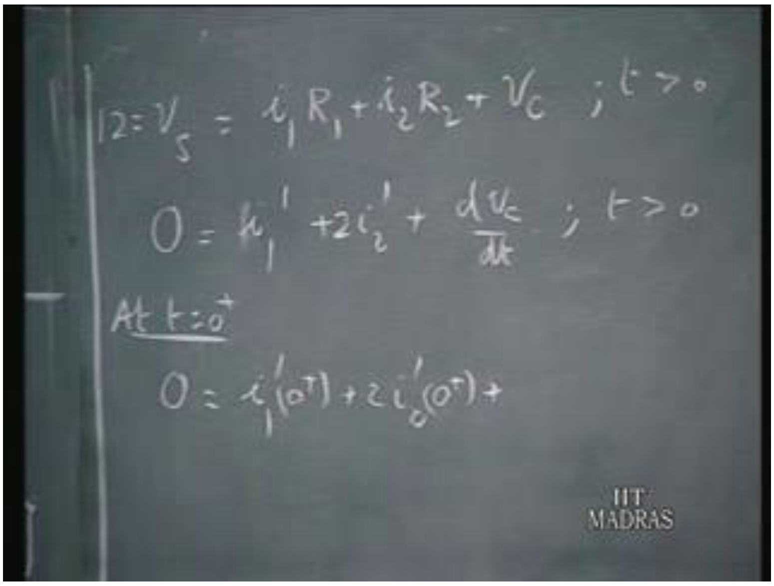

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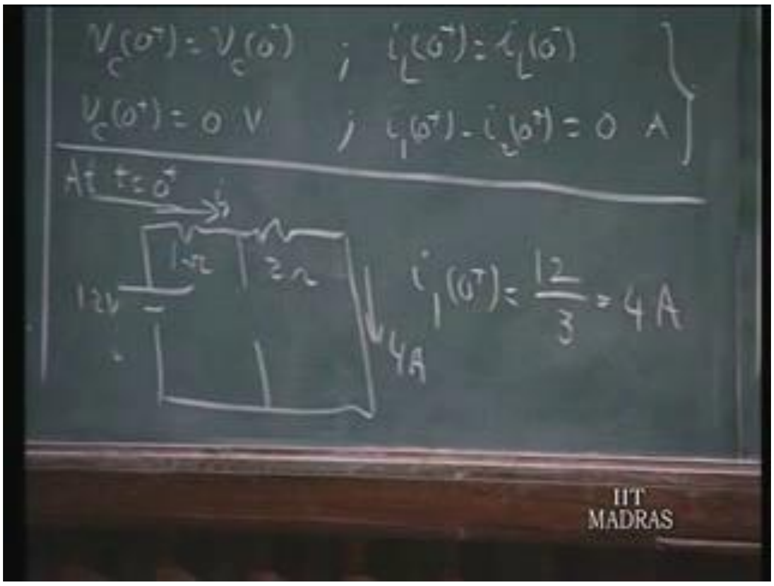

This is i2 0 plus.i2 0 plus is the same as i1 0 plus from this we observe that dVc dt at t equals 0 plus dVc dt 0 plus 4 divided by 2 which is equal to 2C equals half a Ferret, C equals half a Ferret. Therefore, this is equal to 8 Volts per second. Therefore, this substitute dVc dt is equal to 8. So, this will lead me to a result i1 prime 0 plus 2i 2 prime 0 plus equals minus 8.

