Networks and Systems
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Lecture 39
Discrete-Time Systems (2)
Introduction
Solution of Difference Equations.
Impulse response and convolution summation

Having, studied the basic characteristics of discrete time signals and discrete time systems. Let us today, take up the question of solution of the difference equation of a linear time invariant system; through the classical method of solution of such equations. So, in the classical method of solution of difference equations.

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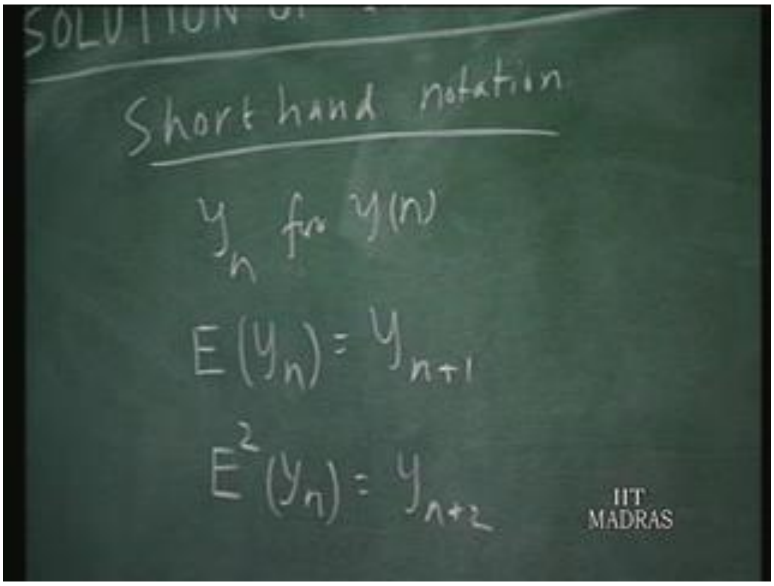

Let us, use some short hand notation let us simply write yn for y of n. So, that it becomes simpler to write secondly, we use operator E operating on yn equals yn plus 1; just like we use D derivative operator in differential equations. Let us use the symbol E; E operating on yn increments the argument by 1. So, E yn is y n plus 1 suppose we write E square yn it means; this is same as y n plus 2 and so on and so forth, So, this the short hand notation we will employ.

Let us illustrate the method of solution of the difference equation by, the classical approach through an example and I will also indicate what we do in a general case on this side along side.

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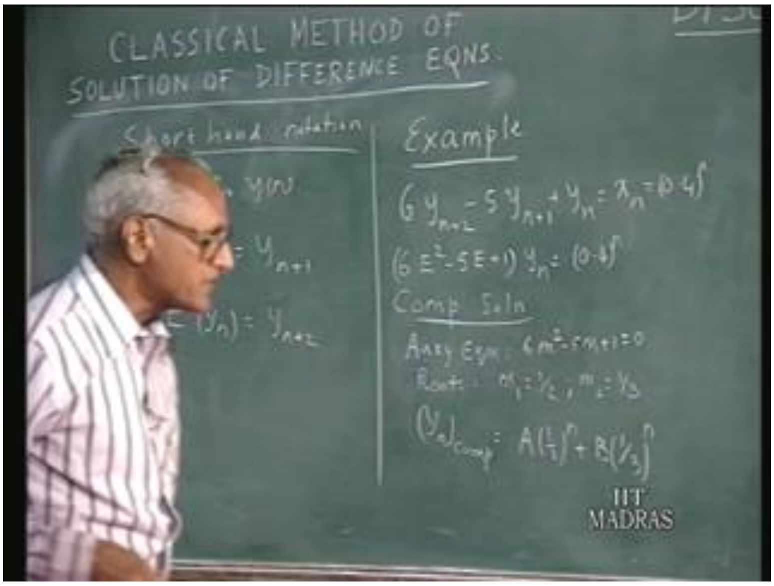

So, let us take an example, in this example: we have 6 yn plus 2 minus 5 yn plus 1 plus yn is a forcing function xn which is point 4 raised to the power of n. Now, in using the short hand notation, we can write this as 6 E square minus 5 E plus 1 operating on yn equals point 4 raised to the power of n. Now, for the solution of this we follow 2 different steps just as, we do in the case of differential equations we find the complementary solution and the particular solution.

So to find the complementary solution that means; the solution of the homogenous equations 6 E square minus 5 E plus 1 times yn equals 0 for that, we form the auxiliary equation by substituting M for E. So, we form the auxiliary equation which is 6 m square minus 5 m plus 1 equals 0. The roots of the auxiliary equation are we have 2 roots m 1 half m 2 one-third these are the 2 roots of the auxiliary equation. Once, we have this the complementary solution yn complementary is written down as A times an arbitrary constant A times half raised to the power of n plus B times one-third raised to the power of n.

You recall that, in the differential equation case if the auxiliary solution auxiliary equation of roots m 1 half and one-third would have written A to the power of half t plus

B to the power of one-third t; instead of that in the difference equation case, we have half to the power of n and one-third to the power of n to put this general framework let me also give some remarks for the general case.

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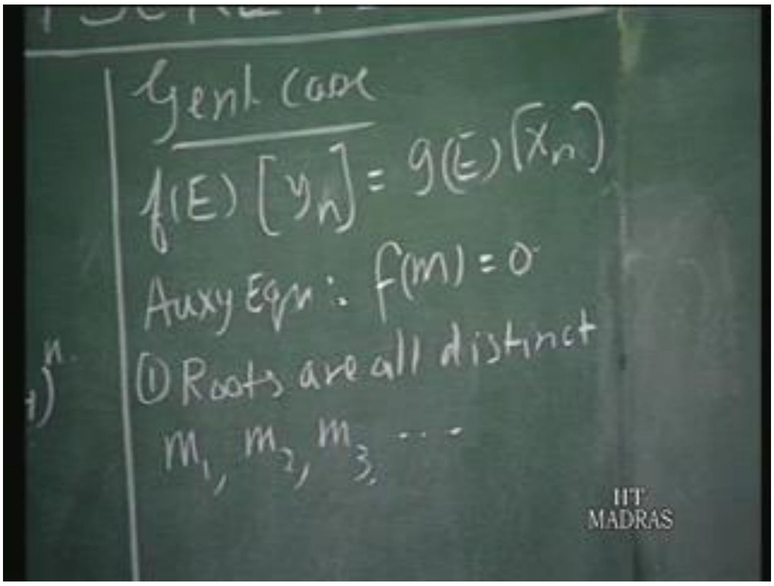

In the general case we may have f of E operating on yn equals a function gE operating on xn that would be a general form of this equation.

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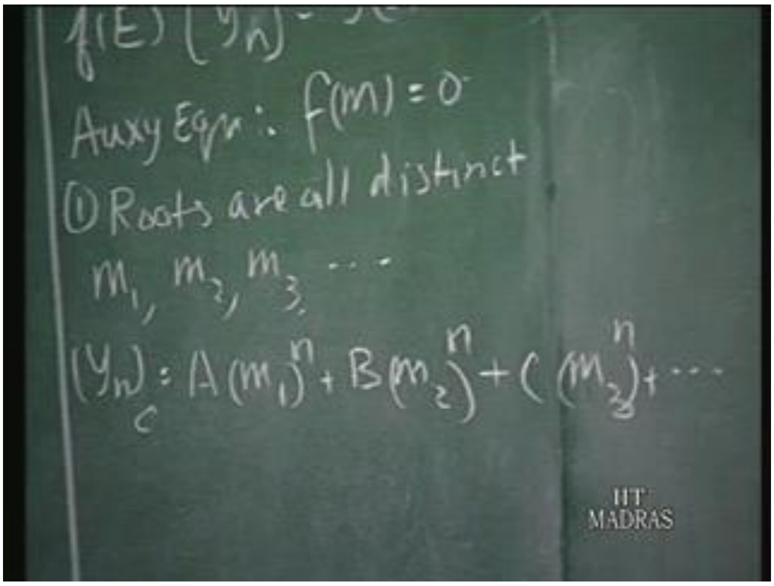

Then; the auxiliary equation would be fm equals 0. Now, we have 2 cases 1 all roots are distinct the roots of this auxiliary equation suppose, they all are distinct say m 1, m 2, m

3 etcetera then; the complementary solution would be having the form A times m 1 raised to the power of n plus B times m 2 raised to power of n plus C times m 3 raised to power of n and so on and so forth.

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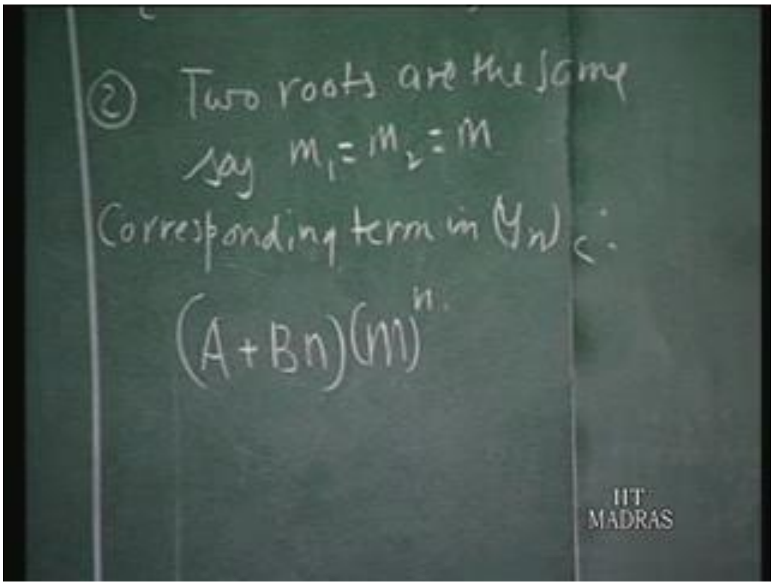

On the other hand suppose: we have some 2 roots or the same say m 1 equals m 2 equals m then; the corresponding term in ync would be instead of A m 1 n plus B m 1 B m 2 n you have A plus B n times. Let us say m to power n.

