

Networks and Systems
Prof V G K Murti
Department of Electrical Engineering
Indian Institute of Technology, Madras

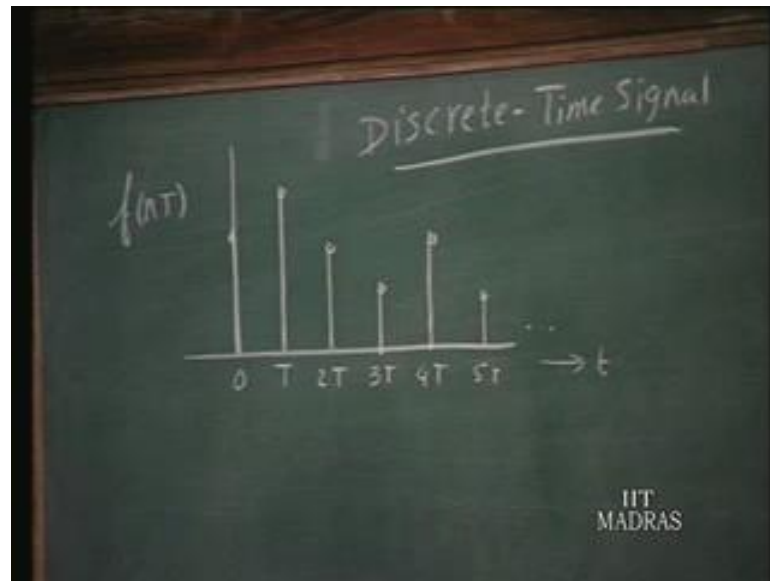
Lecture 38
Discrete-Time Systems (1)
Introduction
Difference Equations of LTI Systems.
Basic Discrete-time Signals

We have been discussing so far the methods of analysis of continuous time systems, where the variable can take values as continuous functions of time. You recall, that we mentioned in the introductory parts or part of this lecture series that in contrast to continuous time signals we have what are called discrete time signals. These signals take values at discrete points, along the x axis that is representing the independent variable and usually this independent variable is time, when we talk about dynamic subsystems.

And so, a discrete time signal has values defined at discrete points along time, discrete points of time. Very often and this is by a large the case that we will be considering, these discrete points are evenly spaced in time. Therefore, the interval between 1 sample of the signal and the next sample is constant, we can call this T this is also called the sampling period.

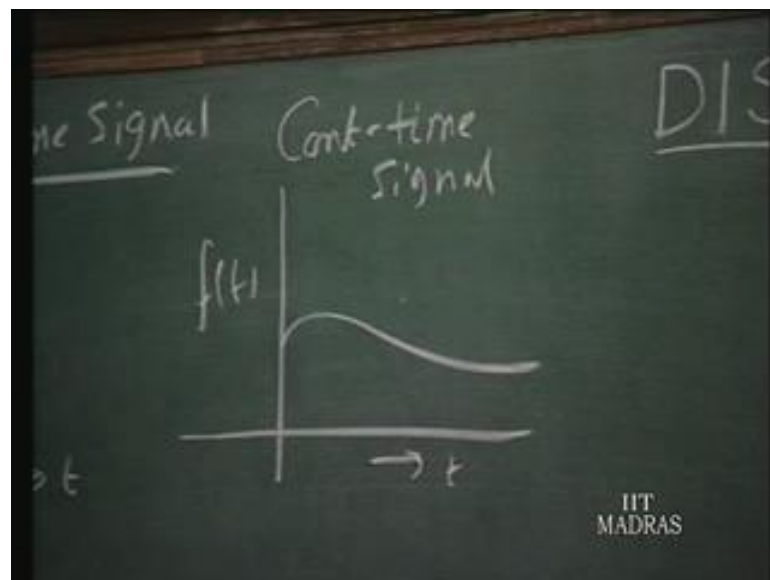
So, in a discrete time signal we have a sequence of values of the signal which are occurring at regular intervals and this can be represented by a means of a figure like this.

(Refer Slide Time 02:56)



So, if we this is discrete time signal we can call it $f(nT)$ this is T $2T$ $3T$ $4T$ $5T$ etcetera. And the amplitude of the signal at different sampling instant is indicated by this vertical lines and this is what we call a discrete time signal.

(Refer Slide Time 03:37)

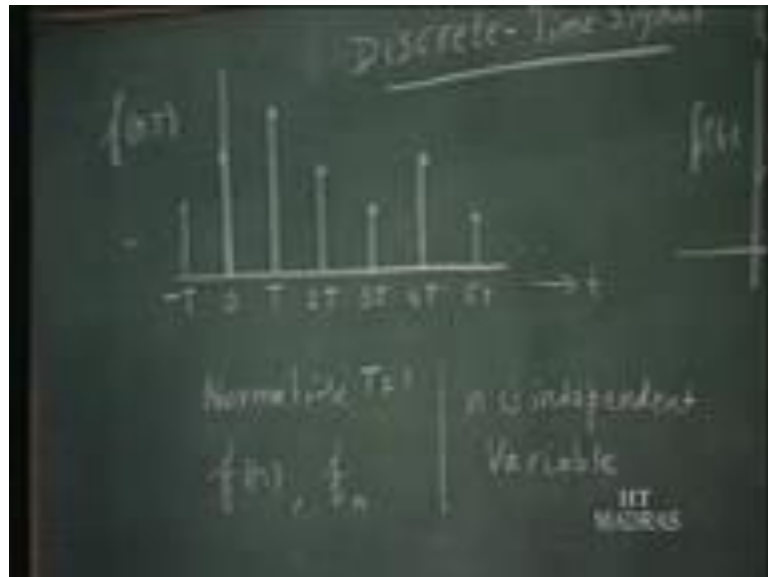


In contrast as you know a continuous time signal f of t is defined for every value of time t that is the distinction between these 2. So, in a discrete time signal you have the function described at discrete points of time and usually it can be of course, negative T also in general. So, we have these signal values with normal occurring at regular points

at regularly spaced points along the time axis. We usually normalize T to be 1 and we will represent this signals as f_n itself.

So, this is a notation which we will normally adopt unless the sampling instant, sampling period is to be other than 1 second we take this to be 1 second and therefore, we can write this as f_n . Another notation for this is $f[n]$, so this is more compact sometimes we use f as a function of n and or f suffix n . In contrast to the continuous time signal, where T is the independent variable here; here the independent variable is n . So, n is the independent variable.

(Refer Slide Time: 05:24)



So, the value of the independent variable will have jumps it can be 1, 2, 3, 4 and so on. Introduce values for the independent variables is what the values that we consider at which this f of n t is described. Such discrete time signals, occurs sometimes naturally for example, if you are talking about the attendance in a class the first class, second class, third class and so forth there is no point in saying 1 and half classes or 2 and half classes.