Now, we need to have another equation this is not enough. Now, we need to add another equation in i1 0 plus i1 prime 0 plus and i2 prime 0 plus. What is the voltage across the inductor? VL equals l times di1 by dt plus di2 by dt minus di2 by dt.That’s what we are having

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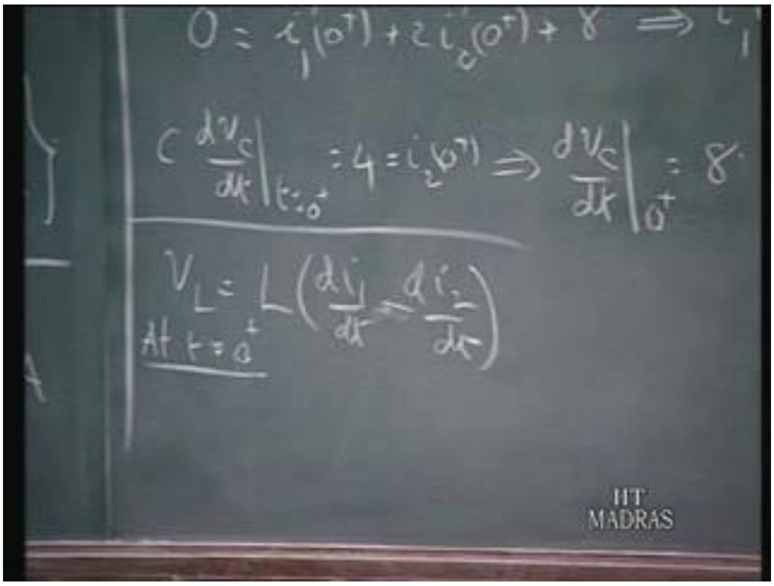

At t equals 0 plus what is the value of the voltage across the inductor. This is equal to VL 0 plus. What is the value of the voltage across the inductor at 0 plus.2 Ohms and a current i1 of 4 Amperes. Therefore, 8 Volts the capacitor voltage is 0 so, at t equals 0 plus this is 8 Volts.

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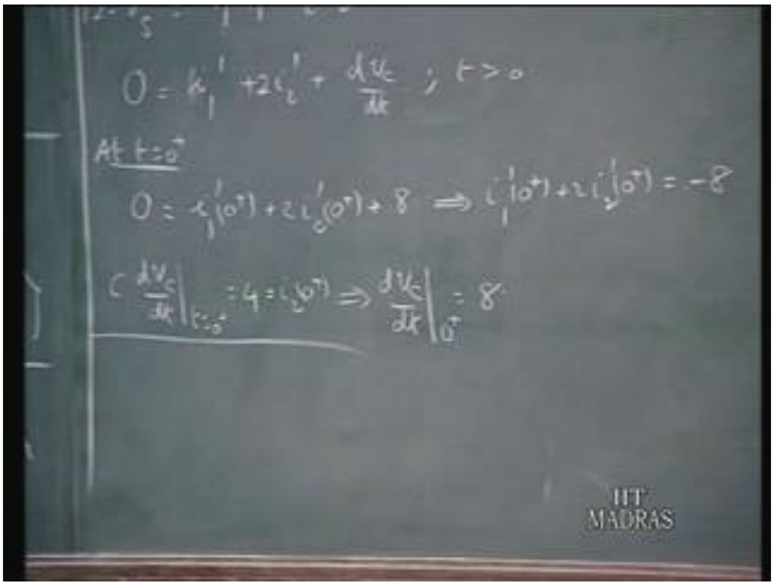

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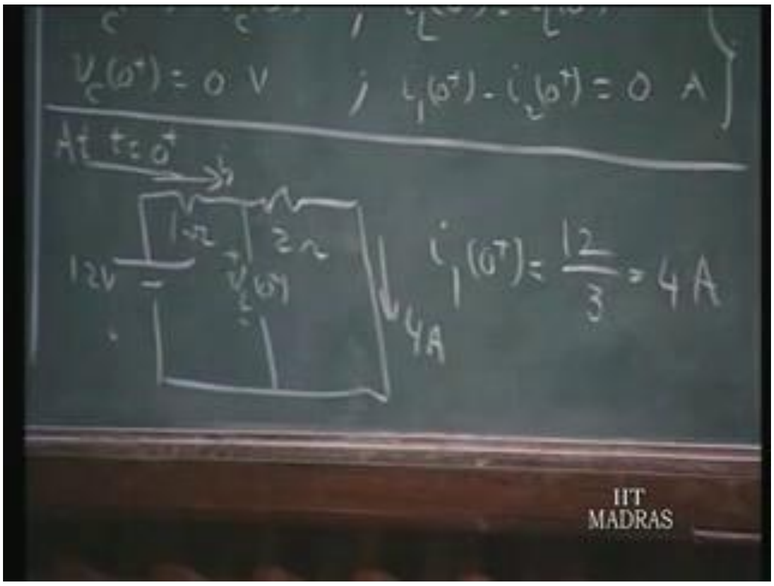