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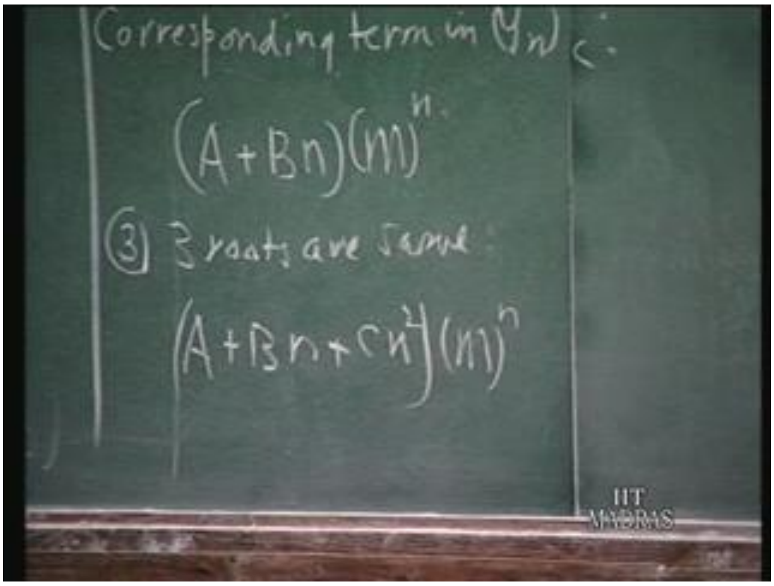

If 3 roots are repeated then you have A plus Bn plus Cn square times the corresponding repeated rule times m to the power of n that is how it goes.

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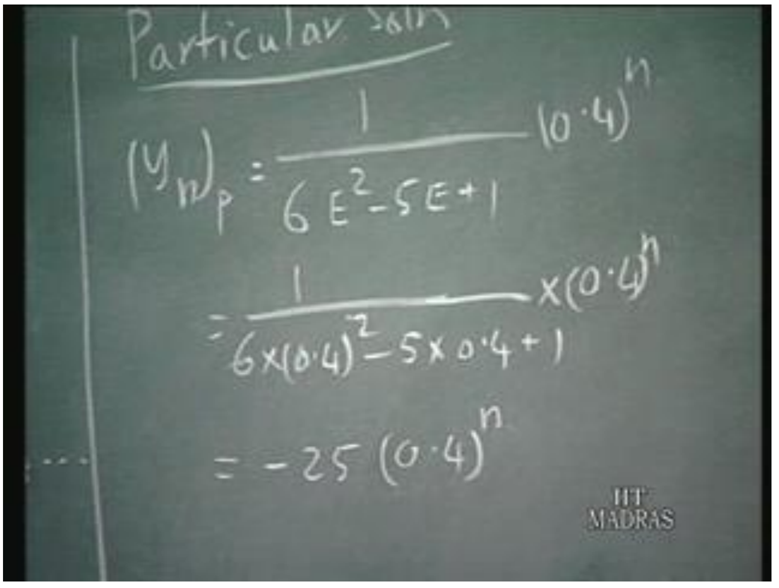

Now let us, take the particular solution. We have yn particular solution; this equation here can be cached in the form yn equals 1 over 6 E square minus 5 E plus 1 operating on 0.4 n. So, when xn is of the form alpha raised to the power of n and you have a function of E operating on that all you have to do is substitute alpha for E. So, this becomes 1 over 6 times 0.4 squared minus 5 times 0.4 plus 1 multiplying 0.4 n; no longer an operator but, just multiplying multiplication of 0.4 n.

So, substituting this the 1.6 therefore, this becomes 0.16 1.96 minus 2 that will turn out be minus 25 times 0.4 n; which would then give the total solution then for yn equals.

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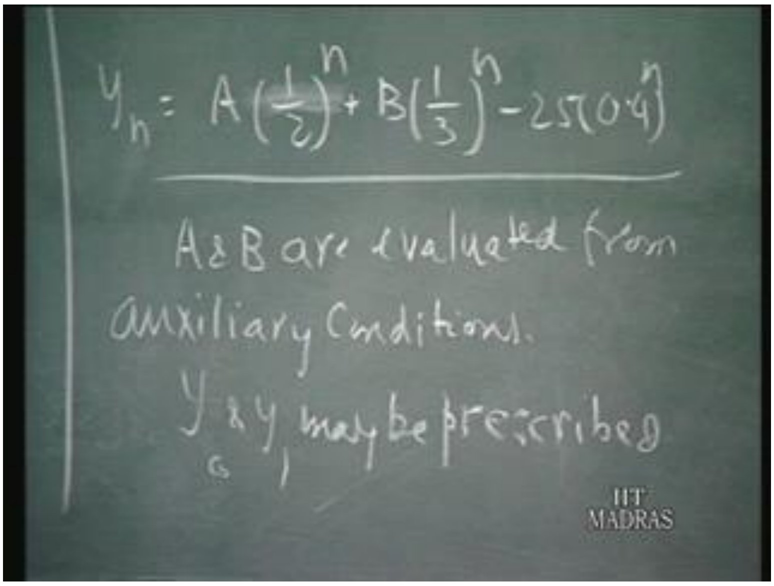

The complementary solution A times point 5 n or half n plus B times one-third n minus 25 times 0.4 n. So, this is the complete solution and A and B are to be evaluated from so called initial conditions, in the case of differential equations here you may you say some auxiliary conditions which are similar to initial conditions. Usually, say y 0 and y 1 may be prescribed.

So, usually y 0 and y 1 are prescribed so, if you know substitute n equals 0 and n equals 1, in this equation and those values are known you can calculate A and B that is how it goes. It may be in it need be 0 at n equals 0 and 1 it could be 2; 2 other equivalents for this function.

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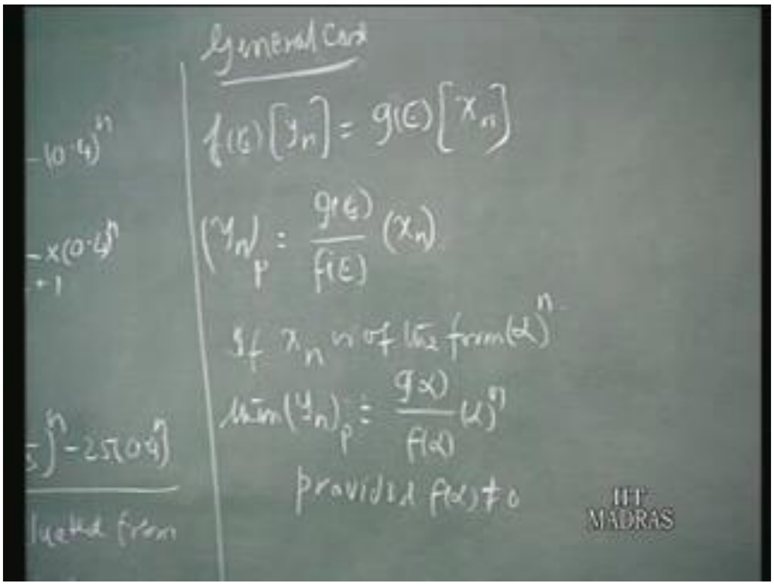

Now let us, put this again some general remarks for this. We may have fE operating on yn equals gE operating on xn. So, yn the particular part of the solution is gE by fE operating on xn. Now, if xn is of the form alpha raised to the power of n. Then, the particular integral solution would be G alpha by f alpha; alpha raised to power of n provided f alpha is not equal to 0 provided f alpha is not equal to 0. There are different rules for different types of forcing function.

We have taken 1 particular example xn is of form of alpha n; we do not spend more time in going through the details of the solution, when the forcing function is of the other forms. We will leave it at that because this is all that we need to worry consider as far as our particular emphasis is concerned. We will emphasize more on the solution of the difference equation through the z transform approach. This is just to give you an idea of the classical approach.

So, we will not pursue this further however, 1 thing you would like to see is if xn is of the form alpha n yn particular is G alpha by f alpha times alpha n that means; the particular integral solution as far the its dependence on the independent variable n is concerned is the same form as xn.

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If xn is z to the power of n then yn the particular integral solution is equal to gz provided fz times z to the power of n. So, that means; the both of them have essentially the same dependence on n. So, this is a characteristic of discrete time system; which we already mentioned. This is characteristics of discrete time system something which we already talked about. A second point which I would like to mention in passing is this is the complementary form of solution.

So, if the complementary form is solution that is force will form a solution is to die out with time this m 1 and m 2 whatever, roots are there for the auxiliary equation must have magnitude less than 1. Because, if n goes to infinity this must decay down to 0 that means the roots of the auxiliary equation must be having a magnitude less than 1 if the complementary solution should die out with time.