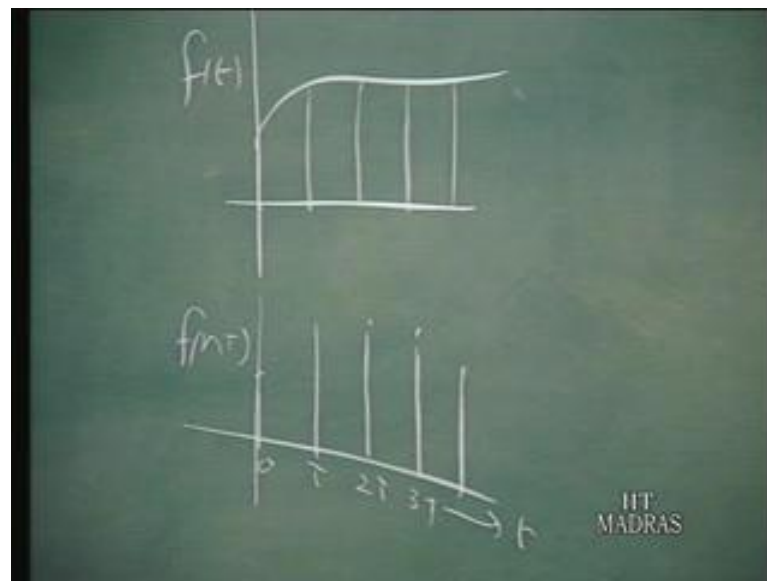
So, it occurs naturally in such a context similarly, if you are talking about the yearly out turn of graduates from a particular university. Then, again each year you have certain number and therefore, there is no meaning attached to the fractional part of an year. Suppose, we have a long ladder network in which we are interested to know the voltage at different nodes.

So, this can be thought of again as a discrete time system, where the voltage is a function of the node number. Now, even here even though the voltage is a function of a discrete node number the independent variable is not time in this case, but it is the position of the node. Even in this case, we will use prefer to as the discrete time system even though the independent variable does not have anything to do with time.

Another example may be if you are interested in finding out or specifying the number of centuries scored by Sachin Tendulkar in each calendar year. So, you have the number of centuries that is the dependent variable and calendar year is the independent variable.

So, you are taking the value of the particular signal at discrete points of time. Now, which is which are spaced apart by 1; all these are examples of discrete time signals which come naturally.

(Refer Slide Time: 07:24)



On the other hand, you may have a situation where we have continuous function of time f of t . And you sample this a discrete points of time and generate from that f of nT , where you take stalk of this continuous variable at discrete points of time. So, this is a sampled version of a continuous time signal. So very often, we have to deal with such discrete time signals which arise from a continuous time signal by sampling process.

For example, you might like to find out record the temperature daily temperature readings at 8 o'clock in the morning. The temperature is a continuous signal but, you're

sampling it at discrete points of time at every day at 8 o'clock. So, this is a sampled version similarly, in we want to send several piece of information over a telemetry channel pressure temperature, humidity so on and so forth. You sample these signals at discrete points of time and all these 1 after another on the same channel.

So, again here again you have sampled version of a continuous time signal. In sample data controlled systems this is a common phenomenon, in sample echo quantity and then use that information to control that quantity. In speech signal, digital signal processing a speech signal is sampled and then the sampled version is a discrete time signal. So, we have basically 2 character 2 different types of situations, where a discrete time signals are encountered.

1 where they occur naturally as the first category we talked about and secondly, where they are generated from a continuous time signal by taking the samples at discrete points of time. So, the term sample is probably applicable to the second category of signals. However, in a loose manner of speaking even when they occur naturally as the first in the first category like the attendance in classes and so on. We refer to the particular value at a given point of time also as a sample in because, it is quite convenient to refer to it in that manner.

(Refer Slide Time: 09:44)



Now, it is also possible for us to just like the independent variable these quantized can take only these definite values. It is possible for us to also specify that magnitude of the

signals also can occupy 1 or some discrete levels. So, instead of varying continuously in the vertical direction we may also specify that the amplitude the value of the signal at any point must take 1 or some specified levels. This it suggests its case is called a digital signal.

For example, if you have got a sampled version of a continuous signal obtained and then this is digitized through analog digital converter depending upon the number of bits we used in the conversion. The output will correspond to a value which again takes discrete values so, that is called a digital signal. A digital signal is again, a discrete time signal but, the amplitude of the signal is confined to certain well defined elements.

As far as the our course is concerned, we will allow this signal amplitude to vary continuously that means, this signal values can vary continuously in the vertical direction. We are only discretizing it only in the as far as the independent variable is concerned. So, we are not talking about digital signals we call we call them discrete time signals. So, that is the difference between discrete time signal and digital signal.

(Refer Slide Time: 11:09)



The thing to keep in mind is that the independent variable here, even if you write f of n T the independent variable is n only not T . So, independent variable is n and usually we associate this with the time axis as the independent variable that is why you call it discrete time signals. But the same theory is applicable as I said to a case for example, if you are trying to find out the voltages at different nodes in a ladder network.

Then, there is no time involved the independent variable is the index of the node then again we still the same techniques that we adopted for a discrete time signal processing be can also be used there.

Now, after having set this then we would like to know what is meant by discrete time system again this is something which we already discussed but, let me review that.

(Refer Slide Time: 12:00)



A discrete time system is 1 which processes discrete time signals. So, we have suppose we have single input, single output system then the discrete time system will be represented by means of a rectangle like this. And the input may be x of n and the output will be y of n . So, that is the input output relation this is the input and this is the output and this is the discrete time system.

So, as far as the single input, single output, discrete time system is concerned you will let in a sequence of values constituting x_n and out comes another sequence of values which is y_n . And this system therefore, takes as input a sequence of values and produces the output another sequence of values. The relation between the y_n and x_n is something which is of interest which will be discussed in later.

But we will be considered with a particular class of discrete time system in our course here, this is the linear time invariant causal system discrete time system is what we are going to talk about much in the same manner as we have done in the case of continuous

time systems. In case of continuous time systems also, we talked about linear time invariant causal systems, here also we are going to talk about linear time invariant causal systems.

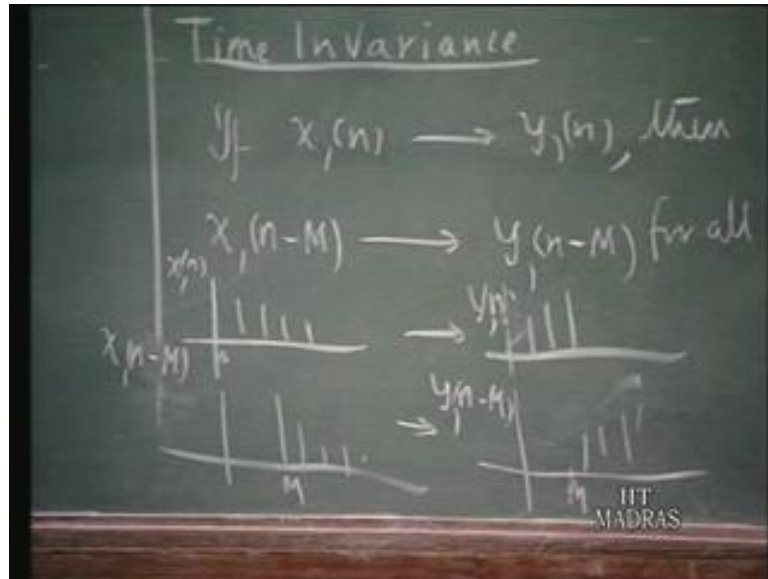
(Refer Slide Time: 14:16)