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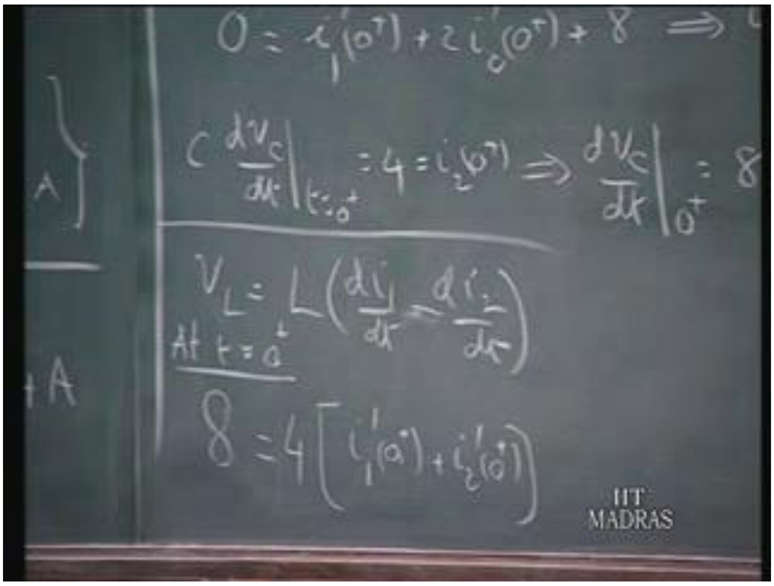

L equals 4 Henrys and this is i1 plus i1 prime 0 plus i2 prime 0 plus. So, you have another equation from this i1 prime 0 plus minus this minus i1 prime 0 plus minus i2 prime 0 plus equals 2. So, you have 2equations.1 is i1 prime 0 plus 2i 2 prime 0 plus is minus 8.

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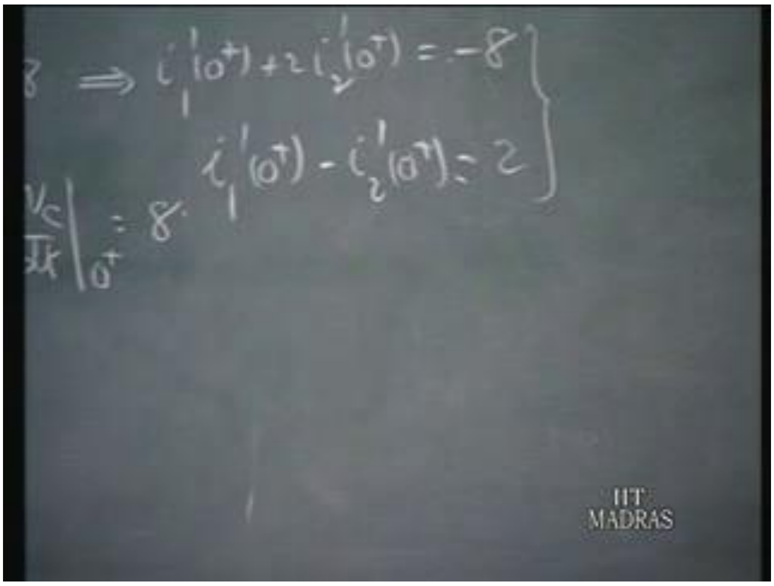

The other equation is i1 prime 0 plus minus i2 prime 0 plus is 2 and the solution for this from these 2 you get i1 prime 0 plus as minus 4 up on 3 Amperes per second. So, you have i1 0 is plus 4 Amperes i1 0 prime is 4 minus 4 by 3 Amperes per second.