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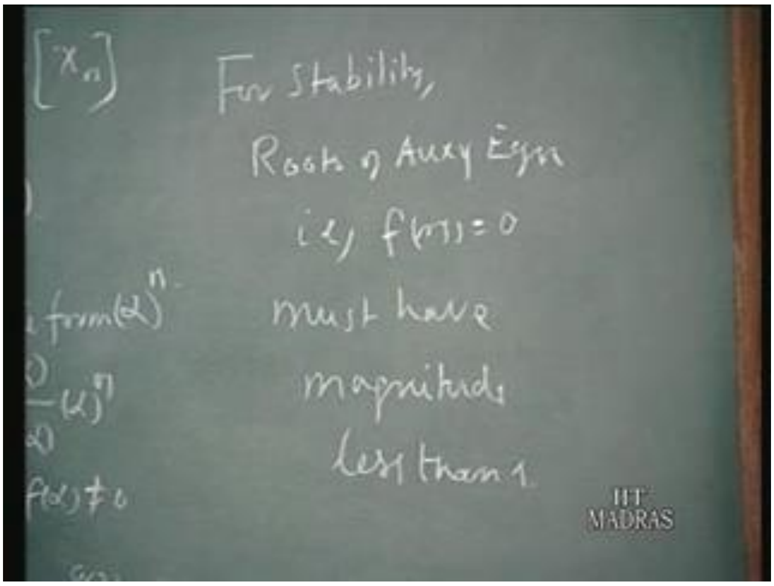

So, for stability we can say for stability the roots of auxiliary equation that is fm equals 0 must have magnitude less than 1. So, that as n goes to infinity that solution must decay down to 0. This is the important thing this corresponds to the real part of the roots being less than 0, in the case of continuous time system here the magnitude of the roots must be less than 1. So, with this background let me quickly work out an example of the application of the classical technique for solution of differential equations to a particular problem.

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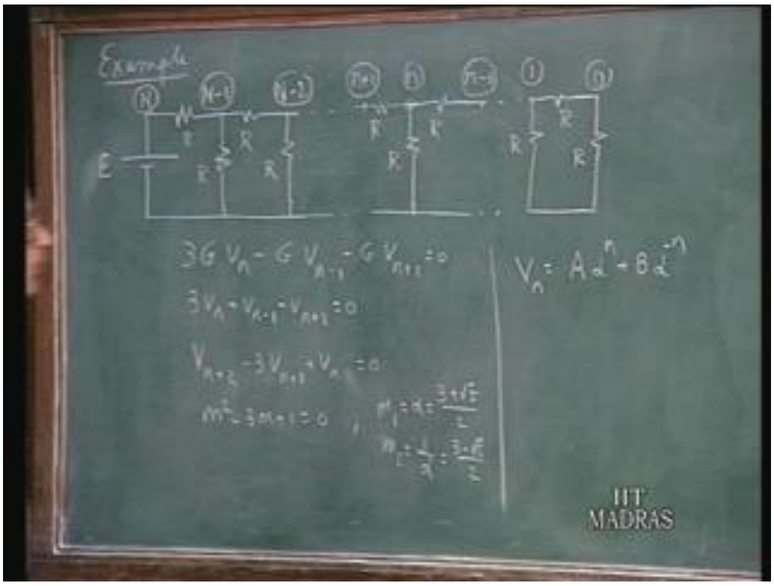

Let us consider, this network which is in the form of ladder network where, all resistance are equal to or all are identical resistors and you have a source here of E volts driving this ladder network. We are interest in finding out the voltage at the n'th node as a function of n. So, let us number the nodes starting from right 0 1 2 3 etcetera up to N. Now, we can form the difference equation which governs the operation of the node voltages by taking a general node n and writing down the node equation at this.

So Vn times conductance of this G plus G plus G 3 times G times Vn minus v times Gn plus 1 minus G times Vn minus 1 must be 0, because at this node there is no source. So we have 3 G times Vn minus G times Vn minus 1 minus G times Vn plus 1 equals 0 where G is the conductance; which is the reciprocal of r, So, we now have the difference equation pertaining to this will be 3 Vn minus Vn minus 1 minus Vn plus 1 equals 0. This can be recast in the form by if you multiply by minus 1 and increment n by 1 we can write this as Vn plus 2 minus 3 Vn plus 1 plus Vn minus 1 equals 0.

So, this is the complete homogenous equation. There is no forcing function and this is the difference equation. Now, note that the independent variable n here is not a this is does not it is not indicating time here but, the independent variable here is a position or the node in the ladder network. So this is in contrast to the discrete time system where n indicates a instant of time but, here it indicates the index of the node so the this the difference equation the auxiliary equation will be e squared m corresponding to e square minus 3 e plus 1 operating on Vn.

We can write this further, if you wish as Vn plus oh i am sorry this is v 3 Vn plus 2 v n plus 1 this is Vn. So, the auxiliary equation corresponding to that will be m square minus 3 m plus 1 equal to 0. The roots of this m 1 equals let us say: alpha which I will write this as 3 plus root 5 upon 2 and m 2 it can be turn out to be 1 over alpha which is 3 minus root 5 by 2. So these are the 2 roots of the auxiliary equation.

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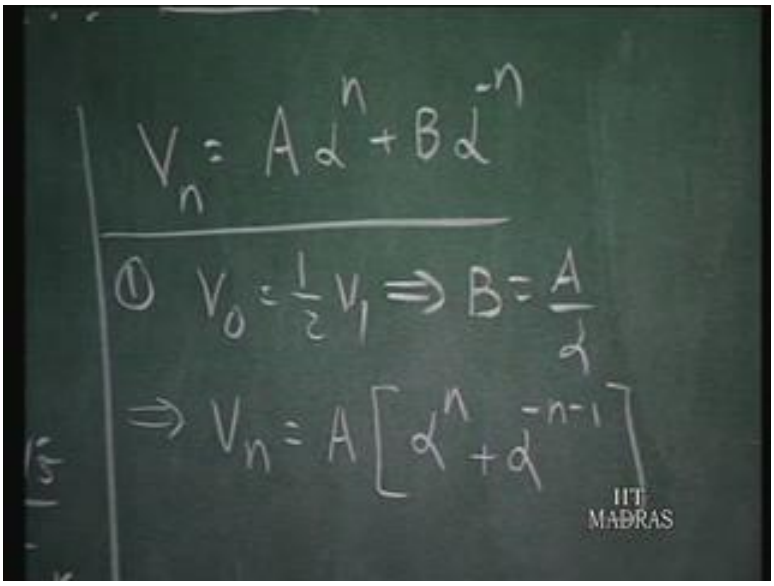

Therefore, the complementary solution for that will turn out to be and that is all the solution, because there is no particular solution, because there is no forcing function. This will turn out to be A alpha n plus B alpha minus n because the 2 roots are the reciprocal of each other. So, this is the general form of the solution of the equation here but, the y here turns out to be the node voltage therefore, I will write this as Vn. So Vn is A e to power of alpha n plus B into e to power of B times alpha to the power of minus n is the general solution.

Now, we have to evaluate A and B knowing the conditions in the network the boundary conditions the auxiliary conditions. We note first v 0 is 1 half of v 1 if this voltage v 1 is there that is final at the end v 0 is 1 half of v 1; because potential divider action across 2 resistors will tell you that v 0 equals 1 half of v 1. So using that we can show that B equals a upon alpha you can substitute. This is this equation and can show that B equals a upon alpha. So, as a consequence Vn turns out to be A times alpha n plus alpha minus n minus 1.

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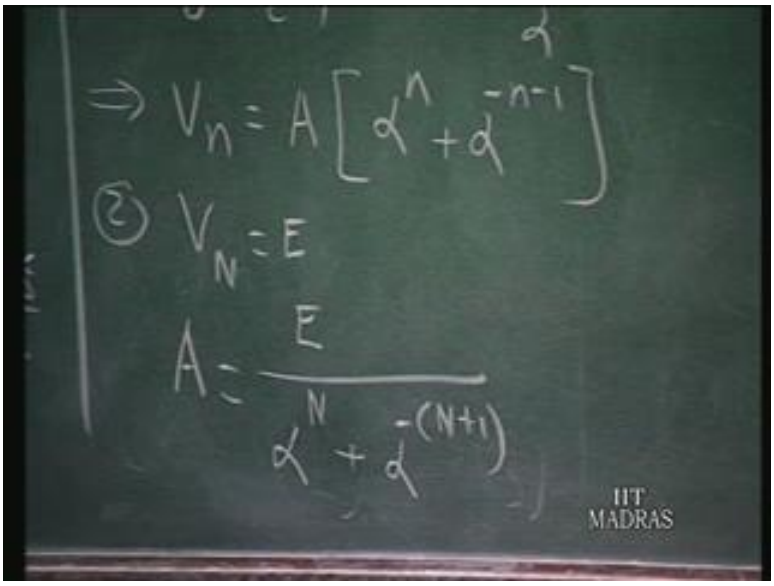

So, we have already evaluated 1 arbitrary constant here the second boundary condition is obtained by noting that VN length equals E. So, VN equals E and using that you can calculate A as E upon alpha n alpha N plus alpha of minus n plus 1. So that is the function that is the value of A.