The image shows a chalkboard with handwritten text. At the top, the word "LINEAR" is written and underlined. Below it, the text reads: "if $x_1(n) \rightarrow y_1(n)$ and $x_2(n) \rightarrow y_2(n)$ then $C_1 x_1(n) + C_2 x_2(n) \rightarrow C_1 y_1(n) + C_2 y_2(n)$ for all constants C_1 & C_2 ". The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

So, before we proceed let us once again review the meanings of these 3 objectives. Linearity what is the implication of linearity as far as the discrete time system is concerned. We will say that, if an input x_1 of n gives rise to an output y_1 of n , both are functions of n . And another input x_2 of n gives rise to y_2 of n then, if the system is linear if $C_1 x_1$ of n a linear combination of these 2 inputs signals will produce an output which is the same form of linear combination of the output quantities for all constants C_1 and C_2 .

So, if you take any 2 constants C_1 and C_2 provided these particular relation is satisfied then we say it is a linear system if not it is a non-linear system. So, much is the same way as we define in the case of continuous system except that in the case of continuous system you write $x_1 t$ produce $y_1 t$ $x_2 t$ $y_2 t$ and $C_1 x_1 t$ plus $C_2 x_2 t$ produces $C_1 y_1 t$ plus $C_2 y_2 t$. Exactly the same except that now, we couch this in expressions which are relevant to discrete time signals.

(Refer Slide Time: 15:45)



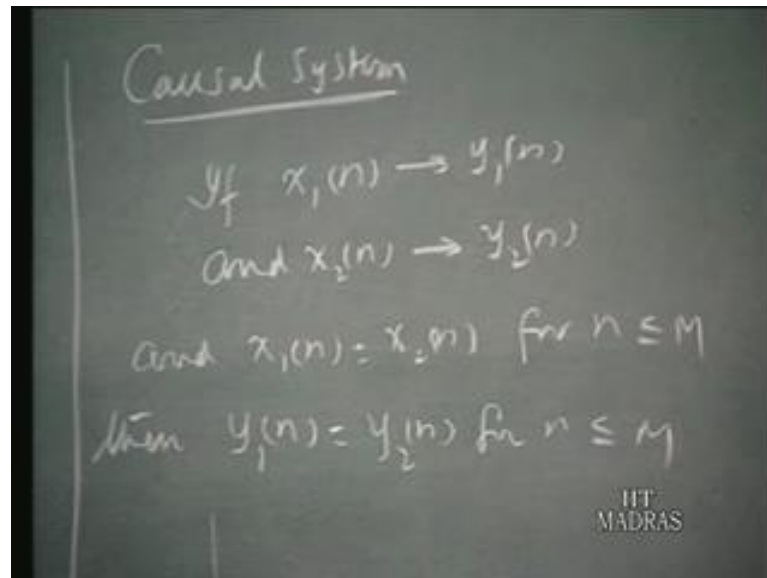
Then, time invariance what is the connotation of time invariance as far as this system is concerned. If $x_1(n)$ produces an output $y_1(n)$ then if the system is kept time invariant, if you delay this $x_1(n)$ or shift it by a certain amount of time in the case of continuous time we said tow. But in this case, we have to delay it by an integral number of sampling in sets therefore, you can say then $x_1(n - M)$, where M is the integer again.

So that means the same signal is delayed M instants of M sampling instants. Then, the output will be a correspondingly delayed output quantity instead of $y_1(n)$ we have $y_1(n - M)$ for all M , for all integral M . It goes without saying that wherever we have an argument here, as far the discrete time systems are concerned the argument of must be an integer. So M goes without saying that M is an integer.

In other words, if I have this is $x_1(n)$ and that produces an output say like that this is $y_1(n)$ then if I delay this by n sampling instants. That is instead of starting from 0 if I start this from M then, the $y_1(n)$ also will correspondingly delayed get delayed by this instant this is M . So, it will be like this so if you delay this input this is $x_1(n)$, this will be $x_1(n - M)$.

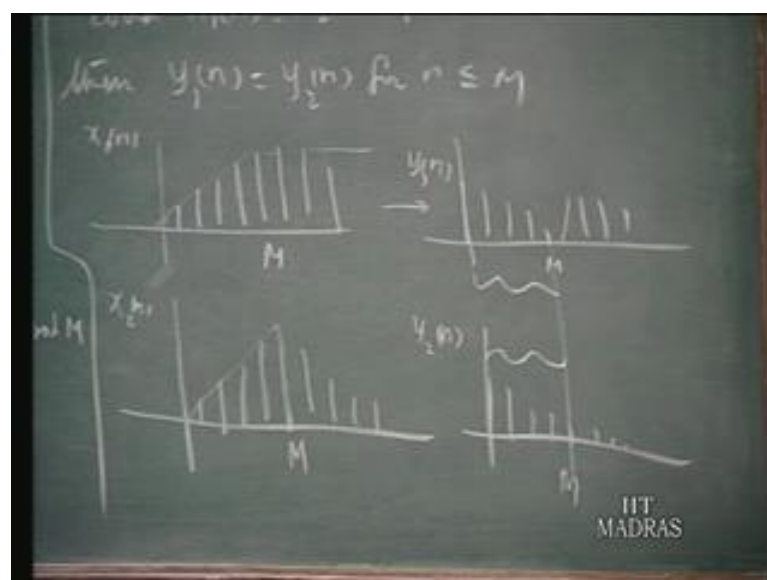
And $y_1(n)$ this is $y_1(n)$ this is $y_1(n - M)$ so this will be the result that means, the same set of samples same sequence of samples will arise but, then it will be delayed it will star from, if this is increasing like this, this will also increase like this; that is the time invariance.

(Refer Slide Time: 18:07)



Thirdly causal system what is meant by causal system? A causal system essentially means if $x_1(n)$ produces $y_1(n)$ and $x_2(n)$ produces $y_2(n)$ furthermore, $x_1(n) = x_2(n)$ for n less than or equal to M . So that means, you have 2 signals which are identical up to M 'th sampling instant then the system is causal we can say that $y_1(n)$ is also equal to $y_2(n)$ for n less than or equal to M .