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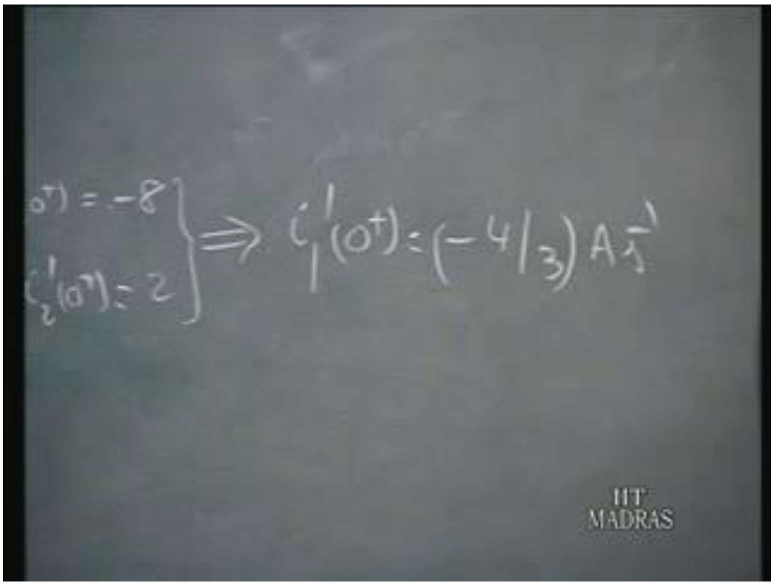

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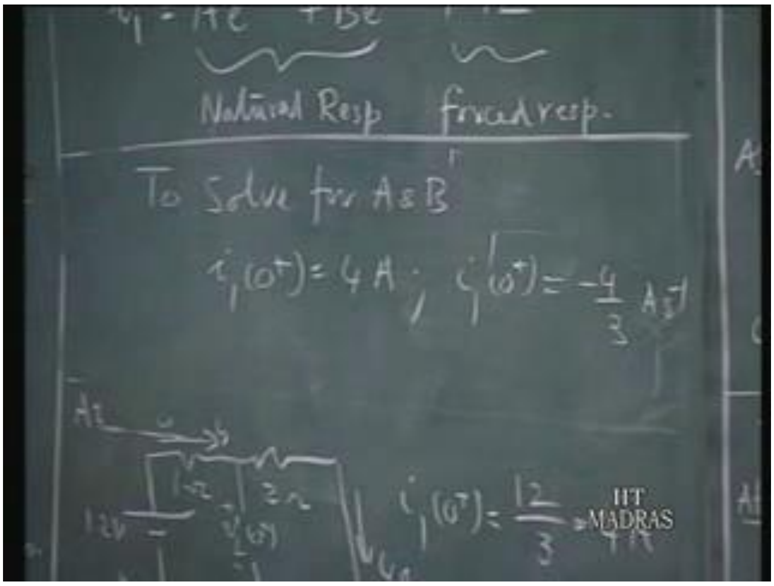

So, you use this information to solve for a and b So, to solve for a and b we use i1 0 plus equals 4 Amperes and i1 prime 0 plus equals minus 4 by 3 Amperes per second. You substitute these values in this expression straight away this value. I will take the derivative of this and time put t equals 0 plus and substitute this value. You solve for a and b you will get, the solution a equals 24, b equals minus 2.

And finally, you get the solution as i1 of t equals 24 e to the power of minus t up on 2 minus 32 e to the power of minus 3 up on 3 plus 12 and this is the solution valid for t greater than 0.That is the final solution. So, let us see what we can gather from through the example. In the differential equation approach, you have 2 partial solutions.