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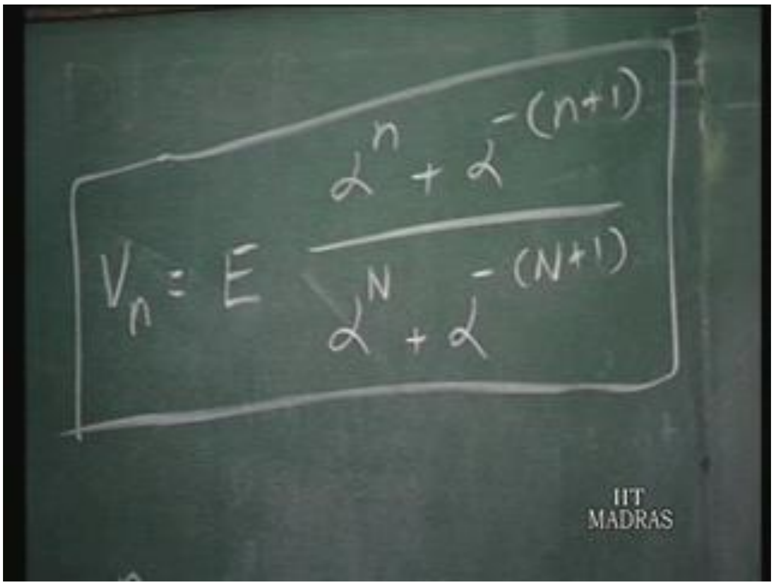

So, substituting this in this expression finally, we end of with the result Vn equals e times alpha n plus alpha minus of n plus 1 divided by alpha times N plus alpha minus n plus 1 that is the final answer for this. So, you observe that we use the difference equation form the auxiliary equation find out the roots corresponding to that and form the general solution, in terms of arbitrary constants; we use the initial conditions corresponding counterpart of initial conditions.

We may call them boundary conditions here and 0 and n and we found out A and B and this is the final solution, in the last lecture I mentioned that a second method of solution of the difference equations is the iterative method or step by step method using the recursive algorithm.

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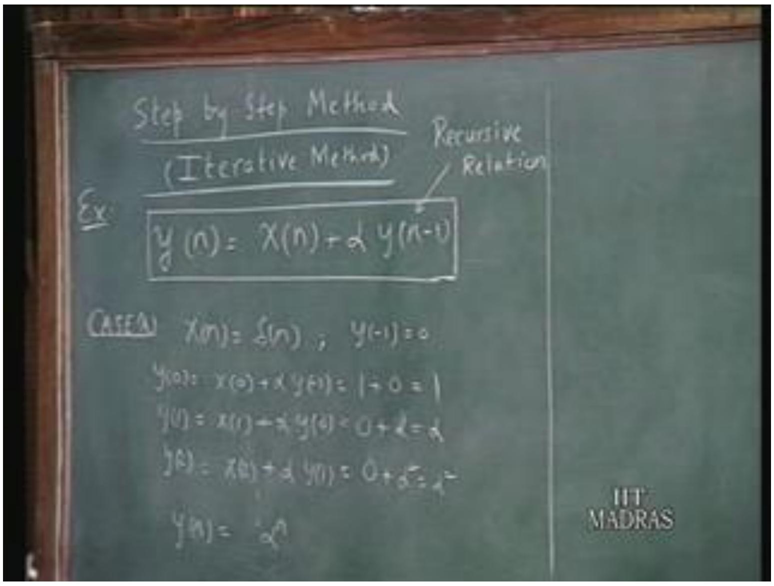

Now let us, let me illustrate this by taking an example suppose: we have difference equation for yn in the form yn equals xn plus alpha yn minus 1 this is a recursive relation; which enables us to calculate yn step by step from the knowledge of the present input and the immediate past output yn minus 1. So, let us take as an example case a where we are given that the forcing function is a unit step delta n and since, we want solve a difference equation of first order we need to know 1 auxiliary condition.

Let us also say that y minus 1 is 0 these are the conditions that are given to us. So, using this information we know y minus 1; we can calculate y 0. So, y 0 substituting n equals 0 here y 0 is x 0 plus alpha times y minus 1 x 0 equals 1 because this is a unit step impulse function unit impulse function has a value 1 at n equals 0 alpha y minus 1 is 0 therefore, this is 1 plus 0 which is equal to 1. Now, we know y 0 so, we can calculate y 1 y 1 again is according to substitute n equals 1 x 1 plus alpha y 0.

So, x 1 plus alpha times y 0 x 1 because xn is a unit impulse x 1 is 0 and y 0 happens to be equal to 1 which we just now calculated. Therefore, this is 0 plus 1 that is also equal to am sorry, alpha times 1 this is alpha times y 0 therefore, this is equal to alpha therefore, this turns out to be alpha y 2 substitutes n equals 2 here x 2 plus alpha times y 1 x 2 again once again is 0 because xn is a impulse function alpha times y 1 which is alpha times alpha square. So, this is equal to alpha square.

So you can now see you can continue this you can get yn easily that yn equals alpha n so the general solution for this is yn equals alpha n. So, this iterative procedure will enable you to calculate the value step by step. Actually, once you get sequence of values for y you may or may not be able to find out a general postponed expression for y of n in this case it turns out that is alpha n may or may not be. So but, this is a numerical procedure suitable for computer implementation because step by step we can proceed. You may or may not be able to find a nice closed form expression for y of n, in this case it is possible for us to do.

So but, it may not be in general possible it is a straight forward procedure it has its value in that it is straight forward happen about through computer implementation and gives numerical results but, a closed form expression is not always guaranteed.

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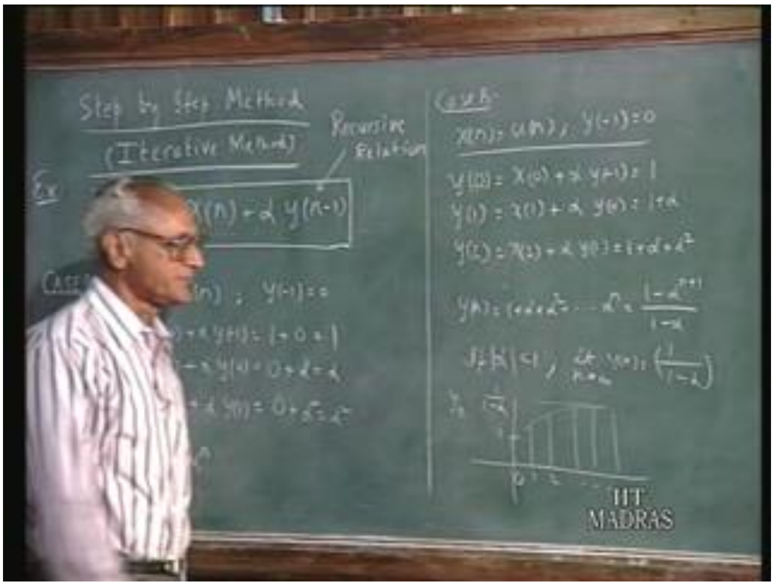

Case B: let us illustrate this further now let us take xn to be un a unit step and once again let us take y minus 1 equals 0. So, proceeding as before y 0 that is in this y 0 is x 0 plus alpha times y of minus 1 x 0 is 1 because xn is un 1 this is 0 therefore, this is 1 y 1 equals x 1 plus alpha times y 0 x 1 happens to be 1, because this un this 1 continues to be 1 alpha times y 0 is equal to 1. Therefore it will be 1 plus alpha y 2 x 2 plus alpha times y 1 so, x 2 is 1 plus alpha times this quantity.

So, 1 plus alpha plus alpha square so you can now, see the trend you can immediately deduce that the yn equals 1 plus alpha plus alpha square up to alpha n which indeed is 1 minus alpha n plus 1 by 1 minus alpha; that is the general expression for yn. So, if alpha quantity happen to be magnitude less than 1 then; limit as n goes to infinity of y n for large n will turn out to be 1 over 1 minus alpha. Because alpha raised to power of n plus 1 tends to be 0. So, the solution for that would be starts with 1.

So, asymptotically reach a value which corresponds to a 1 over 1 minus alpha that is 0 1 2 3 and so on; see, in this difference equation yn minus alpha yn minus 1. So, that means; the roots of the auxiliary equation the roots of the auxiliary equation there is only 1 root because there is only first order equation is equal to alpha. So, for stability alpha has got to be less than 1 so, this builds up like this if alpha happens to be magnitude is greater than 1 it may turn out that this becomes very large it may increase like this.

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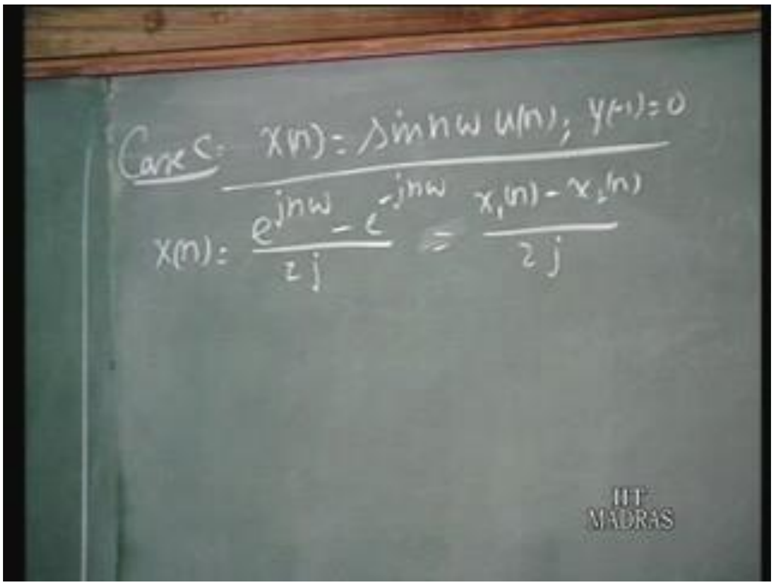

Let us take case C where, we take the input to be a sinusoid sine n omega un and once again let us assume: that y 1 y minus 1 is 0. So, in order to find out the solution for that let me take xn to be e to the power of j n omega minus e to the power of minus j n omega by 2 j. So, I will consider this as y 1 n sorry x 1 n minus x 2 n divided by 2 j so, you can find out the solution for x 1 n find out the solution for x 2 n then; I can find out the solution for xn.