(Refer Slide Time: 19:02)

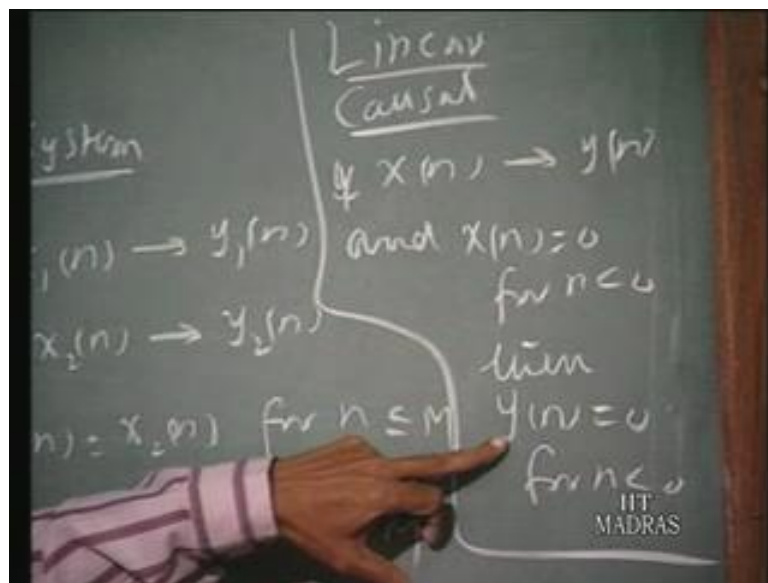


So, what it means, is suppose I have 2 signals: this is say $x_1(n)$ up to M 'th sampling instant this is how it is. Let us, say x_2 which also this have the same behavior up to this

point but, then later it may differ so, this is $x_2[n]$. So, if these 2 are the input signals $x_1[n]$ and $x_2[n]$ are identical up to the M 'th sampling instant then, we can say that whatever output you this produces let us say up to M and then beyond that may be like this.

So, this will also have the same output up to this point and afterwards it may be different so, up to this it is the same. This $y_1[n]$ this is $y_2[n]$ so, these 2 up to that point they are identical they must be identical. This is the meaning of a causal system, for a linear causal system we have a special result.

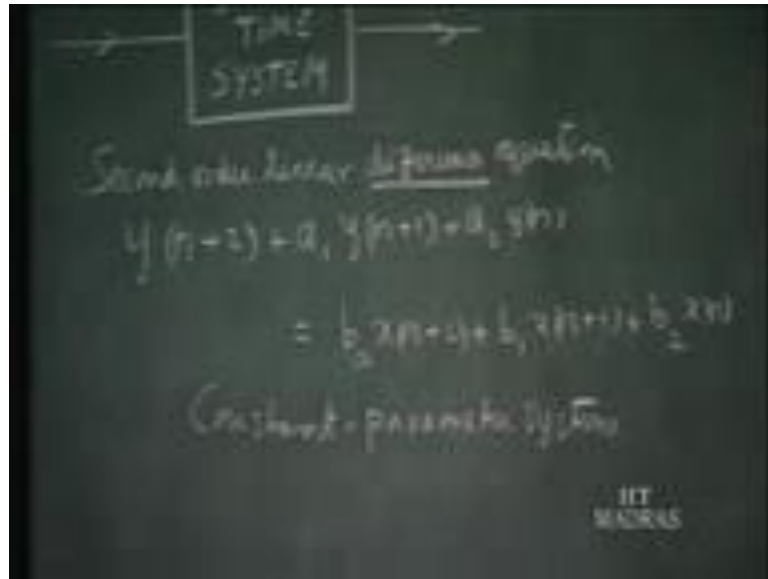
(Refer Slide Time: 20:20)



If linearity is also implied then it means, if $x[n]$ produces $y[n]$ and $x[n]$ is 0 say for n less than 0 then $y[n]$ is also 0 for n less than 0. That is being of causality for the special case of a linear causal system, that is you are applying an input $x[n]$ which is 0 for n less than 0. Then, the output must also be 0 as long as the input is not applied that means, the any output can come after the input is applied not before that.

All this is very similar to what we have discussed in the case of continuous systems. Now, we have put these properties in terms of different notation applicable to discrete time signals. Now, we will work out we will show a few equations how pertaining to discrete time systems and show what are the implications as far as the equations are concerned of the by that virtue of the by the virtue of property of linearity time invariance and causality. Let us, now look at these examples.

(Refer Slide Time: 21:54)



So, this is a representation of discrete time system it takes a sequence of values as input and produce the corresponding output; additional computer for the example can be thought of a discrete time system, a digital filter is 1 such. So, for a linear time invariant and causal discrete time system there will be a linear difference equation connecting the output quantity y of n with the input quantity of x of n .

An example of such a linear difference equation will be something like this y_n plus 2 plus $a_1 y_{n+1}$ plus $a_2 y_n$ equals $b_0 x_{n+2}$ say $b_1 x_{n+1}$ plus $b_2 x_n$. So, the input sequence of values x_n are related to the output sequence in this fashion. This is something like your differential equation except that we do not have derivatives here. But instead of that, the argument is stopped up by 1 unit.

Now, this is said to be a second order difference equation it is said to be the second order because, the difference between the arguments the highest argument or the independent variable which is n plus 2 and the lowest argument is the independent variable is n . The difference between these 2 is 2 therefore, this is said to be a second order linear difference equation.

So, in contrast a differential equation we have what is called a difference equation which governs the operation of a discrete time system. Further the coefficients that are involved a_1 a_2 b_1 b_1 b_0 are constants and that accounts for its time invariant character. So, this is

a constant parameter system, this is the same thing as saying that is the time invariant system. We will see examples, where it fails to be a constant parameter system.

(Refer Slide Time: 24:41)

Second order linear difference equation

$$y(n+2) + a_1 y(n+1) + a_2 y(n) = b_0 x(n+2) + b_1 x(n+1) + b_2 x(n)$$

Constant parameter system

Alternative Form

$$y(n) + a_1 y(n-1) + a_2 y(n-2) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

IIT MADRAS

So, in a linear second order difference equation will be of this type you can also put this alternately by stepping down the argument by 2 units. You can write this as y_n plus $a_1 y_{n-1}$ plus $a_2 y_{n-2}$ equals $b_0 x_n$ plus $b_1 x_{n-1}$ plus $b_2 x_{n-2}$ this is an alternative form. So, the second order difference equation pertaining to a linear time invariant causal system could be either in this form or this form both are equivalent by stepping up this n by 2 you get this, by stepping down by this both are equal.

(Refer Slide Time: 25:26)

General Equation of Nth order

$$y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$$

Recursive Relation.

IIT
MADRAS

In general, we can say y_n is obtained as minus $a_k y_{n-k}$ from $k=1$ to N plus $b_k x_{n-k}$ ranging from $k=0$ to M . So, this is even you can have another value M index also does not matter. So, this is a general equation that means I can write this for example, in this firstly in this particular case y_{n+2} as minus $a_1 y_{n+1}$ minus $a_2 y_n$ and like that.

Similarly, here y_n I can write this as minus $a_1 y_{n-1}$ minus $a_2 y_{n-2}$ plus $b_0 x_n$ plus $b_1 x_{n-1}$ plus $b_2 x_{n-2}$. So, when you shift all except keeping y_n side and take all the other values of the output on the other side you get this, so this is a general equation of n 'th order. So, this is a general form of linear difference equation with constant coefficients with of n 'th order.