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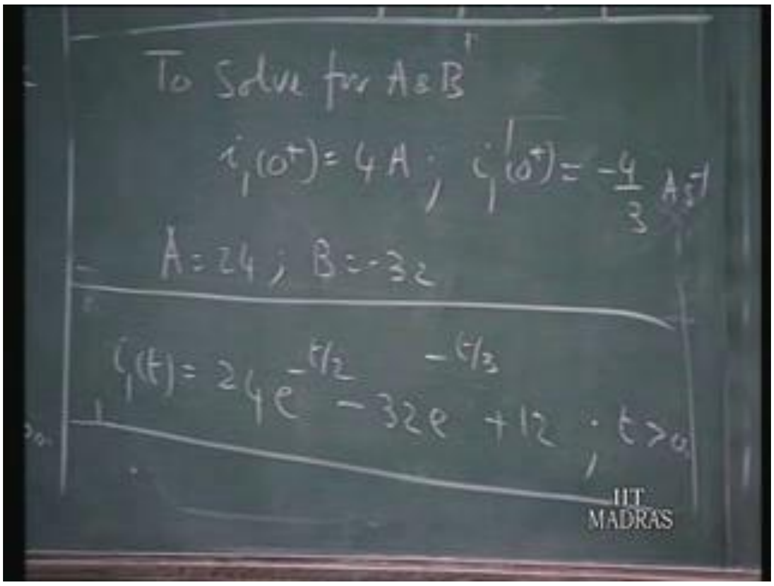

The complementary part of the solution and the particular integral part of the solution, the complementary part of the solution is obtained through, the solution of the Auxiliary equation or also what is called the characteristic equation. And this characteristic equation is independent of the particular response which we have choosing. It depends up on the system as such the composition of the system and the various elements.

The natural frequency, the frequencies that are present in the complementary solution are what are termed the natural frequencies. They are in general complex, in our particular case it may have been real, but in general complex. So, these complex frequencies present in the source free response of the complementary solution are called the natural frequencies.

This is characteristic of the system it does not depend up on the particular forcing function. On the other hand, the particular integral solution is what is called the force response for the solution and it has all the frequencies which are present in the excitation function or the input function. The sum of these will be the total solution and in solving for these we need to have information about the initial conditions of the quantity which you are going to solve for.

In mathematics generally, when you are solve the differential equation these initial conditions for the reason of interest given to you, but in problems of this type we do not have those initial conditions given to you straight away on a platter. What you are given is in a circuit like this some initial conditions pertaining to the regime before the switching operation.

From this you must find out the new initial conditions appropriate to t equals 0 plus pertaining to the particular quantity which you are interested in whereas, the initial conditions are given in terms of the capacitor voltages and inductor currents. So, this requires a lot of manipulations as we have seen and this is really 1 of the troublesome aspects of the classical approach of classical differential equation, approach of the solution of the problems.

Because, if you have imagined that you have a n'th order differential equation here luckily we have only 2 reactive elements. Suppose, you have got 5 reactive elements the differential equation would have been of fifth order and you need to have 5 initial conditions pertaining to any quantity to which you have to solve for not only the initial value, but the first 4 derivatives.

Even for first derivative we had so much trouble. So, you can imagine the type of trouble which you would have had if you had to find out the initial value of the fifth derivative. So, this is really a problem. So, in the classical solution you need to find out the initial conditions appropriate to the particular quantity which you want to solve for and this requires a considerable amount of manipulation.

In this particular example what you have taken, we assume the capacitor voltages and the inductor currents are continuous which is normally the case, but there may arise situations where, the capacitor currents capacitor voltages have to jump from 1 value to another from 0 minus to 0 plus it may have to undergo a jump. And similarly, the inductor currents may have to have a jump from 0 minus value to 0 plus value.

As we have seen in an example when a battery is connected to an uncharged capacitor immediately the voltage has to jump. An impulse current has to exist. So, we may even in the evaluation of the initial conditions it may not be always necessary that Vc 0 minus is equal to Vc 0 plus and IL 0 minus may not necessarily be equal to IL 0 plus. So, such cases also merit our attention.

In the next lecture we will take an example or 2 of where such special initial conditions occur and then, continue with our discussion of some of the elementary concepts that are needed for analysis of systems which we will make use of in our later work.