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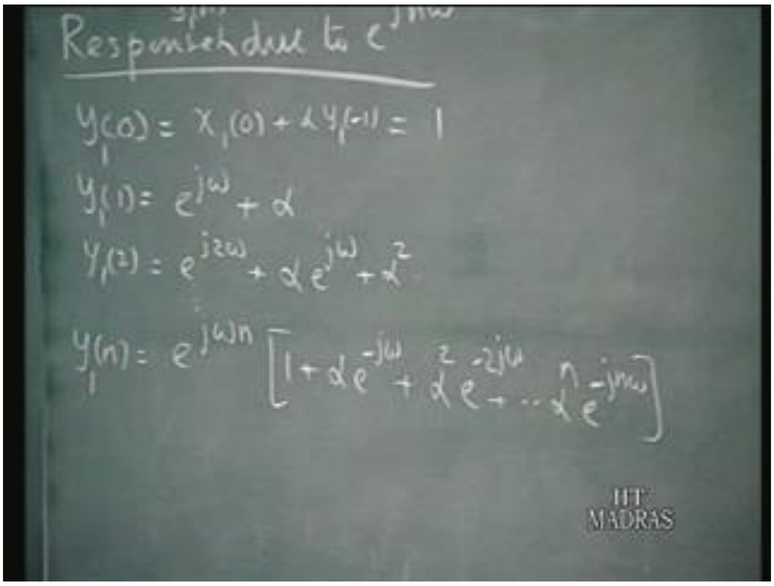

So, let me say I would find the response due to e to the power of j n omega that is due to x 1 m and I will call that response y 1 n. So, y 1 0 equals x 0 x 1 0 in this case plus alpha times y 1 minus 1 which is taken to be 0; which has we assumed as 0 therefore, this turns out to be x 1 0 which happens to be 1 e to the power of j n omega 1 then y 1 1 is e to the power of j omega because that is x 1, when e to the power of j n omega when n equals 1 is e to the power of j omega plus the alpha times the old value which is 1 this is alpha y 1 2 is e to the power of j 2 omega plus alpha times this alpha e to the power of j omega plus alpha square like this it can it will go on.

You can show that in a y 1 n, in a general case is e to the power of j omega n or n j omega times 1 plus alpha e to the power of minus j omega plus alpha square e to the power of minus j 2 omega and so on up to alpha n e to the power of minus j omega.

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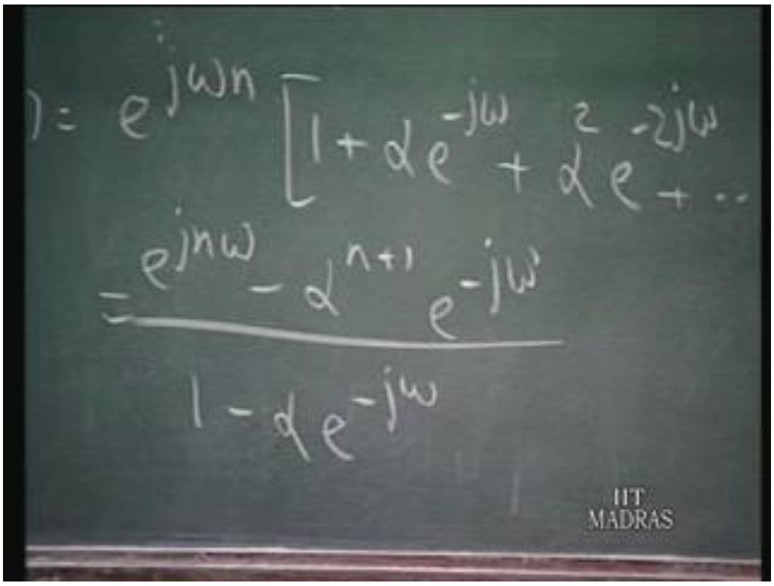

This can be shown to be the general expression for y 1 n and i can show this further to be equal to e to the power of j n omega minus alpha n plus 1 e to the power of minus j omega divided by 1 minus alpha e to the power of minus j omega. Now, it is little messy but, the procedure is straight forward.

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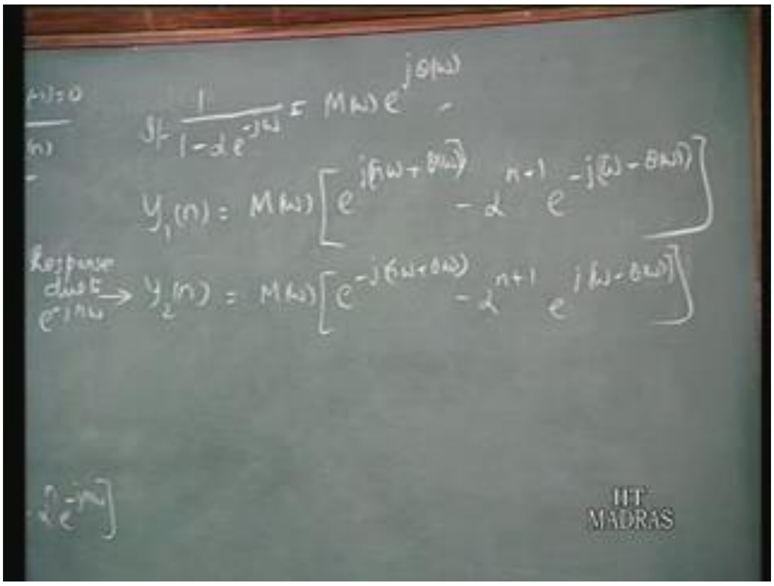

Now suppose I say if 1 over 1 minus alpha e to the power of minus j omega is independent of m of course, so let. So suppose I call this M omega magnitude e to the power of j theta omega. So the angle and magnitudes are functions of omega then y 1 of n this whole thing can be recast in the form; you can put this in this form, you can pull up the details it can be shown to be M omega multiplied by e to the power of j n omega plus theta omega minus alpha n plus 1 e to the power of minus j omega minus theta that is how it will make y 1 of M.

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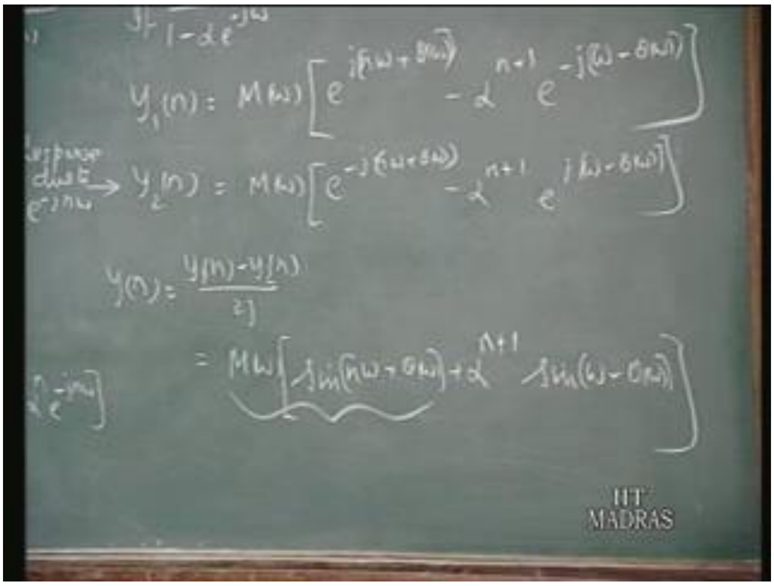

Now, the response this is response to the e to power of jn omega if you calculate y 2 of n; which is the response due to e to the power of minus jn omega which is this other term this will be the complex conjugate of this because e to the power of minus j n omega is the conjugate of e to the power of j n omega. So, this turns out to be M omega e to the power of minus j n omega plus theta omega minus alpha n plus 1 e to the power of j omega minus theta omega. Now, the since the sum of these divided by 2 j must be the total solution, because the input is the sum of these 2 functions the difference of these 2 functions by 2 j.

Therefore, we also have yn equals y 1 n minus y 2 n divided by 2 j and this can be shown to be equal to M omega sine n omega plus theta omega mi plus alpha raised to the power of n minus 1 n plus 1 rather n plus 1 sine omega minus theta omega. So, that is the final solution due to this sinusoidal forcing function you observe here that this is a factor which does not decay, with time you are driving this with a sinusoidal function sine n omega and the output is also a sinusoidal function sine n omega modified in magnitude by M omega by phase theta omega.