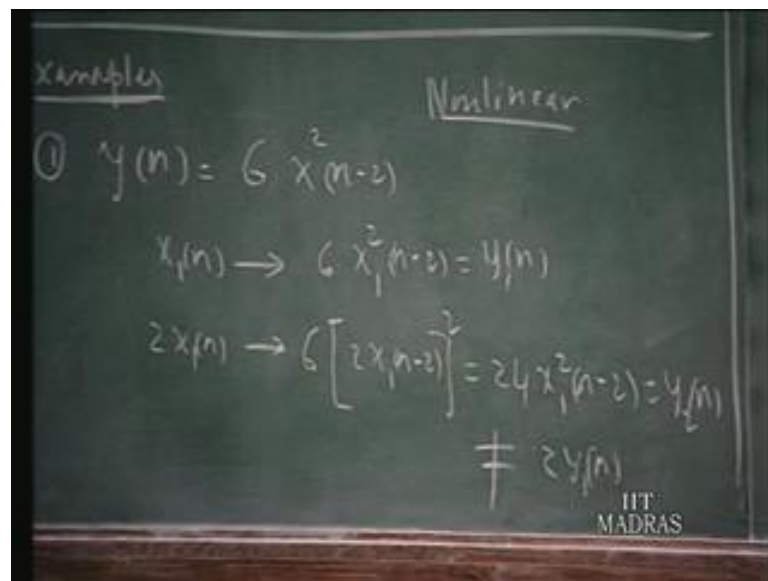
Now, you can see now that this output sample y_n at any point of time can be recursively calculated from knowing the previous output values; previous n output values y_{n-1} to y_{n-2} . If you know all the previous output values up to n sampling instance earlier and here the present input when k is 0 this is $b_0 x_n$ k is 1 $b_1 x_{n-1}$. So that means, any output identify at sampling instant can be computed, from knowing the present input and the past M inputs plus the past n output quantities.

So, this is a kind of recursive relation which enables us to calculate y_n from knowing the past outputs and the present input and the past input. This in this form you can find it if you know the input quantities successively you can calculate the output

quantities. You can out this in the alternative form by trans shifting this on the other side, you get this is in the standard form as shown here.

Now, let us consider some examples of difference equation which may or may not be linear, which may or may not be belong to time invariant discrete systems. But just to bring out the difference between the various types of systems.

(Refer Slide Time: 28:22)

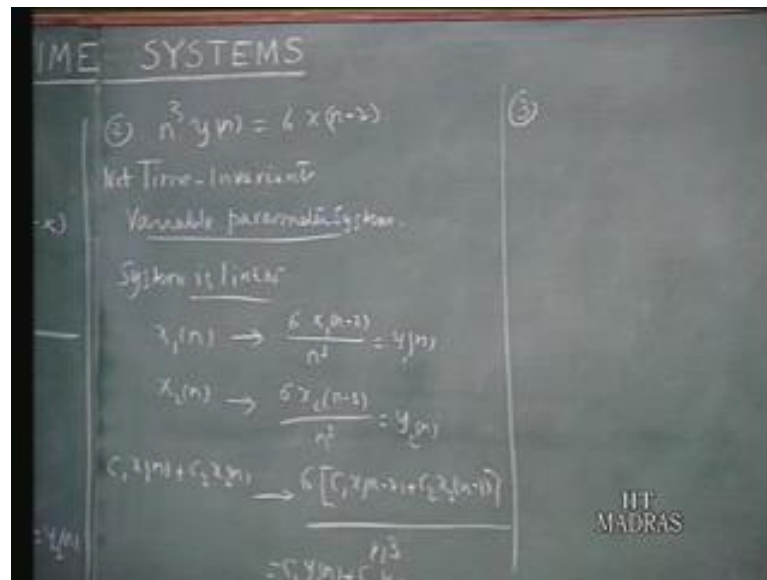


Examples, suppose I have y_n equals $6x^2$ square n minus 2 now, here this does not belong to this standard form where as in this standard form you have y 's and x 's only no squares times are appearing no products; either the input or output quantities. So, if x_n is the input and y_n is the output you have got x^2 here, this holds to be linear this is a non-linear situation. Why it is a non-linear? Suppose I have an x_1 n then it gives rise to an output which is $6x_1^2$ square n minus 2 that is what you get.

Let us, say this is y_1 of n now let me say, if I multiply the input is doubled let us say I have $2x_1$ of n then if the system is linear I must get doubled the output. But what you get here is, since the input $2x_1$ of n the output will be 6 times $2x_1$ n minus 2 whole squared. Because, this x^2 of n minus 2 so, this becomes 24 x_1^2 squared n minus 2 which is not equal to because input is doubled we expect the output also to be doubled, the x this four times actually it is not 2 times it is not equal to 2 this is y_2 of n which is not equal to 2 times y_1 of n ; so, this is non-linear.

So, in a non-linear situation we will not have this linearity this times is a linear system we should all this terms both the independent variable, independent variables and the dependent variables all the terms must be of power 1 square terms poise this.

(Refer Slide Time: 30:26)



Second example we have $n^3 y(n) = 6 x(n-2)$ now, note out $y(n)$ and $x(n)$ they do not have square terms or cube terms so this appears to be linear. But then, the coefficient of $y(n)$ is n^3 instead of a constant therefore, we expect that this is not a time invariant system. So, this is variable parameter system because the coefficient of $y(n)$ is not a constant it is a n^3 this is a variable parameter so time variable not time invariant. Let us, say this is a variable parameter system.

However, the system is linear you recall that in the case of continuous system we say continuous time systems. We said as long as the coefficients of the independent variable or the dependent variable are functions of t then, still it is a linear system; here also, it is a linear system.

Example, suppose I take suppose $x_1(n)$ gives rise to an output which will be after all from this we know the output will be the 6 of $x_1(n-2)$ divided by n^3 , that is your output $y_1(n)$. So, n^3 times $y_1(n)$ of n should be 6 times $x_1(n-2)$ therefore, if $x_1(n)$ is the input the output will be $6 x_1(n-2)$ by n^3 . Suppose, our $x_2(n)$ has the new input you will get naturally $6 x_2(n-2)$ divided by n^3 that will be $y_2(n)$.

Then, it can easily be shown that if I have linear combination $C_1 x_1 + C_2 x_2$ of x_1 and x_2 of n , where C_1 and C_2 are constants then the output will be $C_1 y_1 + C_2 y_2$ of n , which is indeed can be shown to be $C_1 y_1 + C_2 y_2$ of n . So, this satisfies the requirement of a linearity therefore, this system is linear but, it is a variable parameter system.

(Refer Slide Time: 33:15)

③ $y(n) = 3n x(n)$
 Time Variable
 $x_1(n) \rightarrow 3n x_1(n) = y_1(n)$
 $x_2(n) \rightarrow 3n x_2(n)$
 $= x_1(n-M) \quad = 3n x_1(n-M)$
 $= y_2(n)$
 $y_1(n-M) = 3(n-M) x_1(n-M)$
 $\neq y_2(n)$
 IIT MADRAS

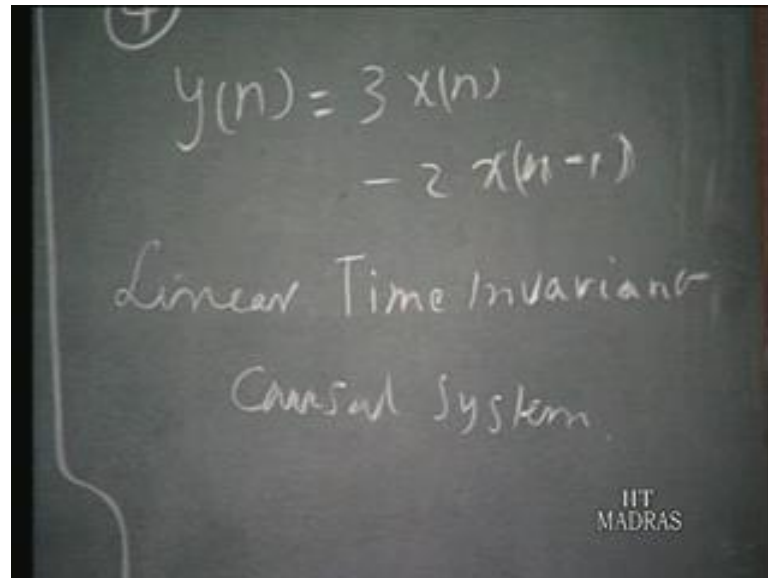
So, variable parameter system obviously will not satisfy the property that if you should be input by a certain amount the output will not also be shifted by a corresponding amount. To illustrate this let us consider y_n is $3n$ times x_n this is again time variable system because, the coefficient is a function of n . Let us see, suppose I have x_1 of n and it give rise to an output which is $3n$ times x_1 of n this is your output y_1 of n .

Now, suppose I take a x_2 of n a new input which is old input delayed by M sampling instants x_2 of n which is same x_1 of n delayed by M sampling instants. So, this all obviously produces an output which is $3n$ times x_2 of n which is $3n$ times x_1 of n minus M that is what you are having, and this is your y_2 of n , this is your y_1 of n .

Now, y_1 of n minus M if you want to see what y_1 of n minus M is this is y of n of M $3n$ times x_1 of n . So, y_1 of n minus M will be 3 times n minus M times x_1 of n minus M and that is different from this, this is not equal to y_2 of n . So, the conclusion is that whenever you have the coefficients of x and y terms as functions of n then it is a linear system, time variable system.

But it does not satisfy the property that we are looking for that when you delay the input by a certain amount the output will also be delayed by the same amount, that property is no longer valid.

(Refer Slide Time: 35:24)


$$y(n) = 3x(n) - 2x(n-1)$$

Linear Time Invariant
Causal System

IIT
MADRAS

Fourth example, suppose I have y_n equals $3x_n$ minus 2 into x_n minus 1 so, this is obviously satisfies all our requirement, it is all the parameters are the coefficients are constants. And this form pertains to the form that you have been talking about, this is a linear time invariant and it will also be a causal system the requirement of a causal system we will talk about in a moment. So, we can take as this is a causal system so, this is the type of system that we are going to talk about and there is no difficulty as far as this is concerned. Now, let us take 1 more example where the system fails to be causal.

(Refer Slide Time: 36:33)

(5) $y(n) = 3x(n-1) + x(n+1)$
Non-Causal System
 $x_1(n) = x_2(n)$ for $n \leq M$
 $x_1(n) \neq x_2(n)$ for $n > M$
IIT
MADRAS

5 suppose, I take y_n as $3x_n$ minus 1 plus x_{n+1} . Now you see, an input output at the n 'th sampling instant is dependent on an input that has occurred 1 sampling instant earlier plus an input that is going to occur later $n+1$. So, the present output depends upon a future input right. So, if there is an input here at that has influenced the output even before the input has occurred therefore, this certainly is a non-causal system.

So, how do you recognize a non-causal system? The highest index that you have got associated with the dependent variable that should not be exceeded by the variable whether the index is the argument or the independent variable. Like you see here in the recursive relation, in this set of terms the highest argument as far as the dependent variable is concerned is n because all others are $n-1$ $n-2$.

The highest variable that you have, for the highest order or for the value of the independent variable is also n k equal to 0 n . So, that is why this is a causal system, if I had if you started k equals minus 1 then it becomes $n+1$ then that fails to be causal. So, since we are talking about the output it depends on the present input and the past inputs, not on the future input. Here we have a case where the output depends upon the future input therefore, it fails to be causal.

To show this more explicitly, suppose I take I have 2 inputs $x_1(n)$ and $x_2(n)$ both are equal for n less than or equal to M and $x_1(n)$ is not equal to $x_2(n)$ for n greater than M suppose I

have 2 inputs. And let us say, let us find out the outputs of these with respect to with the inputs x_1 and x_2 at the M 'th sampling instant.

Therefore, y_1 I want y_1 is the output corresponding to x_1 I want to find this output at the M 'th sampling instant y_1 of M with this input x_1 of M x_1 of M .

(Refer Slide Time: 39:17)

$$y_1(M) = 3x_1(M-1) + x_1(M+1)$$

$$y_2(M) = 3x_2(M-1) + x_2(M+1)$$

$$y_1(M) \neq y_2(M)$$

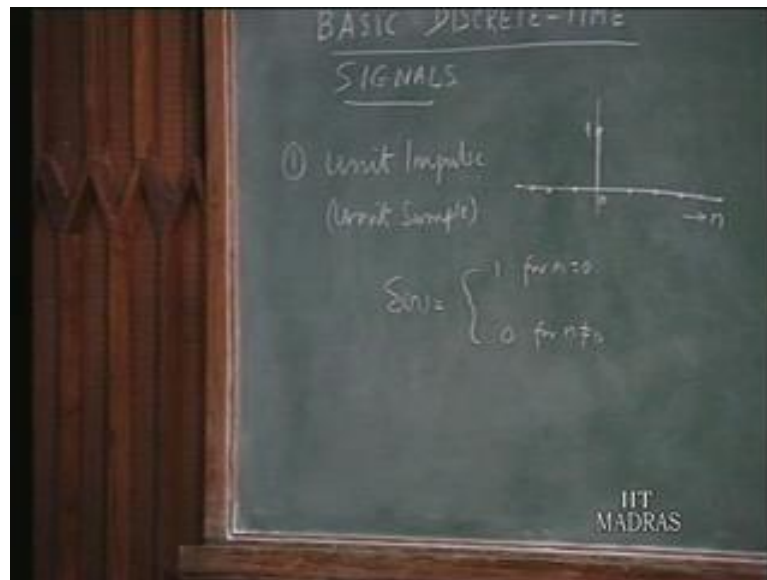
Therefore, according to this y_1 of M will be 3 times x_1 of M minus 1 plus x_1 of M plus 1, y_2 M the output corresponding to the input x_2 n would be 3 times x_2 M minus 1 plus x_2 M plus 1. So, we have 2 inputs which are identical up to the M 'th sampling instant, we would expect the outputs also to be identical up to the M 'th sampling instant M . But when you compare y_1 of M and y_2 M are these equal, if you look at these this is equal to this because, x_1 equal to x_n up to n equals M .