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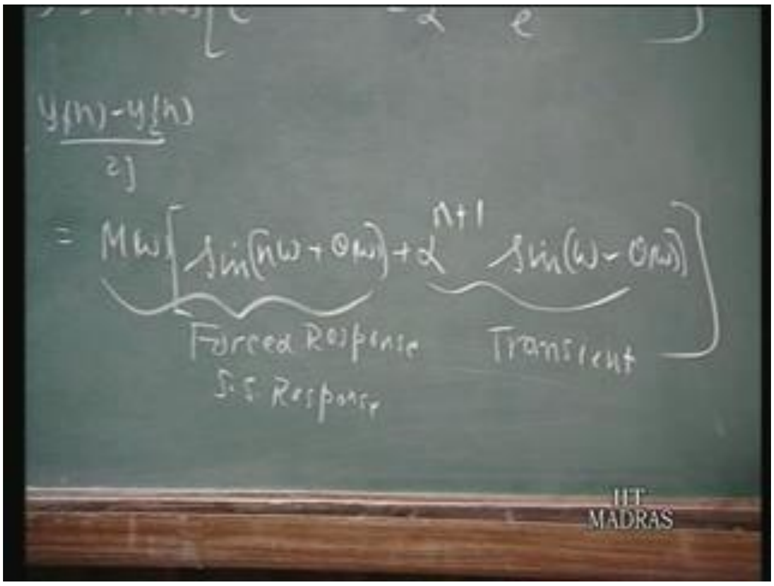

So this is the forced response or the steady state response steady state response where as this portion is transient response or which turns from the which came from the complementary solution. And when n becomes very large alpha if it is a magnitude less than 1 this decays with time. So, you have the forced response and transient time response coming together as far the that is M omega times this is a transient response and M omega times this is a forced response.

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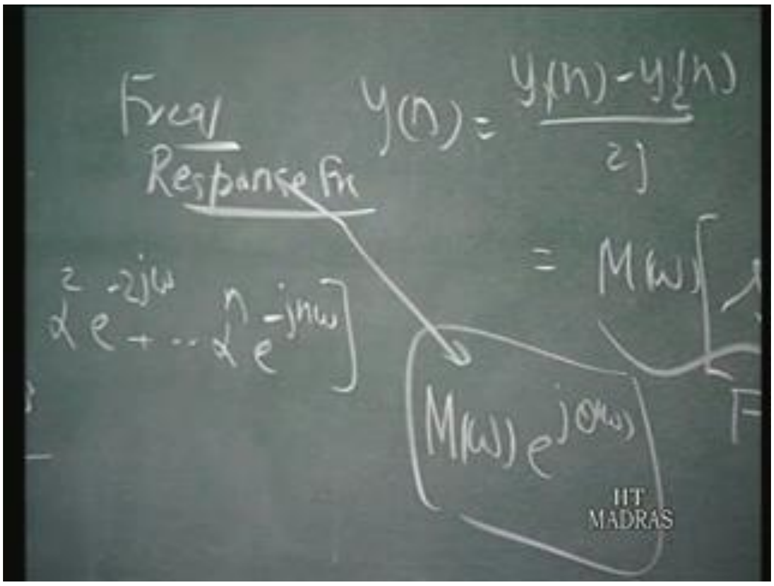

So, you have M omega e to the power of j theta omega is the frequency response function. So, this depends on omega so, if you are driving this with a sinusoidal function the output has also a sinusoid modified in magnitude by M omega by phase theta omega. So, this is the frequency response function which is the function of omega you observe that when you are driving this sinusoid to find out the final solution is a little bit complicated little messy but, the procedure is straight forward.

The only thing is you have to involve yourself with a lot of complex algebra and in this case in all this 3 cases; that you have considered we have taken that even though we are using step by step iterative procedure we are able to find out closed form expressions for y of n. This may or may not be easy in most of the cases you recall that, in the continuous time situation.

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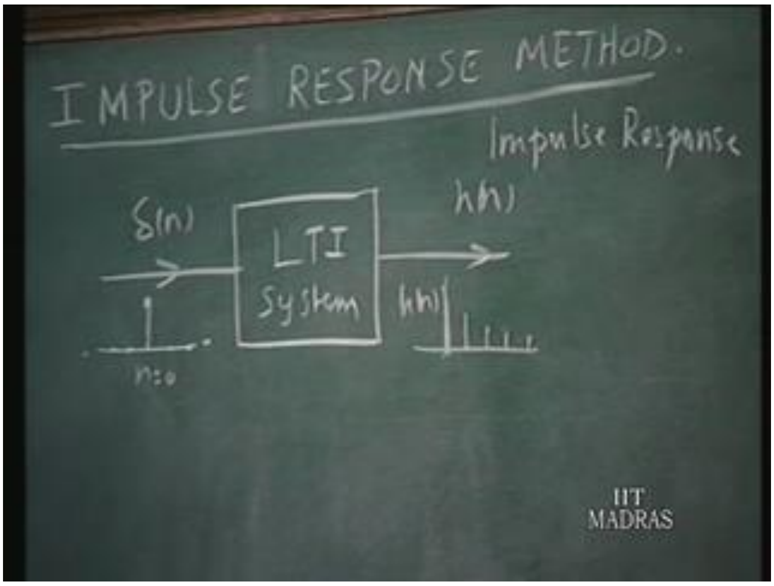

If we knew the impulse response of a linear time invariant system LTI system then; we can find out the response to any input to the convolution integral. We have a similar result in the discrete time case also. So, let us take this a discrete time LTI system a discrete time system linear time invariant discrete time system and let the input be an impulse delta n and let us say: the output is hn which is called the impulse response. So, the input is just a single sample at n equals 0 and you get an impulse response may be like this.

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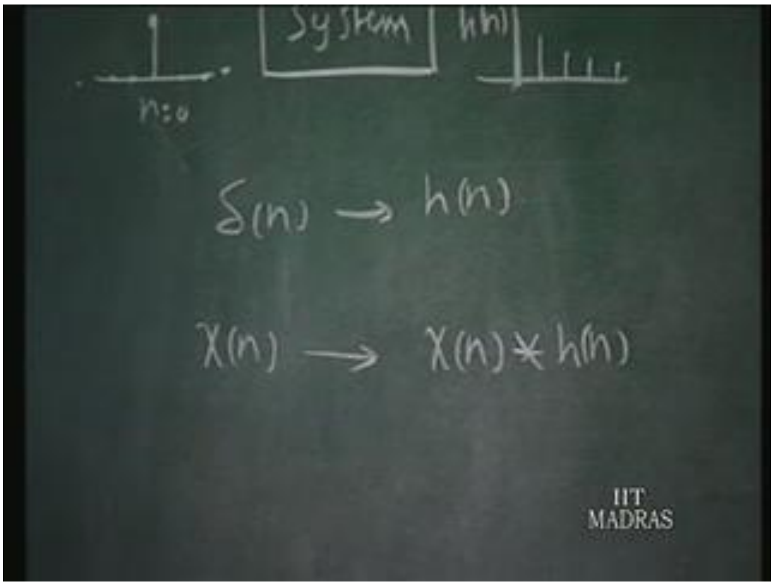

Now, we can represent this delta n gives raise to hn for a general input xn the response would be just as in the case of continuous system the convolution of xn and hn.

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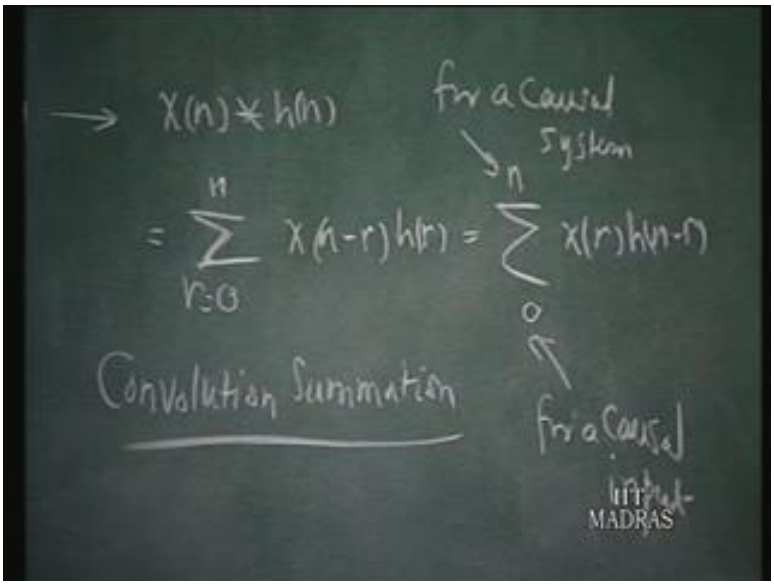

What is meant by the convolution of hn and xn is the, in this case in this continuous case it is a integral here, it is a summation of xn minus r hr summed on r or alternately you can write this as xr hn minus r summed on r. Now, what about the limits of this integral the limits of the integral should be in general from minus infinity to plus infinity but, for a causal system and with a causal input we take this 0 to n.

We take this to 0 because for a causal input xr; if the input is 0 for the negative values of the argument then there is no point in having this summation extending from negative values of r. So, we should take this to be 0. If the system is a causal system impulse is applied at n equals 0 hn is 0 for negative values of time therefore, hn is 0 for n less than 0 for a causal system consequently when r gets beyond n hn minus r becomes 0 because the argument turns out to be negative.

Therefore, we stop the summation up to n for a causal system; after all the summation here also is the same it is the n minus r and r exchange their roles therefore, here also r from 0 to n. So that is the meaning of the convolution this the case of a discrete time system the convolution is the summation. This is called the convolution summation and it is matter of fact, it is simpler than the continuous time case where there is an integration here it is a here it is a purely summation.