But, x_1 n plus M there is no necessity for this to be equal to M plus 1 because, these 2 are equal only up to n equals M , these 2 may not be equal. So, y_1 of M there is no guarantee is not equal to y_1 of M is not equal to y_2 of M therefore, even though the input side remain the same up to the M 'th sampling instant, the outputs are not equal.

Therefore, this is not a causal system in the final conclusion from this is, the argument the highest argument associated with the dependent variable whatever it is the highest argument exceed associated with it the independent variable should not exceed that. Here n plus 1 is exceeding n therefore, this is not a causal system.

So, after having considered the various types of systems and the pertaining equations we would now have a look at the standard signals, discrete time signals which will make use of in considering the performance of discrete time systems.

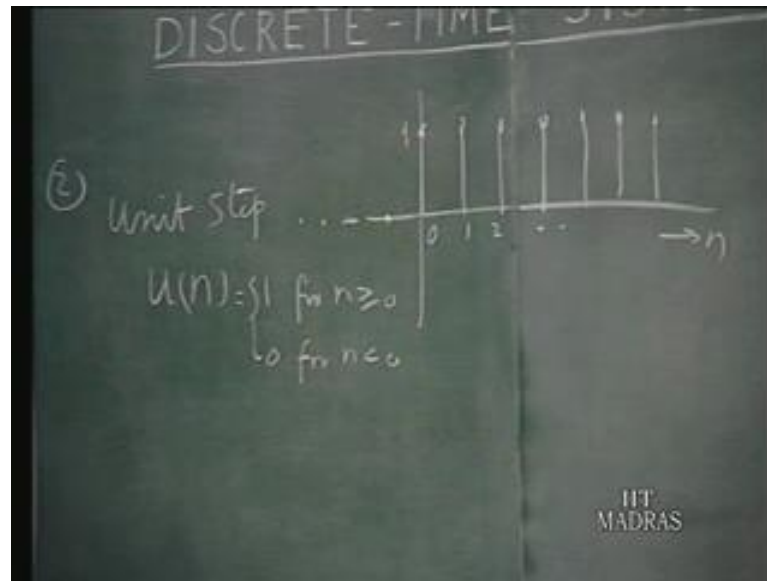
(Refer slide Time: 41:07)



Now, let us look at the basic discrete time signals the first signal that we will talk about is the unit impulse, it is also called unit signal or the unit sample. Basically so, it is 0 everywhere except when n equal 0. So, this is this is called delta n that is symbol that is used for this its equal to 1 for n equals 0 equals 0 for n not equal to 0. So, this is just a single sample occurring at n equals 0; this is called a unit impulse simply called the unit sample or a unit signal.

It is the term impulse is used borrowing the terminology from the continuous time signal, it does not mean the value of this at n equals 0 is infinity its finite it is equal to 1. So, impulse perhaps is a kind of misnomer but, still we will call it unit impulse because, it corresponds to what we have as the unit impulse in the continuous time system. It is symbolized in delta M 1 for n equals 0 for n not equal to 0.

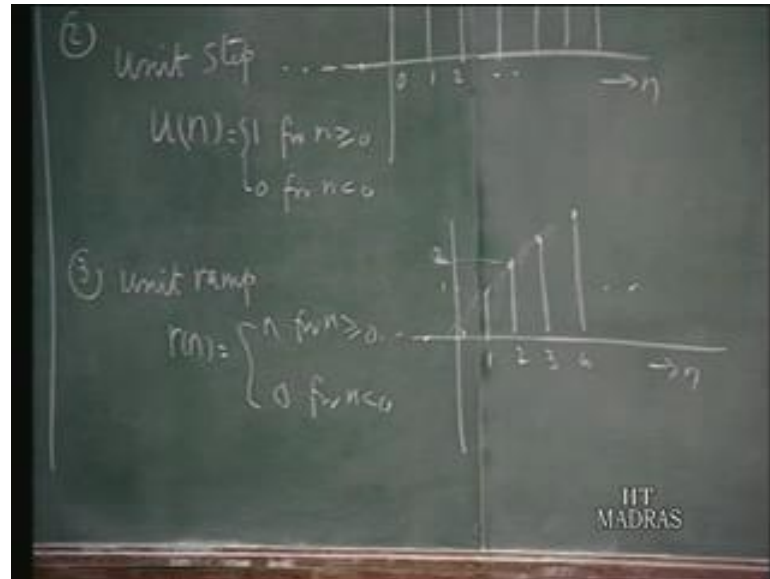
(Refer Slide Time: 42:29)



Now, just like a unit step we also here are the continuous case, here also we have a unit step symbolized as $u(t)$. Now here, the value of this discrete time signal is 1 by through for all non-negative n 0 1 2 etcetera and 0 for negative values of n like that. So, $u(n)$ can be written as 1 for n greater than or equal to 0 0 for n less than 0. So, in contrast with the continuous time case here at n equals 0 we have a definite value which is equal to 1, in the case of continuous time case at n equal at t equals 0 as unit step is concerned.

So, we can take it as 0 or 1 or half or whatever it is it can be left ambiguous. But as far as the periods step in a discrete time case is concerned the value is definite and it is equal to 1 so this value is equal to 1, here also this is equal to 1.

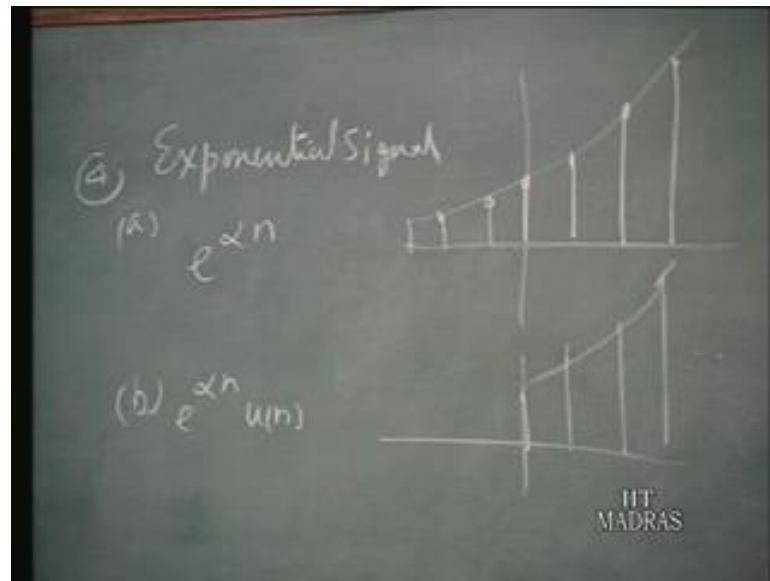
(Refer Slide Time: 43:53)



Now, similarly, you can have a unit ramp it is symbolized as $r(n)$ equals n for n greater than or equal to 0 for negative values of n . So, that would be 0 everywhere but, here it will be 0 like this 1 2 3 4 etcetera up to n . So, at 1 it is equal to 1, at 2 it is equal to 2 and so on and so forth.

So, that is what is called a unit ramp which is somewhat similar to what we have in the case of continuous time system, except in the continuous time system as a function of t it takes off in a way like this. But here, the values of the samples increase proportion in proportion to M .