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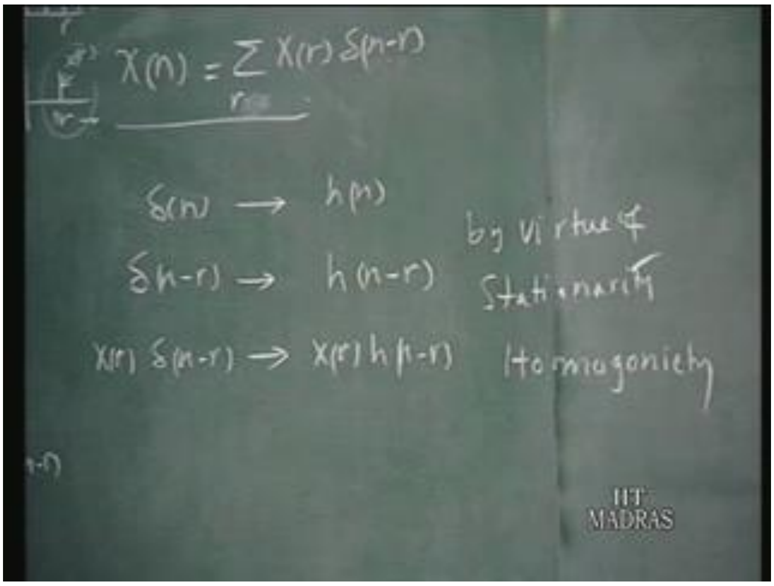

Therefore this is really simple now what is the justification for this result we can justify it in this fashion. First of all we can say that any xn can be written as after all xn is the sample of values a sequence of values like this. So, if you take 1 particular sample standing at place r; I can write this as xr delta n minus r so, xr times delta n minus r is just this sample the value is xr and it is standing at n equals r. So, just this 1 sample of this sequence of samples in x of n can be written as xr times delta n minus r. You want the whole set of samples xn you have to sum up all such values from r 0 onwards from whatever, you have.

So, summed on r xr times delta n minus r summed on r is the seq sequence of samples xn. So, if you want to set the whole lot of the samples you take the r from 0 to infinity if you want to stop at a particular value take from r equal to 0 and stop at that instant. So, any input xn can be thought of as a sequence of number of impulses standing a different values of n 0 1 2 3 and so forth and having magnitudes x 0 x 1 x 2 and so forth. So, any input can be expressed in this fashion.

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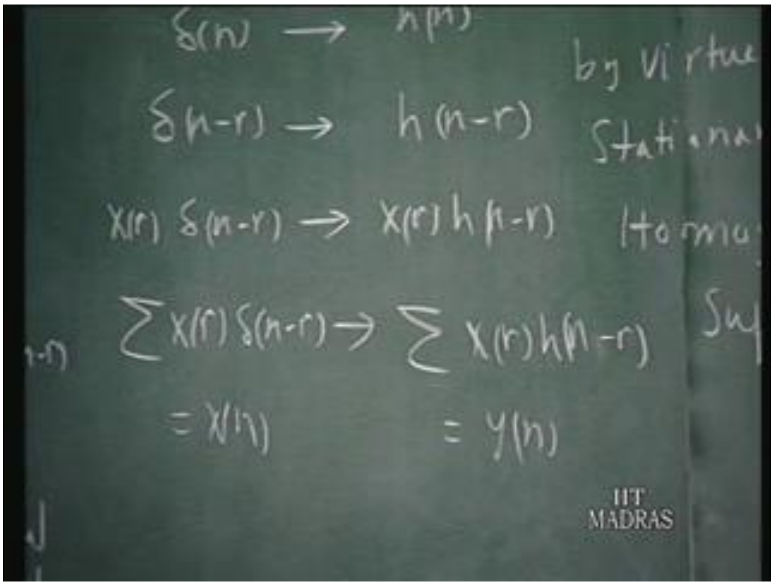

Now, after having said that: we can now say that delta n gives rise to an output hn that is the impulse response, but if you shift this delta by r sampling instants instead of the impulse standing at n equals 0. Suppose, this is delayed by r sampling instants the corresponding output will also be delayed by the same number of sampling instants. This is true by virtue of stationarity by the virtue of the fact that our system is time invariant or having constant parameters, it is a stationary system it is not a time variant system.

But, instead of delta n minus r suppose you have a constant multiplier xr times delta n minus r naturally the response, will be by linearity xr times hn minus r by virtue of homogeneity. The input is multiplied by a constant the output is also multiplied by the same constant. Now, on the other hand what we now have is a whole so lot of this samples not just 1 sample.

So, summation on xr delta n minus r the whole lot will be corresponding summation of the output xr h n minus r. This will be linear superposition and that is the justification after all this is xn therefore, the corresponding result must be yn. So, once you know the impulse response you can find out the response for any arbitrary inputs by this convolution superposition or convolution summation. So, what is it that we are really looking for here.

You have what you are doing here is we have 2 functions discrete time functions xn and hn and their convolution summation is this you keep 1 of them xn changing the independent variable r from n to r xn hn minus r means; you fold it shift it just as you have done in the case of continuous time domain you multiply them that also you do in the same fashion but, the sum the product all those are not integrated now but, they are just summed up that is the only difference we have here.

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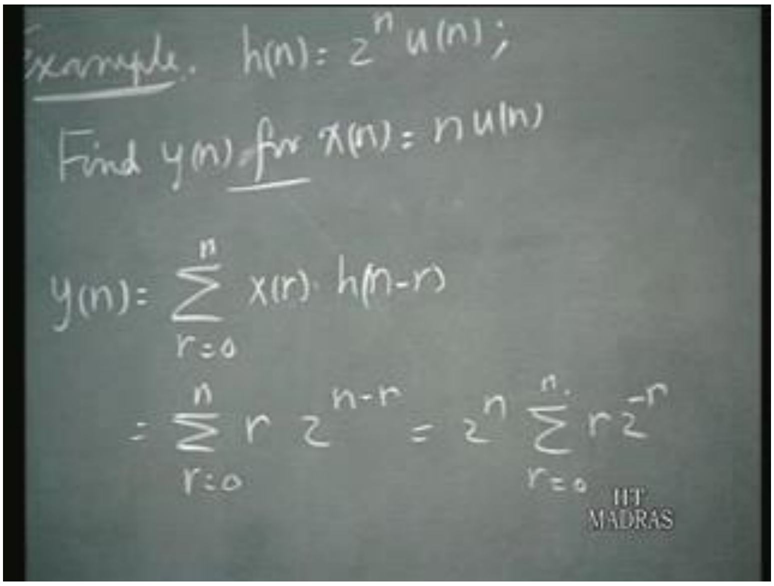

Let us, illustrate this by means; of an example you have let us say hn of a particular LTI system is 2 n un and find yn for an input xn equals n un. Let me say a unit ramp function. So, we can write this as yn equals xr hn minus r r from 0 to infinity .Now, xr is r times ur because anyway r is going from 0 to n ur is going to be 1, in that case therefore, this is r times h n minus r 2 n minus r and in the range of integration range of this summation n minus r is going to be positive, because r is going from 0 to n therefore, we do not have to write un minus r.

Therefore r times 2 n minus r, r from 0 to n this can be further written as 2 to the power of n because; n is a constant as far as this summation is concerned s times 2 to the power minus r, r from 0 to n.

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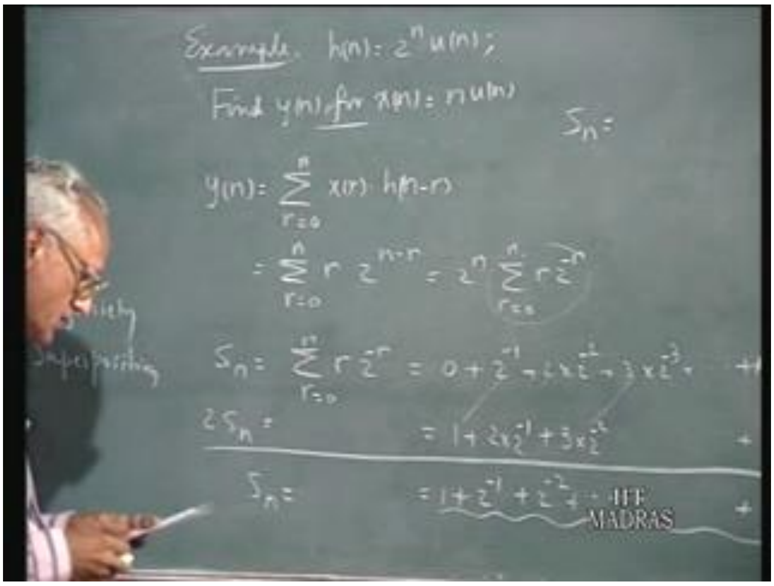

Now, how do we evaluate this summation this summation can be suppose you call that Sn as summation r 2 to power minus r, r from 0 to n this can be written as 0 when r equals 0 2 to the power of minus 1 2 plus 2 times 2 to the power of minus 2 plus 3 times 2 to power of minus 3 and so on and so forth; n minus 1 raised to the power of 2 times minus n plus 1 and finally, n times 2 to the power of n minus n that is all for this. Now, suppose I multiply this by 2; 2 Sn I multiply all this terms by 2.

But, I write this in this fashion 2 times 2 to the power of minus 1 I write this here 1 2 times 2 times 2 to the power of minus 2 that is 2 times 2 to the power of minus 1. So, this goes here this goes here and this will be 3 times 2 to the power of minus 2 like that that is you multiply by this 2 but, put this here similarly, you multiply this by 2 and put this here.

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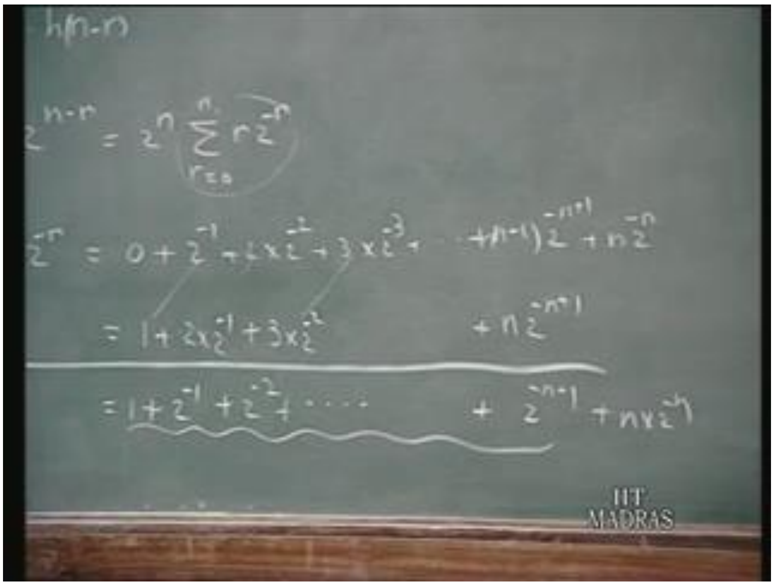

So, n times 2 to the power minus n plus 1 now if you subtract the top row from the bottom row 2 sn minus sn that is sn that is you subtract this values from this. So you get 1 plus 2 minus 1 plus subtract this from this 2 to the power of minus 2 and so on and so forth. Here, you get 2 to the power of minus n plus 1 n minus n minus 1 is of course, 1 plus lastly you have n times 2 to the power of minus n. So, this is geometric progression 1 plus 2 the power of minus 1 2 to the power of minus 2 right up to the power of minus n plus 1.

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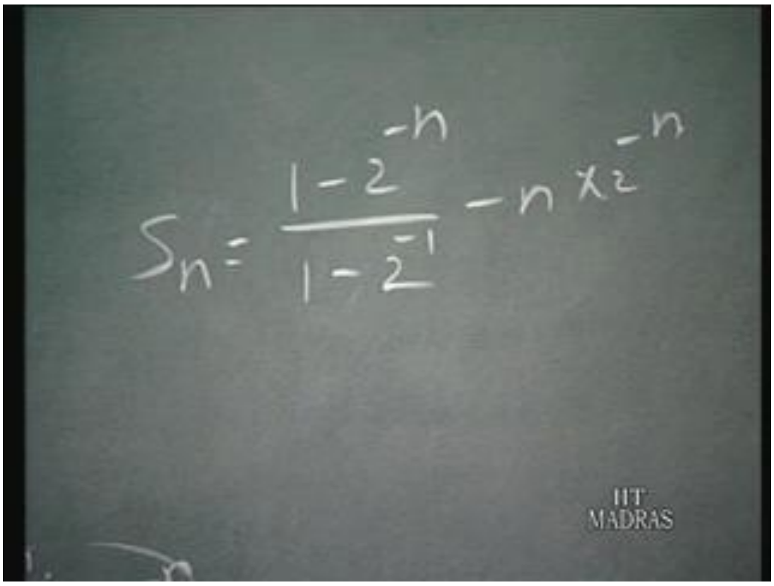

So, that can be shown to be so, sn can now be written as 1 minus 2 to the power of minus n by 1 minus 2 to the power of minus 1. That is the this quantity minus of course, you subtracting this from this therefore, this should be minus shall be minus n times 2 to the power of minus n that is your sn. And once you use that value of sn and substitute in this that is ur sn.

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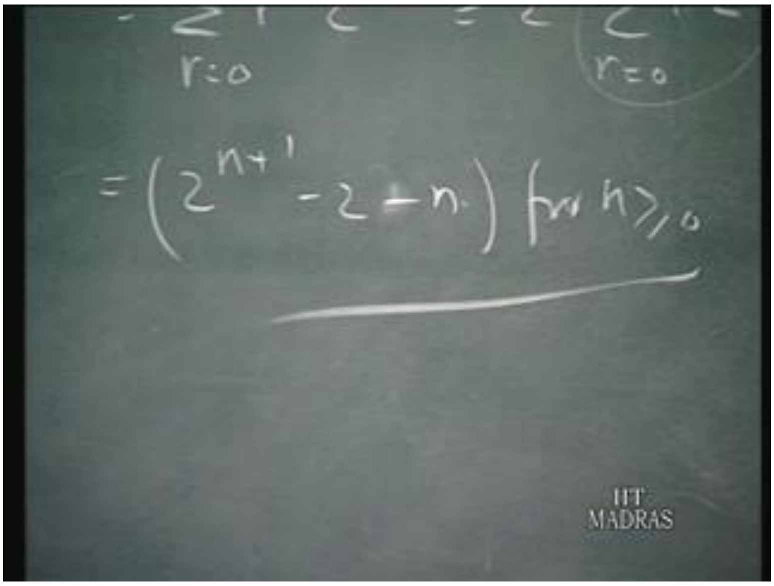

Substitute over here yn can be shown to be 2 to the power of n plus 1 minus 2 minus n for n greater than or equal to 0 that is your solution; which we obtained from this problem using the convolution property.

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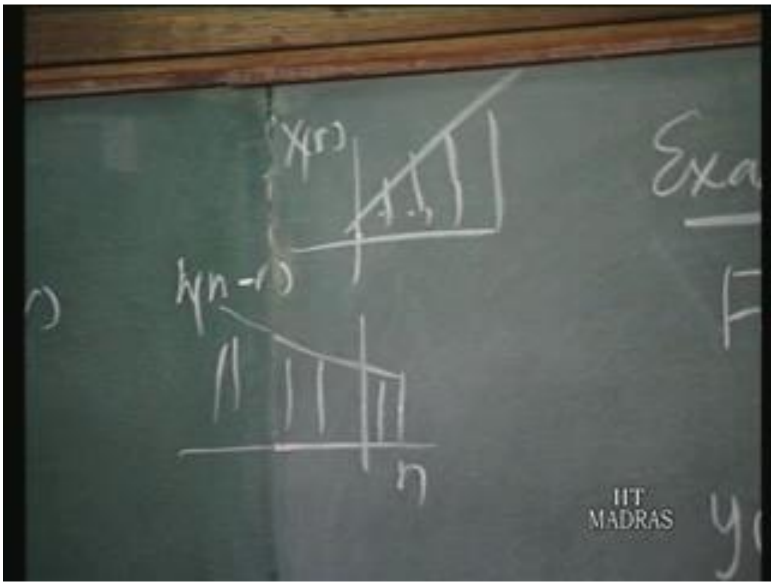

What exactly, you are doing is you recall that hr will be like this you can say that you are keeping 1 quantity xr suppose you are keeping it constant that is x that will be xr and yn minus r will be something like this y hn minus r sorry; hn minus r. So, you shift this depending on the value of n you shift this sequence of samples you multiply this sample with, this sample multiply this sample with this sample, this sample with this sample add up all those products that will be your yn and for different values of n you should shift this.

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So, that means; as far as hn is concerned xr hr you shift it you folded shifted multiplied and then add that means fold shift multiply and then add sum up. So, this is the these are the operation that are involved which correspond to the counterpart in the continuous case where all these are the same except the final stage you integrate but, here you simply add them up. Then you can I would suggest that you work out this problem using this graphical procedure, you establish a row of values corresponding to x of r establish a row values corresponding to hn minus r for a different values of n 0 1 2 3 and so forth.

You multiply the products add them up and do this for different values of n you get a sequence of values for the yn; which you verify will correspond to this in this formula will be in accordance with, this formula as said I would like you to do that as an illustration of the convolution summation. So, in this lecture we have started with the solution of the difference equation, by the classical approach of solution of difference equations you use the complementary solution and the particular solution.

We have taken 1 or 1 special case of how we can find out the particular integral solution; when the forcing function form alpha raised to the power of n in other cases we did not go into that, because we did not want to involve our self in to details of this particular technique. We also saw how the difference equation can be also be solved by the step by step by step procedure using the iterative technique. This will give us a solution in a very simple fashion

However, we may not always end up in a closed form solution or a at least closed form solution may not be quite evident verse; we may or may not be able to find that out. And thirdly we saw how the impulse response of a system can be used to find out the response to any arbitrary input. This involves the convolution summation, which is the counterpart of the convolution integral in the continuous time case.

The technique essentially, is the same you take the product the convolution of 2 time functions. Now, is 1 time function is folded that means; the independent variable is reversed instead of plus r it will be minus r instead of plus n it will be minus n vice versa. And then; you shift it multiply the samples which are at the same position and then add up the all the product that will be the convolution summation.

So, we see that the convolution summation works more or less the same way as the case of continuous time, in situation are more importantly for us. We know that if you know the impulse response of any linear time invariant system, we can find out the response to any arbitrary input both these is a continuous time case or n, in the discrete time case using the convolution property.