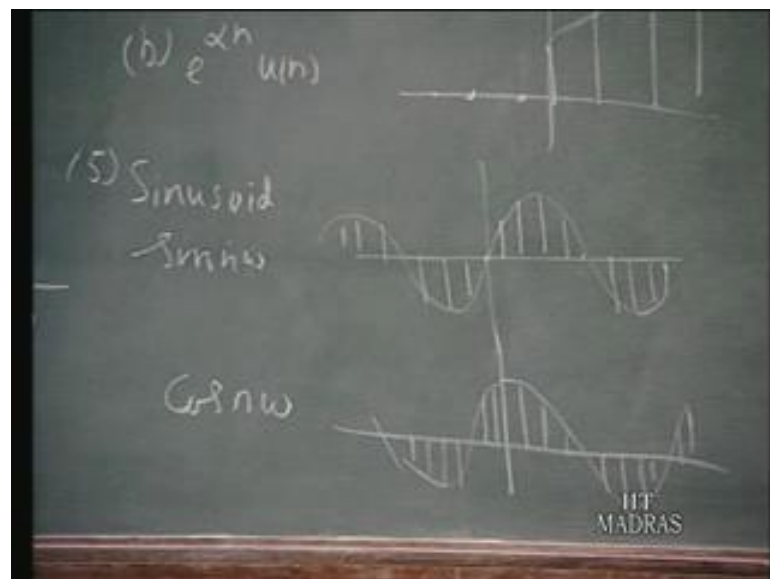
(Refer Slide Time: 44:50)



We can also have an exponential signal e to the power of αn so, you have something like exponentially increasing like this like this e to the power of αn . Now on the other hand, if I have e to the power of αn times u of n that means this is multiplied by a unit step that means, all this samples are cut off and you have only this portion that is obvious.

So, you multiply e to power αn by u_n that means, the negative samples for negative values of n are just removed they will be equal to 0 of course.

(Refer Slide Time: 45:37)

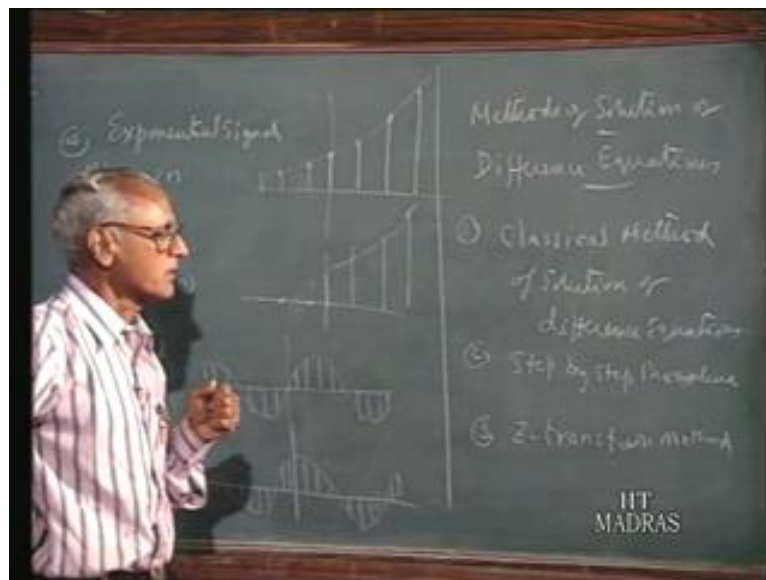


Similarly, you can have a sinusoid. You can have a sine ωn or $n \omega$ which will be in a general case this is the envelope the samples will be like this, all the samples will fall on a envelope which is sine wave. Similarly, if you have $\cos n \omega$ it will be a similar situation but, this will be now a cosine curve.

So, the samples will be again discrete but, there the tips of all samples will fall will follow a sine wave or cosine wave as the case may be sine wave $n \omega$ or ωn you can write may be better call it $n \omega \sin n \omega$ and $\cos n \omega$ or sinusoidal waves. Again, if you take sine $n \omega$ multiplied by u of n this portion be cut off $\cos n \omega$ multiplied by u of n this portion will be cut off.

Now, you see these are the basic type of discrete time signals that we would like to use from this we can build up other signals. For example, e to the power of αn sine $n \omega$ type of signals multiplication of these 2 all those things can be produced. Now, after all we considered the difference equation we would like to find out what are the methods of solution for the difference equations.

(Refer Slide Time: 47:23)



So, methods of solution in time domain methods of solution of difference equations in time domain we will can have we will talk about, first of all classical method of solution of difference equations. Secondly, we can have very elementary procedure step by step procedure using a recursive relation step by step procedure. The third method would be, z transform method.

These are the 3 methods that we are going to discuss, classical differential difference equation solution we have a technique which is corresponding to the solution of a differential equation that you are familiar with. The second method is step by step numerical calculation is convenient because, you have a recursive relation from the values which are known at the present instant of time you can calculate the value of the dependent variable at the next sampling instant and so on which is step by step.

The z transform method corresponds to your Laplace transform technique and we will spend a lot of time on this. As far as the first 2 methods are concerned we will briefly discuss these methods in the next lecture, how to go about solving the difference equation we will consider a very cursory treatment. We will have as far as these 2 signals first 1 is concerned, we will illustrate this step by step procedure nothing much to be said about it further and later on we will talk about this z transform method.

So that, the first 2 methods we will take up in the next lecture. So, in this lecture we have introduced our self to the concept of discrete time signals. We mention discrete time signals are those, which are defined for discrete values of the independent variable usually it is time that is why that is why it is called discrete time signal. It could be any other independent variable like the space coordinate the values of for example, heights at different point along a line.

Now, we indicated a discrete time signal as f as a function of the independent variable n f_n x_n y_n as the case may be. We can also write y subscript n if you like that a that is also an alternative way of doing that and we also, discuss the difference between digital signal and discrete time signal. A digital signal is the 1 which is discrete time signal also, but the amplitudes the values of signal is also quantized is allowed to take only discrete well defined values not any value in between.

Then, we talked about a discrete time system which is the 1 which processes the discrete time signals. We talked about, the linearity property, the time invariance property and the causality property and we will confine our discussion to linear time invariant causal discrete time systems. These are characterized by a difference equation for the n in general, where the order of the difference equation is the difference in the arguments corresponding in the highest argument and the lowest argument associated with the dependent variable.

So we worked out some examples, where these attributes of linearity, causality and time invariance fail and we should we can from look looking at the equation should be able to tell whether these attributes hold or not. That we worked out some examples to illustrate this then finally, we said the solution for this we looked it the basic forms of discrete time signals we define the impulse function, the unit step function, and we saw how a discrete time sinusoid and how discrete time exponential functions look like.

Then, we said the methods of solution of difference equations pertaining to this systems. We will talk about, 3 methods 1 is the approach of the classical difference equation solutions then, second is step by step method and the third is z transform method. We will discuss the first 2 methods in the next lecture and later on we will take up the z transform approach the solution of difference equations.